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MINIMUM DOMINATING DISTANCE ENERGY OF A GRAPH

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Abstract. Recently we introduced the concept of minimum dominating energy[19]. Motivated by this paper, we introduced the concept of minimum dominating distance energy $E_{Dd}(G)$ of a graph G and computed minimum dominating distance energies of a star graph, complete graph, crown graph and cocktail party graphs. Upper and lower bounds for $E_{Dd}(G)$ are also established.

Key words and Phrases: Minimum dominating set, dominating distance matrix, dominating distance eigenvalues, dominating distance energy.

Abstrak. Pada paper sebelumnya pada tahun 2013, kami telah memperkenalkan konsep energi pendominasi minimum. Kami melanjutkan konsep tersebut dengan memperkenalkan konsep energi jarak pendominasi minimum $E_{Dd}(G)$ dari suatu graf G dan menghitung energi jarak pendominasi minimum dari graf bintang, graf lengkap, graf mahkota, dan graf cocktail party. Kami juga mendapatkan batas atas dan bawah untuk $E_{Dd}(G)$.

Kata kunci: Himpunan pendominasi minimum, matriks jarak pendominasi, nilai eigen jarak pendominasi, energi jarak pendominasi.

1. Introduction

The concept of energy of a graph was introduced by I. Gutman [9] in the year 1978. Let G be a graph with n vertices $\{v_1, v_2, ..., v_n\}$ and m edges. Let $A = (a_{ij})$ be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of A, assumed in non increasing order, are the eigenvalues of the graph G. As A is

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real symmetric, the eigenvalues of G are real with sum equal to zero. The energy $E(G)$ of G is defined to be the sum of the absolute values of the eigenvalues of G. i.e.,

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

For details on the mathematical aspects of the theory of graph energy see the reviews[10], paper [11] and the references cited there in. The basic properties including various upper and lower bounds for energy of a graph have been established in [16], and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [5, 6, 7, 12].

Further, studies on maximum degree energy, minimum dominating energy, Laplacian minimum dominating energy, minimum covering distance energies can be found in [18, 19, 20, 21] and the references cited there in.

The distance matrix of G is the square matrix of order n whose (i, j) - entry is the distance (= length of the shortest path) between the vertices v_i and v_j. Let $\rho_1, \rho_2, ..., \rho_n$ be the eigenvalues of the distance matrix of G. The distance energy DE is defined by

$$DE = DE(G) := \sum_{i=1}^{n} |\rho_i|.$$

Detailed studies on distance energy can be found in [3, 4, 8, 13, 14, 22].

2. The Minimum Dominating Distance Energy

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E. A subset D of V is called a dominating set of G if every vertex of V-D is adjacent to some vertex in D. Any dominating set with minimum cardinality is called a minimum dominating set. Let D be a minimum dominating set of a graph G. The minimum dominating distance matrix of G is the $n \times n$ matrix defined by $A_{Dd}(G) := (d_{ij})$, where

$$d_{ij} = \begin{cases} 1 & \text{if } i = j \text{ and } v_i \in D \\ d(v_i, v_j) & \text{otherwise} \end{cases}$$

The characteristic polynomial of$A_{Dd}(G)$ is denoted by $f_n(G, \rho)= det(\rho I - A_{Dd}(G))$. The minimum dominating eigenvalues of the graph G are the eigenvalues of $A_{Dd}(G)$. Since $A_{Dd}(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\rho_1 \geqslant \rho_2 \geqslant \cdots \geqslant \rho_n$. The minimum dominating energy of G is defined as

$$E_{Dd}(G) : = \sum_{i=1}^{n} |\rho_i|$$

Note that the trace of $A_{Dd}(G)$ = Domination Number = k.

Example 1. The possible minimum dominating sets for the following graph G in Figure 1 are i) D_1=$\{v_1, v_5\}$, ii) D_2=$\{v_2, v_5\}$, iii) D_3=$\{v_2, v_6\}$

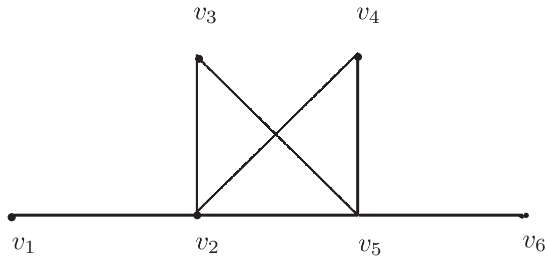

Figure 1

i) $A_{Dd_1}(G) = \begin{pmatrix} 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 0 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 2 & 2 & 1 & 0 \end{pmatrix}$

Characteristic equation is $\rho^6 - 2\rho^5 - 43\rho^4 - 114\rho^3 - 94\rho^2 - 8\rho + 8 = 0$. Minimum dominating distance eigenvalues are $\rho_1 \approx -3.0257, \rho_2 \approx -2, \rho_3 \approx -1.3386, \rho_4 \approx -0.5067, \rho_5 \approx 0.2255, \rho_6 \approx 8.6456$. Minimum dominating distance energy, $E_{Dd_1}(G) \approx 15.7420$

ii) $A_{Dd_2}(G) = \begin{pmatrix} 0 & 1 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 2 & 2 & 1 & 0 \end{pmatrix}$

Characteristic equation is $\rho^6 - 2\rho^5 - 43\rho^4 - 100\rho^3 - 41\rho^2 + 36\rho - 4 = 0$. Minimum dominating distance eigen values are $\rho_1 \approx -3.3028, \rho_2 \approx -2, \rho_3 \approx -1.6445, \rho_4 \approx 0.1431, \rho_5 \approx 0.3028, \rho_6 \approx 8.5015$. Minimum dominating distance energy, $E_{Dd_2}(G) \approx 15.8946$. Therefore, minimum dominating distance energy depends on the dominating set.

3. Minimum Dominating Distance Energy of Some Standard Graphs

Definition 3.1. *The cocktail party graph, is denoted by $K_{n\times 2}$, is a graph having the vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_i\}$ and the edge set $E = \{u_iu_j, v_iv_j : i \neq j\}\bigcup\{u_iv_j, v_iu_j : 1 \leq i < j \leq n\}$.*

Theorem 3.2. *The minimum dominating distance energy of cocktail party graph $K_{n\times 2}$ is $4n$.*

Proof. Let $K_{n\times 2}$ be the cocktail party graph with vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_i\}$. The minimum dominating set of $K_{n\times 2}$ is $D = \{u_1, v_1\}$. Then

$$A_{Dd}(K_{n\times 2}) = \begin{pmatrix} 1 & 2 & 1 & 1 & \dots & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & \dots & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & \dots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & \dots & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & \dots & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 0 \end{pmatrix}$$

Characteristic equation is $\rho^{n-2}(\rho+1)(\rho+2)^{(n-1)}[\rho^2 - (2n+1)\rho + (2n-2)] = 0$
Minimum dominating distance eigenvalues are $\rho = 0$ [$(n-2)$times], $\rho = -1$ [one time], $\rho = -2$ [$(n-1)$ times], $\rho = \dfrac{(2n+1) \pm \sqrt{4n^2 - 4n + 9}}{2}$ [one time each]. So, minimum dominating distance energy is $E_{Dd}(K_{n\times 2}) = 4n$ □

Theorem 3.3. *For any integer $n \geq 3$, the minimum dominating distance energy of star graph $K_{1,n-1}$ is equal to $4n - 7$.*

Proof. Consider the star graph $K_{1,n-1}$ with vertex set $V = \{v_0, v_1, v_2, ..., v_{n-1}\}$, where $deg(v_0) = n - 1$. Minimum dominating set $D = \{v_0\}$. Then

$$A_{Dd}(K_{1,n-1}) = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 0 & 2 & \dots & 2 \\ 1 & 2 & 0 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 2 & \dots & 0 \end{pmatrix}_{n\times n}$$

Characteristic equation is $(\rho + 2)^{n-2}(\rho^2 - (2n-3)\rho + (n-3)) = 0$

The minimum dominating distance eigenvalues are $\rho = -2$ [(n-2) times], $\rho = \dfrac{(2n-3) \pm \sqrt{4n^2-16n+21}}{2}$ [one time each]. So, minimum dominating distance energy is $E_{Dd}(K_{1,n-1}) = 4n-7$. □

Definition 3.4. *The crown graph S_n^0 for an integer $n \geq 2$ is the graph with vertex set $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ and edge set $\{u_iv_j : 1 \leq i, j \leq n, i \neq j\}$. Hence S_n^0 coincides with the complete bipartite graph $K_{n,n}$ with horizontal edges removed.*

Theorem 3.5. *For any integer $n \geq 2$, the minimum dominating distance energy of the crown graph S_n^0 is equal to*

$$7(n-1) + \sqrt{n^2-2n+5}.$$

Proof. For the crown graph S_n^0 with vertex set $V = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$, minimum dominating set is $D = \{u_1, v_1\}$. Then

$$A_{Dd}(S_n^0) = \begin{pmatrix} 1 & 2 & 2 & \ldots & 2 & 3 & 1 & 1 & \ldots & 1 \\ 2 & 0 & 2 & \ldots & 2 & 1 & 3 & 1 & \ldots & 1 \\ 2 & 2 & 0 & \ldots & 2 & 1 & 1 & 3 & \ldots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \ldots & 0 & 1 & 1 & 1 & \ldots & 3 \\ 3 & 1 & 1 & \ldots & 1 & 1 & 2 & 2 & \ldots & 2 \\ 1 & 3 & 1 & \ldots & 1 & 2 & 0 & 2 & \ldots & 2 \\ 1 & 1 & 3 & \ldots & 1 & 2 & 2 & 0 & \ldots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \ldots & 3 & 2 & 2 & 2 & \ldots & 0 \end{pmatrix}_{(2n \times 2n)}$$

Characteristic equation is

$$\rho^{n-2}(\rho+4)^{n-2}[(\rho^2 + (7-n)\rho + (11-3n)][\rho^2 - (3n+1)\rho + (3n-3)] = 0$$

Minimum dominating distance eigenvalues are $\rho = 0[(n-2)\text{times}]$, $\rho = -4$ $[(n-2)\text{times}]$ $\rho = \dfrac{(n-7) \pm \sqrt{n^2-2n+5}}{2}$, [one time each], $\rho = \dfrac{(3n+1) \pm \sqrt{9n^2-6n+13}}{2}$ [one time each]. So, minimum dominating distance energy is

$$E_{Dd}(S_n^0) = 7(n-1) + \sqrt{n^2-2n+5}.$$

□

Theorem 3.6. *For any integer $n \geq 2$, the minimum dominating distance energy of complete graph K_n is $(n-2) + \sqrt{n^2-2n+5}$.*

Proof. For complete graphs the minimum dominating distance matrix is same as minimum dominating matrix [19], therefore the minimum dominating distance energy is equal to minimum dominating energy.

□

4. Properties of Minimum Dominating Eigenvalues

Theorem 4.1. *Let G be a simple graph with vertex set $V = \{v_1, v_2, ..., v_n\}$, edge set E and $D = \{u_1, u_2, ..., u_k\}$ be a minimum dominating set. If $\rho_1, \rho_2, ..., \rho_n$ are the eigenvalues of minimum dominating distance matrix $A_{Dd}(G)$ then*

(i) $\sum_{i=1}^{n} \rho_i = |D|$

(ii) $\sum_{i=1}^{n} \rho_i^2 = 2m + 2M + |D|$ *where* $M = \sum_{i<j,\ d(v_i,v_j)\neq 1} d(v_i, v_j)^2$ *and* $m = |E|$.

Proof. **i)** We know that the sum of the eigenvalues of $A_{Dd}(G)$ is the trace of $A_{Dd}(G)$. Therefore,

$$\sum_{i=1}^{n} \rho_i = \sum_{i=1}^{n} d_{ii} = |D| = k.$$

(ii) Similarly, the sum of squares of the eigenvalues of $A_{Dd}(G)$ is trace of $[A_{Dd}(G)]^2$ Therefore,

$$\begin{aligned}
\sum_{i=1}^{n} \rho_i^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} d_{ji} \\
&= \sum_{i=1}^{n} (d_{ii})^2 + \sum_{i\neq j} d_{ij} d_{ji} \\
&= \sum_{i=1}^{n} (d_{ii})^2 + 2\sum_{i<j} (d_{ij})^2 \\
&= \mid D \mid + 2\sum_{i<j} d(v_i, v_j)^2 \\
&= k + 2m + 2M \quad \text{where } M = \sum_{i<j,\ d(v_i,v_j)\neq 1} d(v_i, v_j)^2
\end{aligned}$$

.

□

Corollary 4.2. *Let G be a (n,m) simple graph with diameter 2 and $D = \{u_1, u_2, ..., u_k\}$ be a minimum dominating set. If $\rho_1, \rho_2, ..., \rho_n$ are the eigenvalues of minimum dominating distance matrix $A_{Dd}(G)$ then*

$$\sum_{i=1}^{n} \rho_i^2 = k + 2(2n^2 - 2n - 3m).$$

Proof. We know that in $A_{Dd}(G)$ there are $2m$ elements with 1 and $n(n-1) - 2m$ elements with 2 and hence corollary follows from the above theorem. □

5. Bounds for Minimum Dominating Energy

Similar to McClelland's [17] bounds for energy of a graph, bounds for $E_{Dd}(G)$ are given in the following theorem.

Theorem 5.1. *Let G be a simple (n,m) graph. If D is the minimum dominating set and $P = |detA_{Dd}(G)|$ then*

$$\sqrt{(2m+2M+k)+n(n-1)P^{\frac{2}{n}}} \leq E_{Dd}(G) \leq \sqrt{n(2m+2M+k)}$$

where k is a domination number.

Proof.

$$\text{Cauchy Schwarz inequality is } \Big(\sum_{i=1}^{n} a_i b_i\Big)^2 \leq \Big(\sum_{i=1}^{n} a_i^2\Big)\Big(\sum_{i=1}^{n} b_i^2\Big)$$

$$\text{If } a_i = 1, b_i = \mid \rho_i \mid \text{ then } \Big(\sum_{i=1}^{n} \mid \rho_i \mid\Big)^2 \leq \Big(\sum_{i=1}^{n} 1\Big)\Big(\sum_{i=1}^{n} \rho_i^2\Big)$$

$$[E_{Dd}(G)]^2 \leq n(2m+2M+k) \qquad \text{[Theorem 4.1]}$$

$$\Longrightarrow E_{Dd}(G) \leq \sqrt{n(2m+2M+k)}$$

Since arithmetic mean is not smaller than geometric mean we have

$$\begin{aligned}\frac{1}{n(n-1)}\sum_{i\neq j} \mid \rho_i \mid\mid \rho_j \mid &\geq \Big[\prod_{i\neq j} \mid \rho_i \mid\mid \rho_j \mid\Big]^{\frac{1}{n(n-1)}} \\ &= \Big[\prod_{i=1}^{n} \mid \rho_i \mid^{2(n-1)}\Big]^{\frac{1}{n(n-1)}} \\ &= \Big[\prod_{i=1}^{n} \mid \rho_i \mid\Big]^{\frac{2}{n}} \\ &= \Big|\prod_{i=1}^{n} \rho_i\Big|^{\frac{2}{n}} \\ &= |detA_{Dd}(G)|^{\frac{2}{n}} = P^{\frac{2}{n}}\end{aligned}$$

$$\therefore \sum_{i\neq j} \mid \rho_i \mid\mid \rho_j \mid \geq n(n-1)P^{\frac{2}{n}} \qquad (1)$$

$$\text{Now consider,} \quad [E_{Dd}(G)]^2 = \Big(\sum_{i=1}^{n} \mid \rho_i \mid\Big)^2$$
$$= \sum_{i=1}^{n} \mid \rho_i \mid^2 + \sum_{i \neq j} \mid \rho_i \mid\mid \rho_j \mid$$
$$\therefore \quad [E_{Dd}(G)]^2 \geq (k + 2m + 2M) + n(n-1)P^{\frac{2}{n}} \qquad \text{[From (1)]}$$
$$i.e., \; E_{Dd}(G) \geq \sqrt{(k + 2m + 2M) + n(n-1)P^{\frac{2}{n}}}$$

□

Theorem 5.2. *If $\rho_1(G)$ is the largest minimum dominating distance eigenvalue of $A_{Dd}(G)$, then*

$$\rho_1(G) \geq \frac{2W(G) + k}{n}$$

where k is the domination number and W(G) is the Wiener index of G.

Proof. Let X be any nonzero vector. Then by [1], We have

$$\rho_1(A_{Dd}) = \max_{X \neq 0} \Big\{ \frac{X' A_{Dd} X}{X'X} \Big\}.$$

Therefore,

$$\rho_1(A_{Dd}) \geq \frac{J' A_{Dd} J}{J'J} = \frac{2\sum_{i<j} d(v_i, v_j) + k}{n} = \frac{2W(G) + k}{n}$$

where J is a unit matrix. □

Lemma 5.3. *Let G be a graph of diameter 2 and $\rho_1(G)$ is the largest minimum dominating distance eigenvalue of $A_{Dd}(G)$, then*

$$\rho_1(G) \geq \frac{2n^2 - 2m - 2n + k}{n}$$

where k is the domination number .

Proof. Let G be a connected graph of diameter 2 and d_i denotes the degree of vertex v_i. Clearly i-th row of A_{dd} consists of d_i one's and $n - d_i - 1$ two's. By using Raleigh's principle, for $J = [1, 1, 1, \cdots, 1]$ we have

$$\rho_1(A_{Dd}) \geq \frac{J' A_{Dd} J}{J'J} = \frac{\sum_{i=1}^{n} [d_i \times 1 + (n - d_i - 1)2] + k}{n} = \frac{2n^2 - 2m - 2n + k}{n}. \qquad □$$

Similar to Koolen and Moulton's [15] upper bound for energy of a graph, upper bound for $E_{Dd}(G)$ is given in the following theorem.

Theorem 5.4. *If G is a (m,n) graph with diameter 2 and* $\dfrac{k+2n^2-2n-2m}{n} \geq 1$ *then*

$$E_{Dd}(G) \leq \frac{k+2n^2-2n-2m}{n} + \sqrt{(n-1)\Big[k+4n^2-4n-6m-\Big(\frac{k+2n^2-2n-2m}{n}\Big)^2\Big]}.$$

Proof. Cauchy-Schwartz inequality is

$$\Big[\sum_{i=2}^{n} a_i b_i\Big]^2 \leq \Big(\sum_{i=2}^{n} a_i^2\Big)\Big(\sum_{i=2}^{n} b_i^2\Big).$$

Put $a_i = 1, b_i =\mid \rho_i \mid$, then

$$\Big(\sum_{i=2}^{n} \mid \rho_i \mid\Big)^2 \leq \sum_{i=2}^{n} 1 \sum_{i=2}^{n} \rho_i^2$$

Then,

$$[E_{Dd}(G) - \rho_1]^2 \leq (n-1)(k+4n^2-4n-6m-\rho_1^2).$$

We have

$$E_{Dd}(G) \leq \rho_1 + \sqrt{(n-1)(k+4n^2-4n-6m-\rho_1^2)}.$$

Let

$$f(x) = x + \sqrt{(n-1)(k+4n^2-4n-6m-x^2)}.$$

For decreasing function, $f'(x) \leq 0$ Then,

$$1 - \frac{x(n-1)}{\sqrt{(n-1)(k+4n^2-4n-6m-x^2)}} \leq 0.$$

We have

$$x \geq \sqrt{\frac{k+4n^2-4n-6m}{n}}.$$

Therefore, $f(x)$ is decreasing in

$$\Big[\sqrt{\frac{k+4n^2-4n-6m}{n}}, \sqrt{k+4n^2-4n-6m}\Big].$$

Clearly,

$$\sqrt{\frac{k+2n^2-2n-2m}{n}} \in \Big[\sqrt{\frac{k+4n^2-4n-6m}{n}}, \sqrt{k+4n^2-4n-6m}\Big].$$

Since $\frac{k+2n^2-2n-2m}{n} \geq 1$, we have

$$\sqrt{\frac{k+2n^2-2n-2m}{n}} \leq \frac{k+2n^2-2n-2m}{n} \leq \rho_1 (\text{by lema } 5.3)$$

Therefore, $f(\rho_1) \leq f\Big(\frac{k+2n^2-2n-2m}{n}\Big)$. Then,

$$\begin{aligned} E_{Dd}(G) &\leq f(\rho_1) \\ &\leq f\Big(\frac{k+2n^2-2n-2m}{n}\Big) \\ &\leq \frac{k+2n^2-2n-2m}{n} \\ &\quad + \sqrt{(n-1)\Big[k+4n^2-4n-6m-\Big(\frac{k+2n^2-2n-2m}{n}\Big)^2\Big]}. \end{aligned}$$

□

Bapat and S. Pati [2] proved that if the graph energy is a rational number then it is an even integer. Similar result for minimum dominating energy is given in the following theorem.

Lemma 5.5. *Let G be a graph with a minimum dominating set D. If the minimum dominating distance energy $E_{Dd}(G)$ is a rational number, then $E_{Dd}(G) \equiv \mid D \mid$(mod 2).*

Proof. Proof is similar to Theorem 5.4 of [19]. □

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