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Preliminary Studies of Thermal Wavelength Approximation in ^{208}Pb and ^{91}Zr hot Nuclei

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Abstract. Level density parameters of ^{208}Pb and ^{91}Zr in equilibrium states have been calculated with a new fugacity approximation namely a thermal wavelength approximation. In this approximation, the fugacity is directly proportional to the nucleon density. In contrast with the constant fugacity, the thermal wavelength approximation gives a simpler way to calculate the nuclei constant radius and the density profile. The calculated ^{208}Pb nuclear density is about 0.17 (fm^{-3}) in which the discrepancy is 0.1% higher than the experimental one. The level density parameters are 14% higher than the experimental results due to neglecting of the shell correction.

Keywords: *Thermal wave length; fugacity; level density parameters; hot nuclei.*

1 Introduction

The hot unstable nucleus has large excitations, which can be created by intermediate energy or high-energy nucleus-nucleus collision [1]. Energy levels of highly excited nuclear systems have small gaps between states. The small gaps lead to the continuum energy levels [2]. The Fermi gas model in nuclear system is the continuum model of nucleus that nucleons are perceived as gas.

The hot nuclei are not thermodynamically stable, they de-excite naturally by emission of nucleons and light particles. This condition is broken down by artificial external pressure so the nuclear equilibrium states are accomplished. The continuum states of nucleon in hot nucleus are occupied by the occupancy probability [3]. In recent calculations such as the refined Thomas Fermi (TF) [4] and level density calculation [5] used occupancy probability as a function of constant fugacity. Base on the first order phase transition, the constant fugacity gives same probability that a system in the grand canonical ensemble has N particles, hence a temperature $T^{3/2}$ is proportional to concerning nuclear density [6].

In this paper the thermal wavelength approximation is assumes to be equal to unity. So that as the fugacity is equal to nuclear density. Once get the calculated converge nuclear density then the entropy and the level density parameter can be obtained. Within the framework of the Liquid Drop Model

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(LDM) formulation [7] the level density parameter must be corrected by the liquid drop shell correction [8].

The paper is organized as follows. In Section II, it briefly reviews the modeling the hot nucleus and present equations relevant for calculating the nuclear density and level density, in which the calculations are based on the thermal wavelength approximation. Results from the calculations are presented in section III and the conclusions are given in section IV.

2 Theoretical Framework

The occupation probability is obtained by minimizing the thermodynamic potential [9] as written in the following equation (1)

$$G = E - TS - \mu A + P\Omega \ . \tag{1}$$

Where G, E, A, Ω are Gibbs potential, the total energy of the system, Helmholtz free energy and the Volume of isolated system respectively. The minimizing process is initiated by setting out the Lagrange multiplier μ, and then by using standard procedure of Lagrange multiplier solution, the occupation probability can be expressed as equation (2) below.

$$f_\tau(\vec{r},\vec{p},T) = \frac{z(r,T)}{1 + z(r,T).exp\left(\left(H_\tau(\vec{r},\vec{p},T) - \mu_\tau\right)\Big/T\right)} \tag{2}$$

In this equation, z and H are fugacity and Hamiltonian of the nuclear system, which can be presented with the equation (3) and (4), respectively

$$z(r,T) = \lambda^3 \rho(r,T) \tag{3}$$

$$H_\tau(\vec{r},\vec{p},T) = \frac{p_\tau^2}{2m_\tau^*} + V_\tau(\vec{r},\vec{p},T) \ . \tag{4}$$

Where V_τ is the effective single particle potential that given by equation (5) [8]

$$V_\tau(\vec{r},\vec{p}) = V_\tau^{(0)}(r) + p^2 V_\tau^{(1)}(r) + V_\tau^{(2)} + \delta_{\tau p} V_C(r) + \frac{P}{\rho(r)} \tag{5}$$

Here P, ρ and τ are the constant external pressure, the nuclear density and the isospin respectively. In later work [8], the last term in equation (5) has the form as $P_o(10\pi/3A)R_u r^2$. The Coulomb potential energy V_c is considered in calculation when the nucleon is proton.

In this work, the nucleon effective mass m_τ^* is approximated by the nucleon rest mass m_τ [10];

$$m_\tau^* = \left[\frac{1}{m_\tau} + 2V_\tau^{(1)} \right]^{-1}$$

$$m_\tau^* \approx m_\tau$$

with the result of that, the potential $V_\tau^{(1)}$ is negligible. V_τ^0 and V_τ^2 are the Seyler-Blanchard type [11,12,13] which they are substituted by

$$V_{nucl}(r,T) = -\frac{V_o}{(1+exp((r-R)/d)} \cdot \rho(r,T). \quad (6)$$

Hence, in equation (5) the effective single particle potential can be written as

$$V_\tau(r,T) = V_{nucl}(r,T) + \frac{P}{\rho(r,T)} + \delta_{\tau p} V_C(r,T). \quad (7)$$

The Coulomb interaction energy density is given by the sum of a direct and an exchange term [14].

$$V_{C,D}(r,T) = e^2 \pi \rho_P(r,T) \int dr' r'^2 \, \rho_P(r',T) \frac{(r+r') - |r-r'|}{rr'} \quad (8.a)$$

$$V_{C,ex}(r,T) = -\frac{3e^2}{4\pi} \left(3\pi^2\right)^{1/3} \rho_P^{3/4}(r,T) \quad (8.b)$$

Both of the proton density and neutron density have been calculated by using the following relationship

$$\rho_\tau(r,T) = \frac{4\pi}{h^3} \left[2m_\tau^*(r)T\right]^{\frac{3}{2}} J_{1/2}(r,T) \quad (9)$$

where $J_{1/2}(r)$ is Fermi integrals, which has the form as below

$$J_k(r,T) = \int_0^\infty \frac{x^k}{1+exp(x-\mu_\tau+V_\tau(r,T))} dx. \quad (10)$$

The total entropy of the nucleus is written as

$$S(T)=\sum\frac{2}{h^3}\int\{[1-f_\tau(r,p,T)]ln[1-f_\tau(r,p,T)]-f_\tau(r,p,T)\,ln(f_\tau(r,p,T))\}d^3r\,d^3p \tag{11}$$

Equation (11) is employed to calculates the level density parameter by using equation

$$a(T)=\frac{S(T)}{T} \tag{12}$$

3 Results and Discussions

The density profiles of the nucleon are calculated by a self-consistent density solutions. The details of the procedure are given in ref. [13]. In Fig. 1, the neutron density profiles of ^{208}Pb and ^{91}Zr at T = 0 MeV are showed. The profiles are saturated at r = 0 fm, where the approximated density is 0.12 neutron.fm^{-3}. As expected, the profiles have Fermi-Dirac distribution shapes. Except the area below the curves, in the same temperature the profiles of ^{208}Pb and ^{91}Zr have the similar shapes.

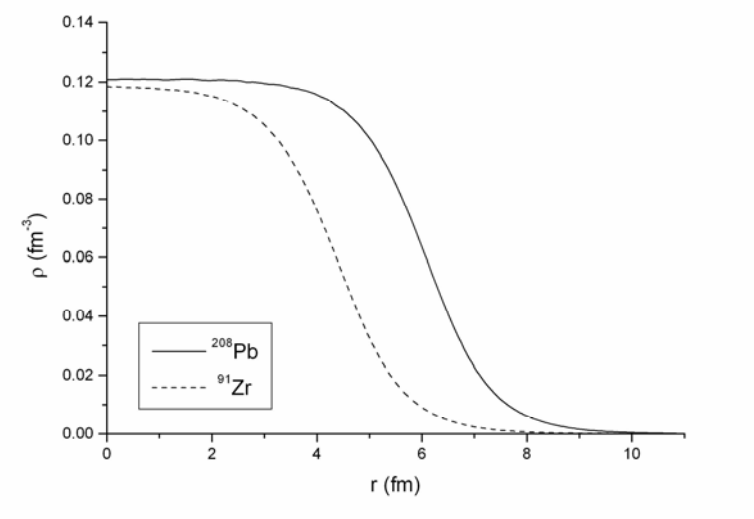

Figure 1 Neutron density profiles at T = 0 MeV.

There are other shapes of the density profiles where the coulomb interactions take part. Fig. 2 shows influence of the coulomb interactions to the density profiles of proton. The density profile of proton curves have no maximum value at r = 0, that indicated the protons are inclined far apart. The results that showed in Fig. 2 are more realistic than in ref. [13].

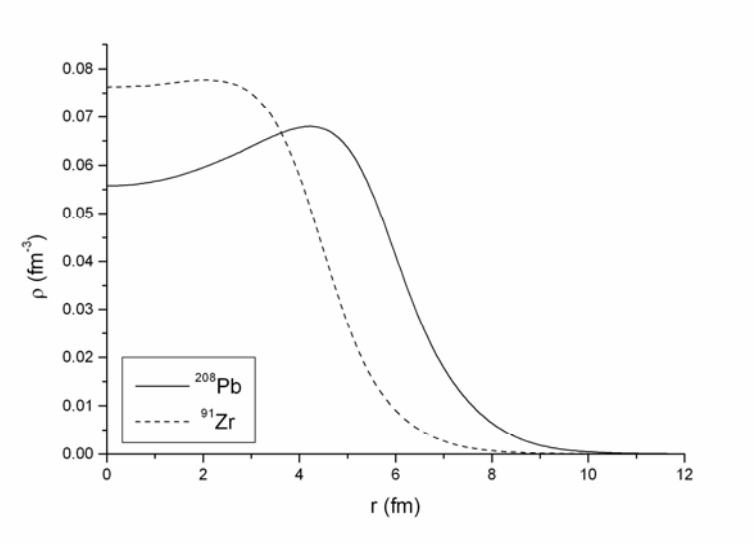

Figure 2 Proton density profiles at $T = 0$ MeV.

Nuclear densities are calculated by proton and neutron density addition. Fig. 3 shows the nuclear densities of ^{208}Pb and ^{91}Zr. Tail parts of these curves have the similar forms especially at r more than the nuclei radii. Because of small coulomb interaction contribution, the nuclear density of ^{91}Zr has flattened curve at r around the central of nucleus. Saturated condition of ^{208}Pb density, which is 0.17 nucl/fm^3 in average, is 0.1% higher than the experimental result.

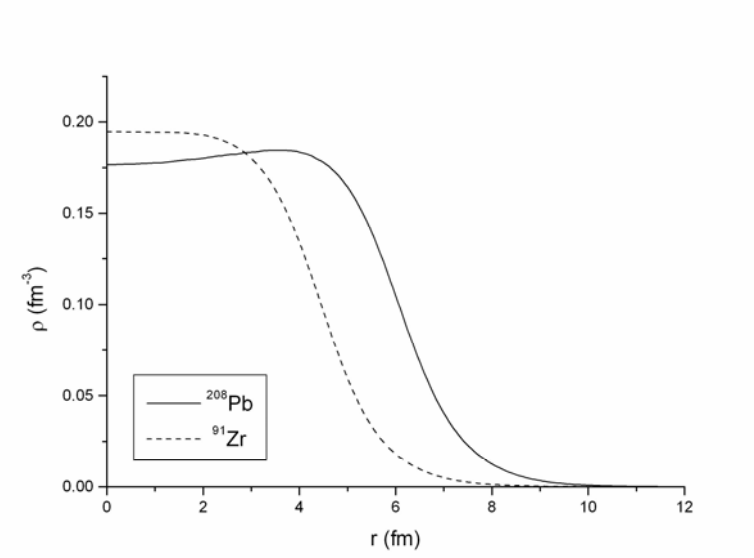

Figure 3 Nuclear density profiles at $T = 0$ MeV.

Fig. 4 shows the neutron nuclei radii constant of ^{208}Pb and ^{91}Zr, the nuclear radii constant gradually decrease with the increment of temperature from 1.245 fm for ^{208}Pb at T = 0 MeV until 1.1775 fm at T = 5 MeV. Because of cut of error, around T = 4.75 MeV the nuclear radii constant of ^{91}Zr increases rapidly. Decreasing of the nuclear radii constant indicates that at the higher temperature both of neutron and proton distribution have diffusive shape. As compare to ref. [13,15], these present works give more sensible results.

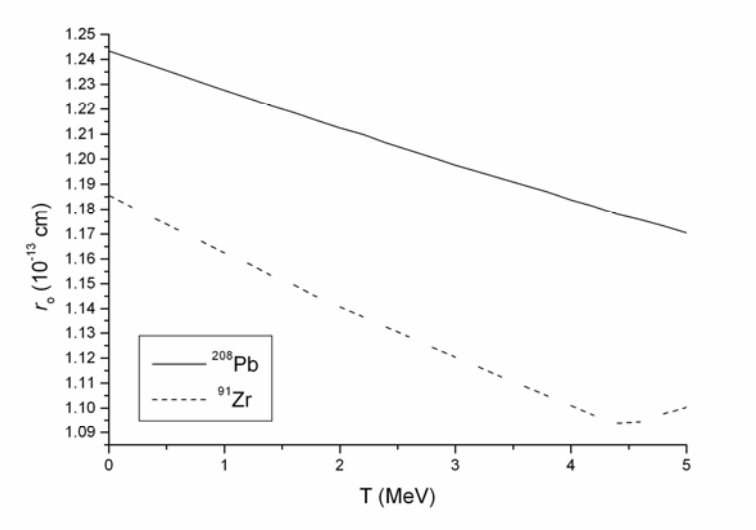

Figure 4 Neutron nuclear radii constant of ^{208}Pb and ^{91}Zr nuclei.

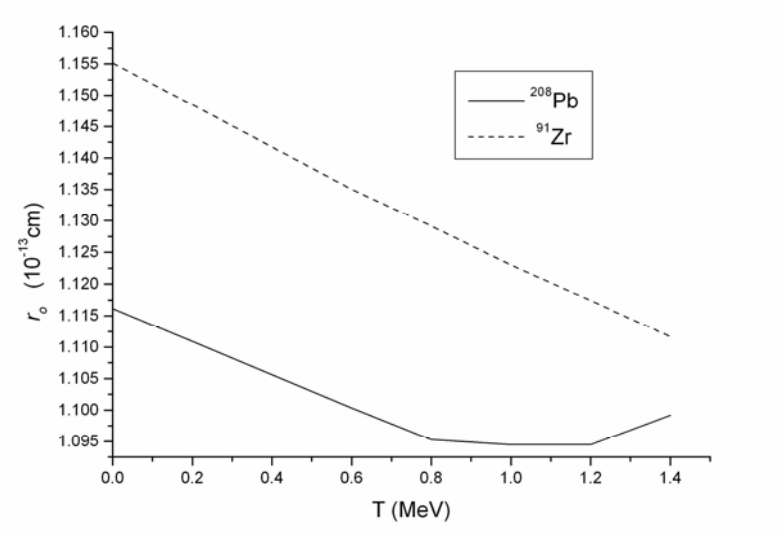

Figure 5 Proton nuclear radii constant of ^{208}Pb and ^{91}Zr nuclei.

Diverse the nuclear radii constant calculation yield occur to proton, where ^{208}Pb nuclear radii constant are higher than that of ^{91}Zr ones. These points occurred because the ^{208}Pb charge distribution has maximum value at 75% of the total are below curve area. At the temperature region between 0.8 MeV and 1.2 MeV the nuclear radii constant are flat. It seems that temperature at 0.8 MeV is the ^{208}Pb maximum limit of nuclear radii constant calculation, with results in the divergence of calculation.

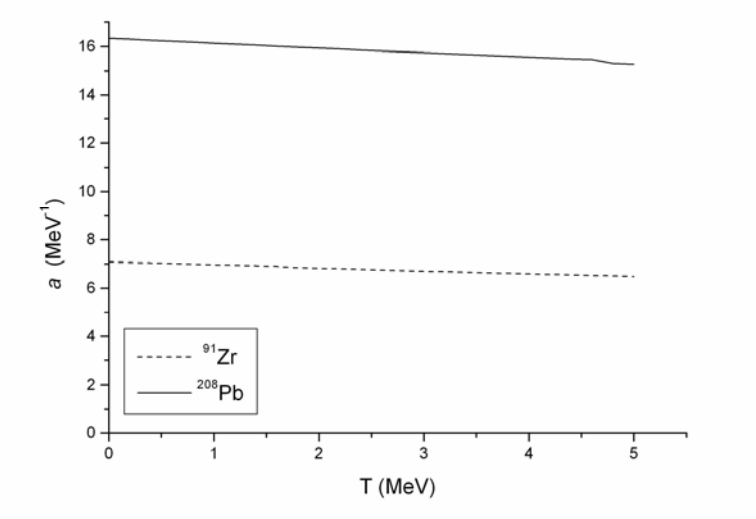

Figure 6 Level density parameters of ^{208}Pb and ^{91}Zr nuclei.

As shown in figure 6, the level density parameter of ^{208}Pb is 16 MeV^{-1} in average and the level density parameter for ^{91}Zr approximately is 7 MeV^{-1} in average. Because of neglecting the shell correction [14], these level density parameters are about 14% higher than the experimental results.

4 Conclusions

This paper has proposed a prescription method to calculated self–consistently the density profile by using the thermal wavelength approximation. Beside the density profile, the approximation is applicable to determine nuclear radii constant of nuclei. With the shell correction, this technique can be chanced to calculate the level density parameters.

Acknowledgments

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