Joint Economic Lot Sizing Optimization in a Supplier-Buyer Inventory System When the Supplier Offers Decremental Temporary Discounts

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Abstract.

This research discusses mathematical models of joint economic lot size optimization in a supplier-buyer inventory system in a situation when the supplier offers decremental temporary discounts during a sale period. Here, the sale period consists of n phases and the phases of discounts offered descend as much as the number of phases. The highest discount will be given when orders are placed in the first phase while the lowest one will be given when they are placed in the last phase. In this situation, the supplier attempts to attract the buyer to place orders as early as possible during the sale period. The buyers will respon these offers by ordering a special quantity in one of the phase. In this paper, we propose such a forward buying model with discount-proportionally-distributed time phases. To examine the behaviour of the proposed model, we conducted numerical experiments. We assumed that there are three phases of discounts during the sale period. We then compared the total joint costs of special order placed in each phase for two scenarios. The first scenario is the case of independent situation – there is no coordination between the buyer and the supplie-, while the second scenario is the opposite one, the coordinated model. Our results showed the coordinated model outperform the independent model in terms of producing total joint costs. We finally conducted a sensitivity analyzis to examine the other behaviour of the proposed model.
Keywords: *supplier-buyer inventory system, forward buying model, decremental temporary discounts, joint economic lot sizing, optimization.*

INTRODUCTION

Nowadays, higher market competition pushes a supply chain to implement a better coordinated inventory strategy that will give benefits for both parties; the supplier and the buyer. One of the strategies is forward buying strategy. In this strategy, the supplier offer discounts to the buyer during a period of sale, called *temporary discount*. The buyer then responds it by ordering a higher or special quantity to cover the demand over a longer period. The discount policy can be a good medium for efficacy inventory coordination between both parties. By offering a temporary price discount, the supplier will increase the cash flow and decrease the inventory phase of items (Sarker and Kindi, 2006). At the same time, the buyer will get benefits from the discounted price so that it will reduce the purchasing costs. The strategy will result in lowering inventory costs, improving asset utilization, and reducing effects on order variability. Spesifically, to implement this strategy, both parties need to seek the joint lot sizes that minimize the total inventory costs. Such a problem is generally called Joint Economic Lot Sizing (JELS) Optimization . In the literature, there are some quantitative models discussed joint lot sizing problems between buyers and suppliers. An early paper of JELS is Goyal (1976). This paper assumes that the lot size method used is Lot-For-Lot policy. Banerjee (1986) presented a paper on JELS models and made a comparison between the solutions of the independent models of each party and those of the integrated model. Moreover, Goyal (1988) developed integrated models where the supplier produced in a multiple integer of the buyer’s order quantity. Pujawan and Kingsman (2002) developed a quantitative model that considers a situation where there are multiple deliveries for one order. The supplier also produced products in a multiple integer of delivery quantity. In this case, an order can be splitted into a number of production runs.

In a different situation, there are some papers discussed the existence of temporary discounts in JELS over a period of sale. The extra decision should be made in this situation is to define the optimum lot size of special orders.

Abad (2003) introduced optimization models for reselling business. In this business, the buyer will resale the items bought from the supplier to end customers. There are two cases concerned in that paper. The first case is a forward buying models in the situation where the supplier offers such discount during the sale period. Thus, in this paper, we attempt to fill in this gap.

In our proposed model, we devide the sale period into *n* phases, are shown in Figure 1. In each phase, the supplier offers different discount. The highest discount will be given if the buyer will place orders in the first phase, while the lowest discount will be offered if the buyer will place in the *n* phase. Thus, the discount will be offered decreasingly over the phased in the sale period. We call this model Joint Economic Lot Sizing with Decreasing Temporary Discount (JELSDTD) Model.

We develop a model of JELSDTD and heuristics algorithms to solve it. Then, we conduct some numerical experiments to show the behaviour of our model. The numerical experiments are done using some data examples modified those from the literature.

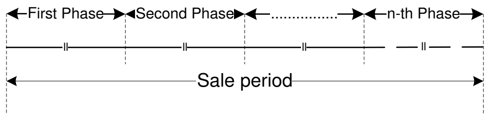

Figure 1. n-phases of the Sale Period

Specifically, we examine the decisions made by the buyer when facing different phases of discounts. We then make a comparison between independent decisions and joint decisions. Finally, we conduct a sensitivity analysis by changing main parameters to further elaborate the behaviour.

The organization of the paper is as follow. In Chapter 2, we formulate the models of JELSDTD in several situations. We discuss the algorithm to solve them. In Chapter 3, we conduct numerical experiments. We finally make a conclusion of this research in Chapter 4.

MODEL FORMULATION

Notation

We use notations as follows:

D : annual demand rate (units/year)
i : holding cost fraction
δ_n : discount price n-th (\$/unit) , where $n = 1,2,...k$
A : ordering cost (\$/order)
C_p : production cost (\$/unit)
C : purchasing cost (\$/unit)
Q : reguler economic order quantity (units/cycle)
Qs_n : special ordering quantity on discount phase n-th (units/order)
T : cycle time for EOQ
T_s : cycle time for special order quantity
T_{sale} : sale period
m : number of regular quantity orders placed special order special placed
r : number of regular quantities orders placed after special order placed
P : production rate (units/year)
S_s : setup cost (\$/*setup*)
TJC : total joint cost (\$)

Assumptions

We define some assumptions as follows:

1. Demand is deterministic and constant

2. Replenishment is instantaneous
3. Ordering costs and holding cost per unit are homogeneoous
4. Ordering costs for both regular and special orders are the same
5. A special order is given only once over the sale period.
6. After buyer conducts a special order, the order quantity backs to economic order quantity with normal purchasing price.

Inventory Models of Buyer and Supplier

The inventory phases in a normal situation in both supplier and buyer sides are illustrated in Figure 2. Here, we assume that the product delivery starts after the whole batch has been produced.

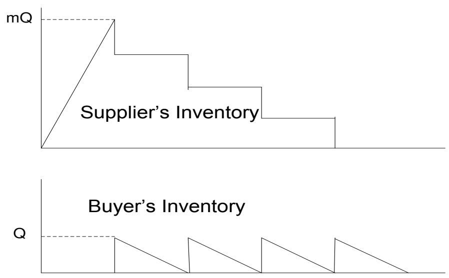

Figure 2 the Inventory Models of Buyer and Supplier in a Normal Situation

The supplier's cost (TC_s) consists of production cost, setup cost and holding cost formulated in equation (1) while the buyer's cost (TC_b) consisting of purchasing costs, ordering costs and holding costs can be formulated in equation (2).

$$TC_s = C_p D + \frac{D}{mQ} S_s + \left(\frac{Q}{2}\left(m\left(1+\frac{D}{P}\right)-1\right)\right) iC_p \quad (1)$$

$$TC_b = CD + A\left(\frac{D}{Q}\right) + iC\frac{Q}{2} \quad (2)$$

The total joint costs then can be formulated as

$$TJC = TC_b + TC_s$$

$$TJC(Q,m) = D\left(C + C_p\right) + \frac{D}{Q}\left(A + \frac{S_s}{m}\right) + i\frac{Q}{2}\left(C - C_p + mC_p\left(1+\frac{D}{P}\right)\right) \quad (3)$$

The optimal order quantity thus can be obtained by seeking the derivative of eq. (3) with respect to Q as follows.

$$Q(m) = \sqrt{\frac{2D\left(A + \frac{S_s}{m}\right)}{i\left(C - C_p + mC_p\left(1+\frac{D}{P}\right)\right)}} \quad (4)$$

Hence, the optimal number of regular quantity orders placed (m^*) is as follows.

$$m^*\left(m^*-1\right) \le \frac{S_s\left(C - C_p\right)}{AC_p\left(\frac{D}{P}+1\right)} \le m^*\left(m^*+1\right) \quad (5)$$

Inventory Models with P Model of with DTime Phase for Every Discount Phase

- The length of sale period is *n*-times reguler cycle time
- The discount offered during the sale period descend as much as the number of phase *(n)* during the sale period. The reduction of discount follows the interest rate.
- The time of every phase for all of discount value are uniform from δ_1 to δ_n $\{T(\delta_1) = T(\delta_2) = ... = T(\delta_n)\}$, are shown in Figure 3.
- Special order time is the same with replenishment time, so when special order is done there is not inventory on hand, are shown in Figure 4

Formulation of Joint Order Special Quantity

Element of the Supplier's cost

The supplier's cost consists of setup cost, holding cost and discount cost. Since the number of production cycle in one year is D/Q_p times, where $Q_p = mQ + Qs_n + rQ$, so that the setup cost in a period can be written as:

$$\frac{D}{Q_p} S_s \qquad (6)$$

The holding cost in a period is multiplication between inventory phase and holding cost (iC_p)

The supplier's inventory phase is given by the area below the bold line. It can be obtained by subtracting the area about the bold line, which represents the time integral of cumulatif delivery quantity, from the area a-b-d-f-g-j-i, which is the time integral of cumulative production for one production cycle. The area of a-b-d-f-g-j-i consists of four parts, i.e., the triangle a-b-c, rectangle b-c-e-d, rectangle f-d-h-g and regtangle g-h-i-j.

The inventory held by the supplier for one cycle is obtained as follows:

$$\frac{(Q_p)^2}{2P} + \frac{mQ(2Q_p - (m+1)Q) + (rQ)\{2Qs_n + \{(r)-1\}Q\}}{2D} \qquad (7)$$

The time average inventory phase throughout the year is given by:

$$\frac{Q_p.D}{2P} + mQ - \frac{m(m+1)Q^2 - rQ\{2Qs_n + (r-1)Q\}}{2Q_p} \qquad (8)$$

Hence, the supplier's holding cost is:

$$iC_p\left(\frac{Q_pD}{2P} + mQ + \frac{m(m+1)Q^2 + rQ(2Qs_n + (r-1)Q)}{2Qs_p}\right) \qquad (9)$$

The supplier's discount cost in one year is:

$$\frac{D}{Q_p}\,\delta_n Qs_n, \text{ for } n = 1 \qquad (10)$$

$$\frac{D}{Q_p}\,\{\,\delta_n Qs_n + Q(\delta_1 + \ldots + \delta_{n-1})\,\}, \text{for } n > 1 \qquad (11)$$

The following expression represents total cost for the supplier

$$TC_s = \frac{D}{Q_p}(S_s - \delta_n Qs_n) + iC_p\left(\frac{Q_pD}{2P} + mQ\right),$$
$$- iC_p\left(\frac{m(m+1)Q^2 - rQ\{2Qs_n + (r-1)\}}{2Q_p}\right)$$

for $n = 1$ (12)

$$TC_s = \frac{D}{Q_p}\left(S_s - (\delta_n Qs_n + Q(\delta_1 + \ldots + \delta_{n-1})\right)$$
$$+ iC_p\left(\frac{Q_pD}{2P} + mQ - \frac{m(m+1)Q^2 - rQ(2Qs_n + (r-1))}{2Q_p}\right)$$

for $n > 1$ (13)

Elements of the Buyer's Cost

The buyer's total cost consists of purchasing cost, ordering cost and holding cost.

Purchasing cost in one year is

$$CD - \frac{D}{Q_p}\,\delta_n Qs_n, \text{ for } n = 1 \qquad (14)$$

$$CD - \frac{D}{Q_p}\,\{\delta_n Qs_n - Q(\delta_1 + \delta_2 + \ldots + \delta_{n-1})\},$$

for $n > 1$ (15)

Ordering cost in one year is:

$$\frac{D}{Q_p} A(1 + m + r) \qquad (16)$$

Holding cost in one year is:

$$\frac{i(C - \delta_n)Qs_n^{\,2} + (m+r)iCQ^2}{2Q_p}, \text{ for } n = 1 \qquad (17)$$

$$\frac{i(C - \delta_n)Qs_n^{\,2} + i\{(n-1)C - \delta_1 - \delta_2 - \ldots - \delta_{n-1}\}Q^2}{2Q_p}$$
$$+ \frac{\{m + r - (n-1)\}iCQ^2}{2Q_p}, \text{ for } n > 1 \qquad (18)$$

Hence, the buyer's total cost can be written as:

$$TC_b = CD - \frac{D}{Q_p}\,\delta_n Qs_n + \frac{D}{Q_p} A(1 + m + r)$$
$$+ \frac{i(C - \delta_n)Qs_n^{\,2} + (m+r)iCQ^2}{2Q_p}$$

for $n = 1$ (19)

$$TC_b = CD - \frac{D}{Q_p}\,\{\delta_n Qs_n + Q(\delta_1 + \delta_2 + \ldots + \delta_{n-1})\}$$
$$+ \frac{D}{Q_p} A(1 + m + r) + \frac{\{m + r - (n-1)\}iCQ^2}{2Q_p}$$
$$+ \frac{i(C - \delta_n)Qs_n^{\,2} + i\{(n-1)C - \delta_1 - \ldots - \delta_{n-1}\}Q^2}{2Q_p}$$

for $n > 1$ (20)

To obtain total joint cost, the supplier's total cost is adding by the buyer's total cost. Hence, the total joint cost can be formulated as:

$$TJC(Qs_n) = \frac{D}{Q_p} S_s + iC_p\left(\frac{Q_p D}{2P} + mQ\right) - iC_p\left(\frac{m(m+1)Q^2 - rQ\{2Qs_n + (r-1)\}}{2Q_p}\right) + CD + \frac{D}{Q_p} A(1+m+r) + \frac{i(C-\delta_n)Qs_n{}^2 + (m+r)iCQ^2}{2Q_p}$$

for $n = 1$ (21)

$$TJC(Qs_n) = \frac{D}{Q_p} S_s + iC_p\left(\frac{Q_p D}{2P} + mQ\right) - iC_p\left(\frac{m(m+1)Q^2 - rQ\{2Qs_n + (r-1)\}}{2Q_p}\right) + CD + \frac{D}{Q_p} A(1+m+r) + \frac{\{m+r-(n-1)\}iCQ^2}{2Q_p} + \frac{i(C-\delta_n)Qs_n{}^2 + i\{(n-1)C - \delta_1 - \ldots - \delta_{n-1}\}Q^2}{2Q_p}$$

for $n > 1$ (22)

Determination of Optimal Joint Special Order Quantity

Optimal joint special order quantity ($Qs_n{}^*$) is obtained by deriving total joint cost ($TJC(Qs_n)$) with respect to special order quantity (Qs_n) and equating it to zero.

To show that spesial order quantity is minimum extreme point, enough prerequisite that must be fulfilled is second derivation from total joint cost ($TJC(Qs_n)$) to special order quantity (Qs_n) is bigger than zero.

NUMERICAL EXAMPLES

We illustrate the behavior of our model using the following numerical examples. In this example, we defined that there are three phases of discounts in a sale period. The parameters used are as follows.

Table 1 Parameters

Notation	Value
D	8000 units/year
C	\$10/unit
A	\$20/order
I	0.3
P	15000 units/year
C_p	\$5/unit
S_s	\$400/setup

Determination of Optimal Order Quantity in Normal Situation

The number of optimal order frequency based on equation (5) obtained is $m^*(m^*-1) \le 13.04 \le m^*(m^*+1)$, thus the optimal order frequency (m^*) that fulfilled above equation is 4. The optimal order quantity $Q*$ is determined using equation (4). The result is as follows:

$$Q(4) = \sqrt{\frac{1920000}{10.7}} = 423.603 = 424\ unit$$

So that, a cycle time is:

$$T_s = \frac{Q}{D} = \frac{424\ unit}{8000\ unit/year} = 0.053\ year = 2.7\ week$$

Joint Decision

In this system, time-phase of every discount phase is assumed to be uniform. Time phase for every discount phase is the same with reguler cycle time, that is 2.7 weeks, so the length of sale period are three times cycle time or 8.1 weeks.

We consider the results of Sarker and Kindi (2006) . We used the value of discount as much as 10% from the purchasing price. This value is used as the direction of discount value in the end of sale period or the last discount phase (*n=k).* So, the value of *k*-phase discount (δ_k) is 10% from the purchasing price. Based on equation 6, percentage of discount value offered in every phase is following:

- The value of the discount in the third phase (δ_3) is 10% from the purchasing price.

- The value of the discount in the second phase (δ_2) is 10,62% from the purchasing price.
- The value of the discount in the first phase o (δ_1) is 11,25% from the purchasing price.

The special order quantity and production quantity every phase of discount value in JELSDTD model can be seen on Tables 2 to 4. While the total joint cost curve from every phase of discount value in *r*=0-3 can be seen in Figure 5.

The numerical results from Tables 2 to 4 and Figure 5 shown that the minimum joint total cost was reached by conducting a special order at the third discount phase and r = 0. This phenomenon showed that the supplier's production quantity are three times than the reguler order quantity plus special order quantity (444.53 units). Hence the supplier must produce 1716.53 units. This gave the total joint cost as much as $ 84481.44.

From the supplier side, the minimum total cost would be obtained if the buyer conducted a special order at the first discount phase, is $4508.81. The minimum special order quantity is 198.13 units. Thus, the special order quantity is lower than reguler order quantity. The calculation results showed that if the buyer conducted a special order earlier and special quantity is smaller, the cost of the supplier is lower. When the buyer conducted a special order earlier of sale period, the supplier will save holding cost. If special order quantity is smaller, the loss of the supplier is smaller too.

From the buyer's perspective, the minimum total cost wouldl be obtained if the buyer conducted a special order at the third discount phase as much as $74571.75 with maximum special order quantity as many as 444.52 units. The calculation results showed that if the buyer placed a special order earlier, the total cost of the buyer will larger, and if the special order quantity is lower, the buyer's cost will be lower as well. By placed a special order at third discount phase, the buyer will save holding cost as many as Q unit at the first and second discount phases as many as Qs_n unit at the third discount phase.

As we knew, the purpose of the supplier offered discount which decremental value is to attract the buyer for conducting special order in early of sale period, so that the supplier will receive more saving holding cost. When the supplier offered a constant discount value, the buyer will place a special order at the end of the sale period. However, the incremental discount value was unable to shift the buyer decision to conduct special order earlier because the cost saving that would be accepted by buying earlier is smaller than the increasing value of the buyer's holding cost.

Independent Decision

The special order quantity and production quantity every phase of discount value, can be seen in Tables 5 to 7. In this policy, the buyer places a special order quantity decision, thus the determination of optimal special quantity will give a minimum the buyer's total cost. A variety of total joint cost curves from every phase of discount value with independent decision in r = 0 - 3 can be seen in Figure 6.

The numerical results at Tables 5 to 7 showed that in the same discount phase dan *r*-value, the buyer's total cost for independent decision is lower than the joint decision, because the purpose of independent decision is minimizing the buyer's total cost. However, the independent decision would produce larger total joint cost than those of the joint decision. From the buyer's perspective, the larger, *r*-value, the larger special order quantity. The reason is because after the buyer placed a special order, the following order return to normal price. Hence, the larger regular order placed after a special order, the larger buyer's total cost. Consequently, to compensate the costs burdened because of the normal price, the buyer would enlarge the special order quantity. This is due to the larger the special order quantity, the larger saving occured in purchasing order..

Figure 6 showed that the minimum total joint cost is reached by conducting special order at the third discount phase and $r = 0$. It is shown that the supplier's production quantity are three times of the regular order quantity plus the special order quantity. The supplier must produce as many as 2165.72 units to yield the total joint cost as much as \$ 84644.49. However, again, the incremental discount value was unable to shift the buyer decision to conduct special order earlier because the cost saving that would be accepted by buying earlier is smaller than the increasing value of the buyer's holding cost.

Comparison between Joint Decision and Independent Decision

Comparison of special order quantity and total joint cost between joint decision and independent decision can be seen in Tabel 8.

Tabel 8 shown that special order quantity with independent decision is bigger than joint decision, it is caused independent decision looking into importance the buyer only , special order quantity is bigger, the buyer is benefit progresively, because the cost is lower. Joint decision and independent decision give the same result for the time of special order should be placed, that is the last discount phase, but optimal total joint cost of joint cost is lower than optimal total joint cost of independent cost. Hence, to accommodate the supplier and the buyer together, special order should be conducted by using joint economic lot size.

Model Behavior Analysis

Model behavior analysis is conducted to see the effects of the parameter changes, which is the discount value, to order schedules, order quantity and the total joint cost. In order to have the buyer buy earlier, the discount value should be larger enough to accept. We changed the discount values using the rules of geometric series with ratio of discount value (ρ) as many as 1, 1.5 and 2.5 and used the discount value in the last phase (n=k) as the basis. Table 9 showed the percentage of discount value in every discount phase. Table 9 showed that in ρ=1, the discount value are constant in all phases. This situation and the results are the same as those of Abad (2003)'s model. From the other two values of ρ, we may see that the buyer would shift the order schedule to the second discount phase if the discount offered in this phase (δ_2) is at least 15%. The order place decision will shift to the first phase if the discount offered in the third phase (δ_3) is 62.5%. We may see here, in this example case, the changes of discount value is quite sensitive to shift the order place decision to the second discount phase. However, to shift the decision to the third discount phase, the supplier needs to offer a very large value of discount to attract the buyer.

CONCLUSION

In this paper, we have developed mathematical models of joint economic lot size optimization in a supplier-buyer inventory system in a situation when the supplier offers decremental temporary discounts during a sale period. In this situation, the supplier attempts to attract the buyer to place orders as early as possible during the sale period.

We have generated numerical examples to show the behavior of the models. It has been shown that the joint decision outperform the independent decision in terms of total joint costs. From the sensitivity analysis, we have concluded that the buyer would buy earlier if the discount value offered should be larger enough to cover the holding cost burdened.

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APPENDICES

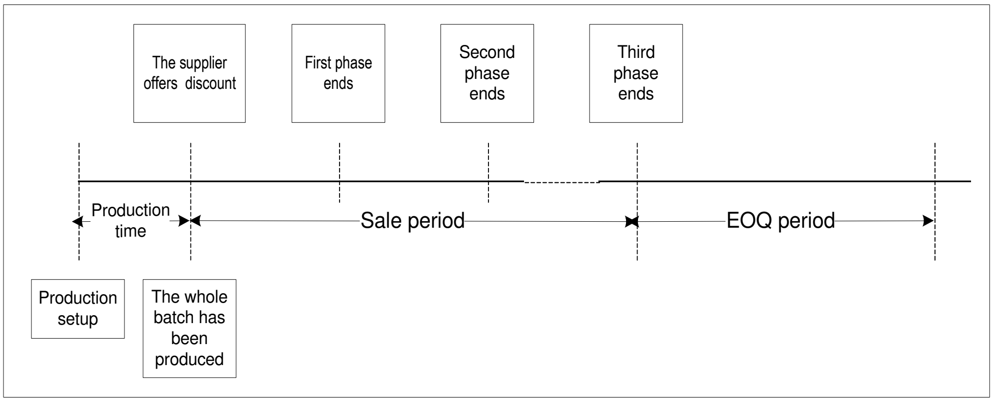

Figure 3 ***Time Frame*** **of the System**

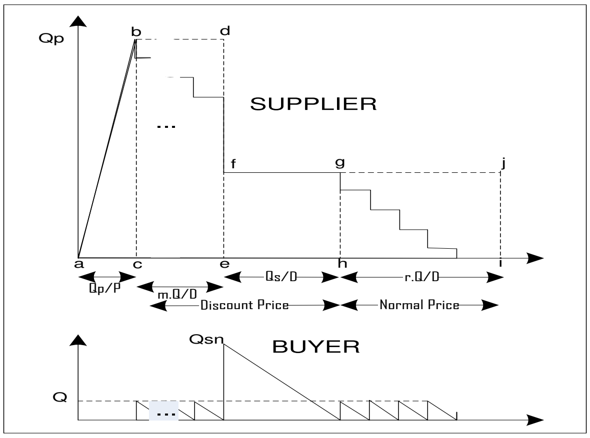

Figure 4 Inventory Model the Supplier-the Buyer with Discount Programs

Table 2 Numerical Results of the First Discount Phase with Joint Decision

r	Qs_1 (unit)	Qp_1 (unit)	TCs_1 ($)	TCb_1 ($)	TJC_1 ($)
0	1049.58	1473.58	9624.41	74984.98	84609.39
1	740.34	1588.34	7607.67	76906.16	84513.83
2	458.58	1730.58	5899.27	78614.17	84513.44
3	198.13	1894.13	4508.81	80078.01	84586.82

Table 3 Numerical Results of the Second Discount Phase with Joint Decision

r	Qs_2 (unit)	Qp_2 (unit)	TCs_2 ($)	TCb_2 ($)	TJC_2 ($)
0	731.75	1579.75	9768.41	74729.50	84497.91
1	451.37	1723.37	7954.80	76542.21	84497.01
2	192.50	1888.50	6452.61	78118.36	84570.97

Table 4 Numerical Results of the Third Discount Phase with Joint Decision

r	Qs_3 (unit)	Qp_3 (unit)	TCs_3 ($)	TCb_3($)	Qs_3(unit)
0	444.53	1716.53	9909.69	74571.75	84481.44
1	187.19	1883.19	8298.99	76256.97	84555.96

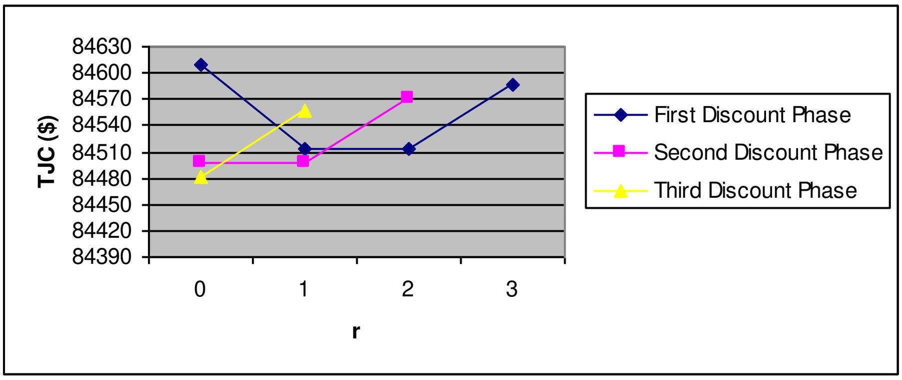

Figure 5 Comparison of $TJC(Qs_n^*)$ among Discount Phase with Joint Decision

Table 5 Numerical Results of the First Discount Phase with Independent Decision

r	Qs_1 (unit)	Qp_1 (unit)	TCs_1 ($)	TCb_1 ($)	TJC_1 ($)
0	1443.94	1867.94	9909.04	74844.49	84753.52
1	1838.59	2686.59	9395.84	75895.24	85291.08
2	2090.42	3362.42	9318.78	76565.74	85884.52
3	2273.07	3969.07	9412.71	77052.05	86464.77

Table 6 Numerical Results of the First Discount Phase with Independent Decision

r	Qs_2 (unit)	Qp_2 (unit)	TCs_2 ($)	TCb_2 ($)	TJC_2 ($)
0	1148.06	1996.06	10066.64	74582.41	84649.05
1	1537.00	2809.00	9602.05	75625.32	85227.37
2	1790.59	3486.59	9540.73	76305.28	85846.01

Table 7 Numerical Results of the First Discount Phase with Independent Decision

r	Qs_3 (unit)	Qp_3 (unit)	TCs_3 ($)	TCb_3($)	Qs_3(unit)
0	893.72	2165.72	10231.43	74413.06	84644.49
1	1267.05	2963.05	9823.64	75421.03	85244.67
2	1517.10	3637.10	9779.09	76096.17	85875.26

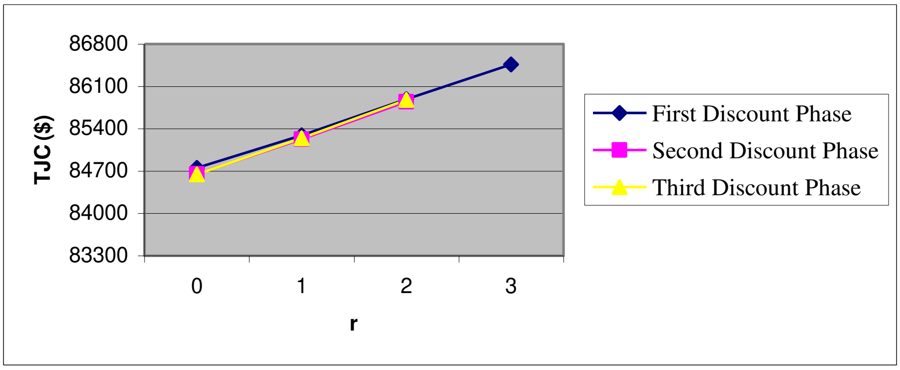

Figure 6 the Comparison of $TJC(Qs_n^*)$ among Discount Phase with Independent Decision

Tabel 8. The Comparison of Qs_n^* and $TJC(Qs_n^*)$ between Joint Decision and Independent Decision

Discount Level	JOINT		INDEPENDENT	
	Qs (unit)	TJC ($)	Qs (unit)	TJC ($)
1	458.58	84513.44	1443.94	84753.52
2	451.37	84497.01	1148.06	84649.05
3	444.53	84481.44	893.72	84644.49

Table 9 Calculation Result of δ_n

Ratio	Discount Value		
	δ_1	δ_2	δ_3
1	10%	10%	10%
1.5	22.5%	15%	10%
2.5	62.5%	25%	10%

Table 10 Calculation Result of Qs_n^* and $TJC(Qs_n^*)$ with Increasing of δ_n

Ratio	Discount Level					
	First		Second		Third	
	Qs_1 (unit)	TJC_1 ($)	Qs_2 (unit)	TJC_2 ($)	Qs_3 (unit)	TJC_3 ($)
1	457.83	84513.84	449.85	84500.06	445.37	84484.38
1.5	1120.85	84477.85	747.99	84456.16	437.50	84456.84
2.5	1515.64	83892.81	781.80	84334.90	414.81	84377.44

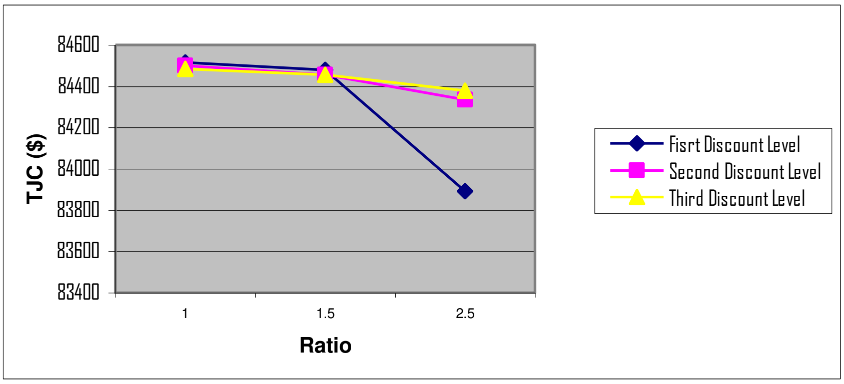

Figure 7 The comparison of $TJC(Qs_n^*)$ among Discount Phase with Increasing of δ_n