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INVARIANT RANDOM APPROXIMATION IN NONCONVEX DOMAIN

R. Shrivastava, H. K. Nashine and C. L. Dewangan

Abstract. Random fixed point results in the setup of compact and weakly compact domain of Banach spaces which is not necessary starshaped have been obtained in the present work. Invariant random approximation results have also been determined as its application. In this way, random version of invariant approximation results due to Mukherjee and Som [13] and Singh [17] have been given.

1. INTRODUCTION

Probabilistic functional analysis is an important mathematical discipline because of its applications to probabilistic models in applied problems. Random operator theory is needed for the study of various classes of random equations. The theory of random fixed point theorems was initiated by the Prague school of probabilistic in the 1950s. The interest in this subject enhanced after publication of the survey paper by Bharucha Reid [5]. Random fixed point theory has received much attention in recent years (see, e.g. [2, 14, 15, 16, 18]).

Interesting and valuable results applying various random fixed point theorems appeared in the literature of approximation theory. In this direction, some of the authors are Beg and Shahzad [1, 3, 4], Khan et al. [8], Lin [11], Tan and Yuan [18] and Papageorgion [15, 16]. In the subject of best approximation, one often wishes to know whether some useful property of the function being approximation is inherited by the approximating function.

In fact, Meinardus [12] was the first who observed the general principle and employed a fixed point theorem to established the existence of an invariant ap-

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proximation. Later on, a number of results were developed along this direction under different conditions (see, e.g. [9, 12, 17]).

The aim of this paper is to establish existence of random fixed point as random best approximation for compact and weakly compact domain of Banach spaces which is not necessary starshaped. To achieve the goal, the contractive jointly continuous family property given by Dotson [6] has been used. By doing so, random version results of certain invariant approximation theorems obtained by Mukherjee and Som [13] and Singh [17] have been obtained.

2. PRELIMINARIES

In the material to be produced here, the following definitions have been used:

Definition 2.1. *[14]. Let* $(\Omega, \mathcal{A})$ *be a measurable space and* $\mathcal{X}$ *be a metric space. Let* $2^{\mathcal{X}}$ *be the family of all nonempty subsets of* $\mathcal{X}$ *and* $\mathcal{C}(\mathcal{X})$ *denote the family of all nonempty compact subsets of* $\mathcal{X}$*. Now, we call a mapping* $\mathcal{F} : \Omega \to 2^{\mathcal{X}}$ *measurable (respectively, weakly measurable) if, for any closed (respectively, open) subset* $\mathcal{B}$ *of* $\mathcal{X}, \mathcal{F}^{-1}(\mathcal{B}) = \{w \in \Omega : \mathcal{F}(w) \cap \mathcal{B} \neq\} \in \mathcal{A}$*. Note that, if* $\mathcal{F}(w) \in \mathcal{C}(\mathcal{X})$ *for every* $w \in \Omega$*, then* $\mathcal{F}$ *is weakly measurable if and only if measurable.*

A mapping $\xi : \Omega \to \mathcal{X}$ is called a measurable selector of a measurable mapping $\mathcal{F} : \Omega \to 2^{\mathcal{X}}$, if ξ is measurable and, for any $w \in \Omega, \xi(w) \in \mathcal{F}(w)$. A mapping $f : \Omega \times \mathcal{X} \to \mathcal{X}$ is called a random operator if for any $x \in \mathcal{X}, f(., x)$is measurable. A measurable mapping $\xi : \Omega \to \mathcal{X}$ is called a random fixed point of a random operator $f : \Omega \times \mathcal{X} \to \mathcal{X}$ if for every $w \in \Omega, \xi(w) = f(w, \xi(w))$. A random operator $f : \Omega \times \mathcal{X} \to \mathcal{X}$ is continuous if, for each $w \in \Omega, f(w, .)$ is continuous.

Definition 2.2. *Let* $\mathcal{M}$ *be a nonempty subset of a Banach space* $\mathcal{X}$ *. For* $x_0 \in \mathcal{X}$*, define*

$$d(x_0, \mathcal{M}) = \inf_{y \in \mathcal{M}} \|x_0 - y\|$$

and

$$\mathcal{P}_{\mathcal{M}}(x_0) = \{y \in \mathcal{M} : \|x_0 - y\| = d(x_0, \mathcal{M})\}.$$

Then an element $y \in \mathcal{P}_{\mathcal{M}}(x_0)$ *is called a best approximant of* x_0 *of* $\mathcal{M}$*. The set* $\mathcal{P}_{\mathcal{M}}(x_0)$ *is the set of all best approximants of* x_0 *of* $\mathcal{M}$*.*

Further, definition providing the notion of contractive jointly continuous family introduced by Dotson [6] may be written as:

Definition 2.3. *[6]. Let* $\mathcal{M}$ *be a subset of metric space* $(\mathcal{X}, d)$ *and* $\Delta = \{f_\alpha\}_{\alpha \in \mathcal{M}}$ *a family of functions from* $[0, 1]$ *into* $\mathcal{M}$ *such that* $f_\alpha(1) = \alpha$ *for each* $\alpha \in \mathcal{M}$*. The* *family Δ is said to be contractive if whenever there exists a function $\theta : (0,1) \to (0,1)$ such that for all $\alpha, \beta \in \mathcal{M}$ and all $\in (0,1)$ we have*

$$d(f_\alpha(t), f_\beta(t)) \le \theta(t) d(\alpha, \beta).$$

The family is said to be jointly continuous if $t \to t_0$ in $[0,1]$ and $\alpha \to \alpha_0$ in $\mathcal{M}$ imply that $f_\alpha(t) \to f - \alpha_0(t_0)$ in $\mathcal{X}$.

Definition 2.4. *[6]. Let $\mathcal{M}$ be a subset of metric space $(\mathcal{X}, d)$ and Δ is a family as in Definition 2.3, then Δ is said to be jointly weakly continuous if $t \to t_0$ in $[0,1]$ and $\alpha \xrightarrow{w} \alpha_0$ in $\mathcal{M}$ imply that $f_\alpha(t) \xrightarrow{w} f_{\alpha_0}(t_0)$ in $\mathcal{M}$.*

The following result would also be used in the sequel:

Theorem 2.1. *[14]. Let $(\mathcal{X}, d)$ be a Polish space and $\mathcal{T} : \Omega \times \mathcal{X} \to \mathcal{X}$ be a continuous random operator. Suppose there is some $h \in (0,1)$ such that for $x, y \in \mathcal{X}$ and $w \in \Omega$, we have*

$$d(\mathcal{T}(w,x), \mathcal{T}(w,y)) \le h \max\{d(x,y), d(x, \mathcal{T}(w,x)), d(y, \mathcal{T}(w,y)), \frac{1}{2}[d(x, \mathcal{T}(w,x)) + d(y, \mathcal{T}(w,y))]\}.$$

Then $\mathcal{T}$ have a random fixed point.

3. MAIN RESULTS

We first prove, random fixed point result for compact subset of Banach space which is not necessary starshaped.

Definition 2.1. *Let $\mathcal{X}$ be a Banach space and $\mathcal{M}$ be a subset of $\mathcal{X}$. Let $\mathcal{T} : \Omega \times \mathcal{M} \to \mathcal{M}$ be continuous random operator. Suppose $\mathcal{M}$ is nonempty compact and admits a contractive and jointly continuous family Δ. If $\mathcal{T}$ satisfies for $x, y \in \mathcal{M}, w \in \Omega$ and $t \in (0,1)$*

$$\|\mathcal{T}(w,x) - \mathcal{T}(w,y)\| \le \max\{\|x-y\|, dist(x, f_{\mathcal{T}(w,x)}(t)), dist(y, f_{\mathcal{T}(w,y)}(t)), \frac{1}{2}[dist(x, f_{\mathcal{T}(w,y)}(t)) + dist(y, f_{\mathcal{T}(w,x)}(t))]\}, \quad (1)$$

then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ such that $\xi(w) = \mathcal{T}(w, \xi(w))$ for each $w \in \Omega$.

Proof. Choose $k_n \in (0,1)$ such that $\{k_n\} \to 1$ as $n \to \infty$. Then for each n, define a random operator $\mathcal{T}_n : \Omega \times \mathcal{M} \to \mathcal{M}$ as

$$\mathcal{T}_n(w,x) = f_{\mathcal{T}(w,x)}(k_n). \quad (2)$$

Then $\mathcal{T}_n$ is a well-defined map from $\mathcal{M}$ into $\mathcal{M}$ and $\mathcal{T}_n$ is continuous because of the joint continuity of $f_x(t)(x \in \mathcal{M}, t \in [0,1])$. It follows from 1 and 2 that

$$\begin{aligned}\|\mathcal{T}_n(w,x) - \mathcal{T}_n(w,y)\| &= \|f_{\mathcal{T}(w,x)}(k_n) - f_{\mathcal{T}(w,y)}(k_n)\| \\ &\leq \phi(k_n)\|\mathcal{T}(w,x) - \mathcal{T}(w,y)\| \\ &\leq \phi(k_n)\max\{\|x-y\|, dist(x, f_{\mathcal{T}(w,x)}(k_n)), \\ &\quad \frac{1}{2}[dist(x, f_{\mathcal{T}(w,y)}(k_n)) + dist(y, f_{\mathcal{T}(w,y)}(k_n)), \\ &\quad dist(y, f_{\mathcal{T}(w,x)}(k_n))]\} \\ &\leq \phi(k_n)\max\{\|x-y\|, \|x-\mathcal{T}_n(w,x)\|, \|y-\mathcal{T}_n(w,y)\|, \\ &\quad \frac{1}{2}[\|x-\mathcal{T}_n(w,y)\| + \|y-\mathcal{T}_n(w,x)\|]\}\end{aligned}$$

i.e.,

$$\begin{aligned}\|\mathcal{T}_n(w,x) - \mathcal{T}_n(w,y)\| \leq \phi(k_n)\max\{&\|x-y\|, \|x-\mathcal{T}_n(w,x)\|, \|y-\mathcal{T}_n(w,y)\|, \\ &\frac{1}{2}[\|x-\mathcal{T}_n(w,y)\| + \|y-\mathcal{T}_n(w,x)\|]\}\end{aligned}$$

for all $x, y \in \mathcal{M}, w \in \Omega$ and $\phi(k_n) \in (0,1)$.
By the continuity of $\mathcal{T}_n(.,x)(x \in \mathcal{M})$, the inverse image of any open subset $\mathcal{K}$ of $\mathcal{M}$ is open in $w = [0,1]$ and hence Lebsegue measurable. Thus each $\mathcal{T}_n(.,x)$ is a random operator. By Theorem 2.1, $\mathcal{T}_n$ has a random fixed point ξ_n of $\mathcal{T}_n$ such that $\xi_n(w) = \mathcal{T}_n(w, \xi_n)$ for all $n \in N$.
For each n, define $\mathcal{G}_n : \Omega \to \mathcal{C}(\mathcal{M})$ by $\mathcal{G}_n = cl\{\xi(w) : i \geq n\}$ where $\mathcal{C}(\mathcal{M})$ is the set of all nonempty compact subset of $\mathcal{M}$.
Let $\mathcal{G} : \Omega \to \mathcal{C}(\mathcal{M})$ be a mapping defined as $\mathcal{G}(w) = \cap_{n=1}^{\infty}\mathcal{G}(w)$. Then, by a result of Himmelberg [7, Theorem 4.1] we see that $\mathcal{G}$ is measurable. The Kuratowski and Ryll-Nardzewski selection Theorem [10] further implies that G has a measurable selector $\xi : \Omega \to \mathcal{M}$. We now show that ξ is the random fixed point of $\mathcal{T}$. We first fix $w \in \Omega$. Since $\xi(\Omega) \in \mathcal{G}(w)$, there exists a subsequence $\{\xi_m(w)\}$ of $\{\xi_n(w)\}$ that converges to $\xi(w)$; that is $\xi_m(w) \to \xi(w)$. Since $\mathcal{T}_m(w, \xi_m(w)) = \xi_m(w)$, we have $\mathcal{T}_m(w, \xi_m(w)) \to \xi(w)$.
Proceeding to the limit as $m \to \infty, k_m \to \infty$ and by using joint continuity,

$$\mathcal{T}_m(w, \xi_m(w)) = f_{\mathcal{T}(w,\xi_m(w))}(k_m) \to f_{\mathcal{T}}(w, \xi(w))(1) = \mathcal{T}(w, \xi(w)).$$

This completes the proof.□

An immediately consequence of the Theorem 3.2 is as follows:

Corollary 3.1. *Let $\mathcal{X}$ be a Banach space and $\mathcal{M}$ be a subset of $\mathcal{X}$. Let $\mathcal{T} : \Omega \times \mathcal{M} \to \mathcal{M}$ be continuous random operator. Suppose $\mathcal{M}$ is nonempty compact and admits* *a contractive and jointly continuous family* Δ*. If* $\mathcal{T}$ *satisfies for* $x, y \in \mathcal{M}, w \in \Omega$ *and* $t \in (0,1)$

$$\|\mathcal{T}(w,x) - \mathcal{T}(w,y)\| \leq \max\{\|x-y\|, dist(x, f_{\mathcal{T}(w,x)}(t)), dist(y, f_{\mathcal{T}(w,y)}(t)), \frac{1}{2}dist(x, f_{\mathcal{T}(w,x)}(t)), \frac{1}{2}dist(y, f_{\mathcal{T}(w,x)}(t))\}, \quad (3)$$

then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ *such that* $\xi(2) = \mathcal{T}(2, \xi(w))$ *for each* $w \in \Omega$.

As an application of Theorem 3.2, we have following results on invariant approximations:

Theorem 3.3. *Let* $\mathcal{X}$ *be a Banach space and* $\mathcal{T} : \Omega \times \mathcal{X} \to \mathcal{M}$ *be continuous random operator. Let* $\mathcal{M} \subseteq \mathcal{X}$ *such that* $\mathcal{T}(w,.) : \partial\mathcal{M} \to \mathcal{M}$*, where* $\partial\mathcal{M}$ *stands for the boundary of* $\mathcal{M}$*. Let* $x_0 \in \mathcal{X}$ *and* $x_0 = \mathcal{T}(w, x_0)$*. Suppose* $\mathcal{D} = \mathcal{P}_{\mathcal{M}}(x_0)$ *is nonempty compact and admits a contractive and jointly continuous family* Δ*. If* $\mathcal{T}$ *satisfies for* $x, y \in \mathcal{D} \cup \{x_0\}, w \in \Omega$ *and* $t \in (0,1)$

$$\|\mathcal{T}(w,x) - \mathcal{T}(w,y)\| \leq \begin{cases} \|x - x_0\|, \ if \ y = x_0, \\ \max\{\|x-y\|, dist(x, f_{\mathcal{T}(w,x)}(t)), dist(y, f_{\mathcal{T}(w,y)}(t)), \\ \frac{1}{2}[dist(x, f_{\mathcal{T}(w,y)}(t)) + dist(y, f_{\mathcal{T}(w,x)}(t))]\}, \ if \ y \in \mathcal{D}, \end{cases} \quad (4)$$

then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ *such that* $\xi(w) = \mathcal{T}(w, \xi(w))$ *for each* $w \in \Omega$.

Proof. Let $y \in \mathcal{D}$. Also, if $y \in \partial\mathcal{M}$ then $\mathcal{W}(w,y) \in \mathcal{M}$, because $\mathcal{T}(w, \partial\mathcal{M}) \subseteq \mathcal{M}$ for each $w \in \Omega$. Now since $x_0 = \mathcal{T}(w, x_0)$,

$$\|\mathcal{T}(w,y) - x_0\| = \|\mathcal{T}(w,y) - \mathcal{T}(w,x_0)\| \leq \|x - x_0\|,$$

yielding thereby $\mathcal{T}(w,y) \in \mathcal{D}$; consequently $\mathcal{D}$ is $\mathcal{T}(w,.)$-invariant, that is, $\mathcal{T}(w,.) \subseteq \mathcal{D}$. Now, Theorem 3.2 guarantees that there exists a measurable map $\xi : \Omega \to \mathcal{D}$ such that $\xi(w) = \mathcal{T}(w, \xi(w))$ for each $w \in \Omega$.□

Next, an immediate consequence of the Theorem 3.3 is as follows:

Corollary 3.2. *Let* $\mathcal{X}$ *be a Banach space and* $\mathcal{T} : \Omega \times \mathcal{M} \to \mathcal{M}$ *be continuous random operator. Let* $\mathcal{M} \subseteq \mathcal{X}$ *such that* $\mathcal{T}(w,.) : \partial\mathcal{M} \to \mathcal{M}$*, where* $\partial\mathcal{M}$ *stands for the boundary of* $\mathcal{M}$*. Let* $x_0 \in \mathcal{X}$ *and* $x_0 = \mathcal{T}(w, x_0)$*. Suppose* $\mathcal{D} = \mathcal{P}_{\mathcal{M}}(x_0)$ *is nonempty compact and admits a contractive and jointly continuous family* Δ*. If* $\mathcal{T}$ *satisfies for* $x, y \in \mathcal{D} \cup \{x_0\}, w \in \Omega$ *and* $t \in (0,1)$

$$\|\mathcal{T}(w,x) - \mathcal{T}(w,y)\| \leq \begin{cases} \|x - x_0\|, \ if \ y = x_0, \\ \max\{\|x-y\|, dist(x, f_{\mathcal{T}(w,x)}(t)), dist(y, f_{\mathcal{T}(w,y)}(t)), \\ \frac{1}{2}[dist(x, f_{\mathcal{T}(w,y)}(t)) + dist(y, f_{\mathcal{T}(w,x)}(t))]\}, \ if \ y \in \mathcal{D}, \end{cases} \quad (5)$$

then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ *such that* $\xi(w) = \mathcal{T}(w, \xi(w))$ *for each* $w \in \Omega$.

An analogue of the Theorem 3.2 for weakly compact subset is as follows:

Theorem 3.4. *Let* $\mathcal{X}$ *be a Banach space and* $\mathcal{M}$ *be a subset of* $\mathcal{X}$. *Let* $\mathcal{T} : \Omega \times \mathcal{M} \to \mathcal{M}$ *be weakly continuous random operator. Let* $\mathcal{M}$ *is nonempty separable weakly compact and admits a contractive and jointly weakly continuous family* Δ.*If* $\mathcal{T}$ *satisfies (3.1) for* $x, y \in \mathcal{M}, w \in \Omega$ *and* $t \in (0, 1)$, *then there exists a measurable map* $\xi : \Omega \to \mathcal{M}$ *such that* $\xi(w) = \mathcal{T}(w, \xi(w))$ *for each* $w \in \Omega$, *provided* $\mathcal{I} - \mathcal{T}(w, .)$ *is demiclosed at zero for each* $w \in \Omega$, *where* $\mathcal{I}$ *is a identity mapping.*

Proof. For each $n \in \mathbb{N}$, define $\{k_n\}, \{T_n\}$ as in the proof of the Theorem 3.2. Also, we have

$$\|\mathcal{T}_n(w, x) - \mathcal{T}_n(w, y)\| \leq \phi(k_n) \max\{\|x - y\|, \|x - \mathcal{T}_n(w, x)\|, \|y - \mathcal{T}_n(w, y)\|, \frac{1}{2}[\|x - \mathcal{T}_n(w, y)\| + \|y - \mathcal{T}_n(w, x)\|]\}$$

for all $x, y \in \mathcal{M}, w \in \Omega$, and $\phi(k_n) \in (0, 1)$. Since weak topology is Hausdorff and $\mathcal{M}$ is weakly compact, it follows that $\mathcal{M}$ is strongly closed and is a completely metric space. Thus, weak continuity of $\mathcal{R}$, joint weakly continuous family Δ and Theorem 2.1 guarantee that there exists a random fixed point ξ of $\mathcal{T}_n$ such that $\xi_n = \mathcal{T}_n(w, \xi_n(w))$ for each $w \in \Omega$.

For each n, define $\mathcal{G}_n : \mathcal{WC}(\mathcal{M})$ by $\mathcal{G}_n = w - cl\{\xi_i(w) : i \geq n\}$, where $\mathcal{WC}(\mathcal{M})$ is the set of all nonempty weakly compact subset of $\mathcal{M}$ and $w - cl$ denotes the weak closure. Define a mapping $\mathcal{G} : \Omega \to \mathcal{WC}(\mathcal{M})$ by $\mathcal{G}(w) = \cap_{n=1}^{\infty} \mathcal{G}_n(w)$. Because $\mathcal{M}$ is weakly compact and separable, the weak topology on $\mathcal{M}$ is a metric topology. Then by result of Himmelberg [7, Theorem 4.1] implies that $\mathcal{G}$ is $w-$measurable. The Kuratowski and Ryll-Nardzewski selection Theorem [10] further implies that $\mathcal{G}$ has a measurable selector $\xi : \Omega \to \mathcal{M}$. We now show that ξ is the random fixed point of $\mathcal{T}$. We first fix $w \in \Omega$. Since $\xi(w) \in \mathcal{G}(w)$, therefore there exists a subsequence $\{\xi_m(w)\}$ of $\{\xi_n(w)\}$ that converges weakly to $\xi(w)$; that is $\xi_m(w) \xrightarrow{w} \xi(w)$. Now,

$$\begin{aligned} \xi_m(w) - \mathcal{T}(w, \xi_m(w)) &= \mathcal{T}_m(w, \xi_m(w)) - \mathcal{T}(w, \xi_m(w)) \\ &= f_{\mathcal{T}(w, \xi_m(w))}(k_m) - \mathcal{T}(w, \xi_m(w)). \end{aligned}$$

Since $\mathcal{M}$ is bounded and $k_m \to 1$, it follows from joint weakly continuity that

$$\begin{aligned} \xi_m(w) - \mathcal{T}(w, \xi_m(w)) &= f_{\mathcal{T}(w, \xi(w))}(1) - \mathcal{T}(w, \xi(w)) \\ &= \mathcal{T}(w, \xi(w)) - \mathcal{T}(w, \xi(w)) \\ &= 0. \end{aligned}$$

Now, $y_m = \xi_m(w) - \mathcal{T}(w, \xi_m(w)) = (\mathcal{I} - \mathcal{T})(w, \xi_m(w))$ and $y_m \to 0$. Since $(\mathcal{I} - \mathcal{T})(w, .)$ is demiclosed at 0, so $0 \in (\mathcal{I} - \mathcal{T})(w, \xi(w))$. This implies that $\xi(w) = \mathcal{T}(w, \xi(w))$. This completes the proof.□

An immediate consequence of the Theorem 3.4 is as follows:

Corollary 3.3. *Let $\mathcal{X}$ be a Banach space and $\mathcal{M}$ be a subset of $\mathcal{X}$. Let $\mathcal{T} : \Omega \times \mathcal{M} \to \mathcal{M}$ be weakly continuous random operator. Suppose $\mathcal{M}$ is nonempty separable weakly compact and admits a contractive and jointly weakly continuous family Δ. If $\mathcal{T}$ satisfies (3.3) for $x, y \in \mathcal{M}, w \in \Omega$ and $t \in (0,1)$, then there exists a measurable map $\xi : \Omega \to \mathcal{M}$ such that $\xi(w) = \mathcal{T}(w, \xi(w))$ for each $w \in \Omega$, provided $\mathcal{I} - \mathcal{T}(w, .)$ is demiclosed at zero for each $w \in \Omega$, where $\mathcal{I}$ is a identity mapping.*

As an application of Theorem 3.4, we have following results on invariant approximations:

Theorem 3.5. *Let $\mathcal{X}$ be a Banach space and $\mathcal{T} : \Omega \times \mathcal{X} \to \mathcal{X}$ be weakly continuous random operator. Let $\mathcal{M} \subseteq \mathcal{X}$ such that $\mathcal{T}(w, .) : \partial\mathcal{M} \to \mathcal{M}$, where $\partial\mathcal{M}$ stands for the boundary of $\mathcal{M}$. Let $x_0 \in \mathcal{X}$ and $x_0 = \mathcal{T}(w, x_0)$. Suppose $\mathcal{D} = \mathcal{P}_{\mathcal{M}}(x_0)$ is nonempty separable weakly compact and admits a contractive and jointly weakly continuous family Δ. Further, suppose $\mathcal{T}$ satisfies the condition (3.4) for$x, y \in \mathcal{D} \cup \{x_0\}, w \in \Omega$ and $t \in (0,1)$. Then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ such that $\xi(w) = \mathcal{T}(w, \xi(w))$ for each $w \in \Omega$, provided $\mathcal{I} - \mathcal{T}(w, .)$ is demiclosed at zero for each $w \in \Omega$, where $\mathcal{I}$ is a identity mapping.*

Proof. It follows from the proof of the Theorem 3.3.

Next, an immediate consequence of the Theorem 3.5 is as follows:

Corollary 3.4. *Let $\mathcal{X}$ be a Banach space and $\mathcal{X} : \Omega \times \mathcal{X} \to \mathcal{X}$ be weakly continuous random operator. Let $\mathcal{M} \subseteq \mathcal{X}$ such that $\mathcal{T}(w, .) : \partial\mathcal{M} \to \mathcal{M}$, where $\partial\mathcal{M}$ stands for the boundary of $\mathcal{M}$. Let $x_0 \in \mathcal{X}$ and $x_0 = \mathcal{T}(w, x_0)$. Suppose $\mathcal{D} = \mathcal{P}_{\mathcal{X}}(x_0)$ is nonempty separable weakly compact and admits a contractive and jointly weakly continuous family Δ. Further, suppose $\mathcal{T}$ satisfies the condition (3.5) for $x, y \in \mathcal{D} \cup \{x_0\}, w \in \Omega$ and $t \in (0,1)$. Then there exists a measurable map $\xi : \Omega \to \mathcal{D}$ such that $\xi(w) = \mathcal{T}(w, \xi(w))$ for each $w \in \Omega$, provided $\mathcal{I} - \mathcal{T}(w, .)$ is demiclosed at zero for each $w \in \Omega$, where $\mathcal{I}$ is a identity mapping.*

Remark 3.1. *In the light of the comment given by Dotson [6] and Khan et al. [9] if $\mathcal{M} \subseteq \mathcal{X}$ is $p-$starshaped and $f_\alpha(t) = (1-t)p + t\alpha, (\alpha \in \mathcal{M}, t \in [0,1])$, then $\{f_\alpha\}_{\alpha \in \mathcal{M}}$ is a contractive jointly continuous family with $\phi(t) = t$. Thus the class of subsets of $\mathcal{X}$ with the property of contractiveness and joint continuity contains the class of starshaped sets which in turn contain the class of convex sets.*

Remark 3.2. *Theorem 3.3, Corollary 3.2, Theorem 3.5 and Corollary 3.4 generalize and give random version of the result due to Mukherjee and Som [13].*

Remark 3.2. *With the Remark 3.1, Theorem 3.3, Corollary 3.2, Theorem 3.5 and Corollary 3.4 generalize and give random version of the result of Singh [17] without star- shapedness condition of domain.*

REFERENCES

1. I. Beg and N. Shahzad, “Random fixed points and approximations in random convex metric spaces”, *Appl. Math. Stoch. Anal.* **6** (1993), 237–246.
2. I. Beg and N. Shahzad, “Random fixed points of random multivalued operators on polish spaces”, *Nonlinear Anal.* **20** (1993), 835–847.
3. I. Beg and N. Shahzad, “Applications of the proximity map to Random fixed points theorems in Hilbert space”, *J. Math. Anal. Appl.* **196** (1995), 606–613.
4. I. Beg and N. Shahzad, “On invariant random approximations”, *Approx. Theory Appl.* **12** (1996), 68–72.
5. A.T. Bharucha-Reid, “Fixed point theorem in probabilistic analysis”, *Bull. Amer. Math. Soc.* **82** (1976), 641–645.
6. W.G.Jr. Dotson, “On fixed point of nonexpansive mappings in nonconvex sets”, *Proc. Amer. Math. Soc.* **38** (1973), 155–156.
7. C.J. Himmerberg, “Measurable relations”, *Fund. Math.* **87** (1975), 53–72.
8. A.R. Khan, A.B. Thaheem, and N. Hussain, “Random fixed points and random approximations”, *South. Asian Bull. Math.* **27** (2003), 289–294.
9. A.R. Khan, A. Latif, A. Bano, and N. Hussain, “Some results on common fixed points and best approximation”, *Tamkang J. Math.* **36:1** (2005), 33–38.
10. K. Kuratoski and C. Ryll-Nardzewski, “A general theorem on selectors”, *Bull Acad. Pol. Sci. Ser. Sci.Math. Astron. Phys.* **13** (1965), 397–403.
11. T.C. Lin, “Random approximations and random fixed point theorems for nonsself maps”, *Proc. Amer.Math. Soc.* **103** (1988), 1129–1135.
12. G. Meinardus, “Invarianze bei Linearen Approximationen”, *Arch. Rational Mech. Anal.* **14** (1963), 301–303.
13. R.N. Mukherjee and T. Som, “A note on an application of a fixed point theorem in approximation theory”, *Indian J. Pure Appl. Math.* **16:3** (1985), 243–244.
14. D. O’Regan, N. Shahzad, and R.P. Agrawal, “Random fixed point theory in spaces with two metrices”, *J. Appl. Math. Stoch. Anal.* **16:2** (2003), 171–176.
15. N.S. Papagergiou, “Random fixed point theorems for measurable multifunctions in Banach spaces”, *Proc. Amer. Math. Soc.* **97** (1986), 507–514.
16. N.S. Papagergiou, “Random fixed points and random differential inclusions”, *Inter. J. Math. Math. Sci.* **11** (1988), 551–560.
17. S.P. Singh, “An application of a fixed point theorem to approximation theory”, *J. Approx. Theory* **25** (1979), 89–90.
18. K.K. Tan and X.Z. Yuan, “Random fixed point theorems and approximation in cones”, *J. Math. Anal. Appl.* **185** (1994), 378–390.

R. Shrivastava: Department of Mathematics, Disha Institute of Managment and Technology, Satya Vihar, Gyanganga - Chandrakhuri Marg, Distt. Raipur - 492101(Chhattisgarh), India.
E-mail: hemantnashine@rediffmail.com

H.K. Nashine: Department of Mathematics, Disha Institute of Managment and Technology, Satya Vihar, Gyanganga - Chandrakhuri Marg, Distt. Raipur - 492101(Chhattisgarh), India.
E-mail: nashine_09@rediffmail.com

C. L. Dewangan: Department of Mathematics, Govt. Chhattisgarh P G College, Raipur - 492101(Chhattisgarh), India.
E-mail: ritushri_7@rediffmail.com.