J. Indones. Math. Soc.
Vol. 16, No. 2 (2010), pp. 105–113.

COMMON-EDGE SIGNED GRAPH OF A SIGNED GRAPH

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Abstract. A *Smarandachely k-signed graph* (*Smarandachely k-marked graph*) is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$ where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ is a function, where each $\overline{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is abbreviated a *signed graph* or a *marked graph*. The *common-edge graph* of a graph $G = (V, E)$ is a graph $C_E(G) = (V_E, E_E)$, where $V_E = \{A \subseteq V; |A| = 3$, and A is a connected set$\}$ and two vertices in V_E are adjacent if they have an edge of G in common. Analogously, one can define the *common-edge signed graph* of a signed graph $S = (G, \sigma)$ as a signed graph $C_E(S) = (C_E(G), \sigma')$, where $C_E(G)$ is the underlying graph of $C_E(S)$, where for any edge (e_1e_2, e_2e_3) in $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. It is shown that for any signed graph S, its common-edge signed graph $C_E(S)$ is balanced. Further, we characterize signed graphs S for which $S \sim C_E(S)$, $S \sim L(S)$, $S \sim J(S)$, $C_E(S) \sim L(S)$ and $C_E(S) \sim J(S)$, where $L(S)$ and $J(S)$ denotes line signed graph and jump signed graph of S respectively.

Key words and Phrases: Smarandachely k-signed graphs, Smarandachely k-marked graphs, balance, switching, common-edge signed graph, line signed graph, jump signed graph.

2000 Mathematics Subject Classification: 05C22
Received: 26-05-2010, accepted: 23-11-2010.

Abstrak. Sebuah *graf bertanda-k Smarandachely* (*Smarandachely k-marked graph*) adalah sebuah pasangan terurut $S = (G, \sigma)$ $(S = (G, \mu))$ dimana $G = (V, E)$ adalah *graf pokok (underlying graph) dari* S dan $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ adalah sebuah fungsi, dimana tiap $\overline{e}_i \in \{+, -\}$. Kemudian, sebuah graf bertanda-k Smarandachely disingkat dengan sebuah graf bertanda *marked graph*. *Graf sekutu-sisi* dari sebuah graf $G = (V, E)$ adalah sebuah graf $C_E(G) = (V_E, E_E)$, dimana $V_E = \{A \subseteq V; |A| = 3$, dan A adalah sebuah himpunan terhubung$\}$ dan dua titik di V_E bertetangga jika mereka mempunyai sebuah sisi sekutu di G. Secara analog, kita dapat mendefinisikan *graf bertanda sekutu-sisi* dari sebuah graf bertanda $S = (G, \sigma)$ sebagai sebuah graf bertanda $C_E(S) = (C_E(G), \sigma')$, dimana $C_E(G)$ adalah graf pokok dari $C_E(S)$, dimana untuk suatu sisi (e_1e_2, e_2e_3) di $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. Pada paper ini, akan ditunjukkan bahwa untuk setiap graf bertanda S, graf bertanda sekutu-sisi $C_E(S)$ adalah seimbang. Lebih jauh, kami mengkarakterisasi graf bertanda S untuk $S \sim C_E(S)$, $S \sim L(S)$, $S \sim J(S)$, $C_E(S) \sim L(S)$ dan $C_E(S) \sim J(S)$, dimana $L(S)$ dan $J(S)$ masing-masing menyatakan graf bertanda garis dan graf bertanda lompat dari S.

Kata kunci: Graf bertanda-k Smarandachely, seimbang, pertukaran, graf bertanda sekutu-sisi, graf bertanda garis, graf bertanda lompat.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is to refer to [7]. We consider only finite, simple graphs free from self-loops.

A *Smarandachely k-signed graph* (*Smarandachely k-marked graph*) is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$ where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \to (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ is a function, where each $\overline{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is called abbreviated a *signed graph* or a *marked graph*. A *signed graph* is an ordered pair $S = (G, \sigma)$, where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \to \{+, -\}$ is a function. A signed graph $S = (G, \sigma)$ is *balanced* if every cycle in S has an even number of negative edges (See [8]). Equivalently, a signed graph is balanced if product of signs of the edges on every cycle of S is positive.

A *marking* of S is a function $\mu : V(G) \to \{+, -\}$; A signed graph S together with a marking μ is denoted by S_μ.

The following characterization of balanced signed graphs is well known.

Proposition 1.1. (**E. Sampathkumar** [10]) *A signed graph $S = (G, \sigma)$ is balanced if and only if there exists a marking μ of its vertices such that each edge uv in S satisfies $\sigma(uv) = \mu(u)\mu(v)$.*

The idea of switching a signed graph was introduced by Abelson and Rosenberg [1] in connection with structural analysis of marking μ of a signed graph S. Switching S with respect to a marking μ is the operation of changing the sign of every edge of S to its opposite whenever its end vertices are of opposite signs. The signed graph obtained in this way is denoted by $\mathcal{S}_\mu(S)$ and is called *μ-switched signed graph* or just *switched signed graph*. Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *isomorphic*, written as $S_1 \cong S_2$ if there exists a graph isomorphism $f : G \to G'$ (that is a bijection $f : V(G) \to V(G')$ such that if uv is an edge in G then $f(u)f(v)$ is an edge in G') such that for any edge $e \in G$, $\sigma(e) = \sigma'(f(e))$. Further a signed graph $S_1 = (G, \sigma)$ *switches* to a signed graph $S_2 = (G', \sigma')$ (or that S_1 and S_2 are *switching equivalent*) written $S_1 \sim S_2$, whenever there exists a marking μ of S_1 such that $\mathcal{S}_\mu(S_1) \cong S_2$. Note that $S_1 \sim S_2$ implies that $G \cong G'$, since the definition of switching does not involve change of adjacencies in the underlying graphs of the respective signed graphs.

Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *weakly isomorphic* (see [17]) or *cycle isomorphic* (see [18]) if there exists an isomorphism $\phi : G \to G'$ such that the sign of every cycle Z in S_1 equals to the sign of $\phi(Z)$ in S_2. The following result is well known (See [18]):

Proposition 1.2. (**T. Zaslavasky** [18]) *Two signed graphs S_1 and S_2 with the same underlying graph are switching equivalent if and only if they are cycle isomorphic.*

2. Common-edge Signed Graph of a Signed Graph

In [4], the authors define *path graphs* $P_k(G)$ of a given graph $G = (V, E)$ for any positive integer k as follows: $P_k(G)$ has for its vertex set the set $\mathcal{P}_k(G)$ of all distinct paths in G having k vertices, and two vertices in $\mathcal{P}_k(G)$ are adjacent if they represent two paths $P, Q \in \mathcal{P}_k(G)$ whose union forms either a path P_{k+1} or a cycle C_k in G.

Much earlier, the same observation as above on the formation of a line graph $L(G)$ of a given graph G, Kulli [9] had defined the *common-edge graph* $C_E(G)$ of G as the *intersection graph* of the family $\mathcal{P}_3(G)$ of 2-paths (i.e., paths of length two) each member of which is treated as a set of edges of corresponding 2-path; as shown by him, it is not difficult to see that $C_E(G) \cong L^2(G)$, for any isolate-free graph G, where $L(G) := L^1(G)$ and $L^t(G)$ denotes the t^{th} *iterated line graph* of G for any integer $t \geq 2$.

In this paper, we extend the notion of $C_E(G)$ to realm of signed graphs: Given a signed graph $S = (G, \sigma)$ its *common-edge signed graph* $C_E(S) = (C_E(G), \sigma')$ is that signed graph whose underlying graph is $C_E(G)$, the common-edge graph of G, where for any edge (e_1e_2, e_2e_3) in $C_E(S)$, $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$. This differs from the common-edge signed graph defined in [15].

Further a signed graph is a common-edge signed graph if there exists a signed graph S' such that $S \cong C_E(S')$.

Proposition 2.1. *For any signed graph $S = (G, \sigma)$, its common-edge signed graph $C_E(S)$ is balanced.*

Proof. Let $\sigma^{'}$ denote the signing of $C_E(S)$ and let the signing σ of S be treated as a marking of the vertices of $C_E(S)$. Then by definition of $C_E(S)$ we see that $\sigma'(e_1e_2, e_2e_3) = \sigma(e_1e_2)\sigma(e_2e_3)$, for every edge (e_1e_2, e_2e_3) of $C_E(S)$ and hence, by Proposition 1.1, the result follows. □

For any signed graph $S = (G, \sigma)$, its common edge signed graph is balanced. However the converse need not be true. The following result gives a sufficient condition for a signed graph to be a common-edge signed graphs.

Theorem 2.2. *A connected signed graph $S = (G, \sigma)$ is a common-edge signed graph if there exists a consistent marking μ of vertices of S such that for any edge uv, $\sigma(uv) = \mu(u)\mu(v)$ and its underlying graph G is a common-edge graph. Conversely if S is a common edge signed graph, then S is balanced.*

Proof. Suppose that there exists a consistent marking μ of vertices of S such that for any edge uv, $\sigma(uv) = \mu(u)\mu(v)$ and G is a common-edge graph. Then there exists a graph H such that $C_E(H) \cong G$. Now consider the signed graph $S' = (L(H), \sigma')$, where for any edge $e = (uv, vw)$ in $L(H)$, $\sigma'(e)$ is the marking of the corresponding vertex uvw in $C_E(H) = G$. Then S' is balanced since the edges in any cycle C of S' which corresponds to a cycle in S and the marking μ is a consistent marking. Thus S' is a line signed graph. That is there exists a signed graph S'' such that $S'' \cong L(S')$. Then clearly $C_E(S) \cong S''$.

Conversely, suppose that $S = (G, \sigma)$ is a common edge signed graph. That is there exists a signed graph $S' = (G', \sigma')$ such that $C_E(S) \cong S'$. Consider $L(S') = (L(G'), \sigma'')$ where $\sigma''(uv, vw) = \sigma(uv)\sigma(vw)$. Now consider the marking $\mu : V(G) \to \{+, -\}$ defined by $\mu(uvw) = \sigma''(uv, vw)$. Then by definition for any edge$e = (uvw, vwx)$ in S, where $uv, vw, wx \in E(G')$, $\sigma(e) = \sigma'(uv)\sigma'(wx) = \sigma'(uv)\sigma'(vw)\sigma'(vw)\sigma'(wx) = \sigma''(uv, vw)\sigma''(vw, wx) = \mu(uvw)\mu(vwx)$. Hence by Proposition 1.1, S is balanced. □

For any positive integer k, the k^{th} iterated common-edge signed graph, $C_E^k(S)$ of S is defined as follows:

$$C_E^0(S) = S,\ C_E^k(S) = C_E(C_E^{k-1}(S))$$

Corollary 2.3. *For any signed graph $S = (G, \sigma)$ and any positive integer k, $C_E^k(S)$ is balanced.*

In [15], the author characterized those graphs that are isomorphic to their corresponding common-edge graphs.

Proposition 2.4. (**D. Sinha** [15]) *For a simple connected graph $G = (V, E)$, $G \cong C_E(G)$ if and only if G is a cycle.*

We now characterize those signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 2.5. *For any signed graph $S = (G, \sigma)$, $S \sim C_E(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proof. Suppose $S \sim C_E(S)$. This implies, $G \cong C_E(G)$ and hence by Proposition 2.4, we see that the graph G is 2-regular. Now, if S is any signed graph with underlying graph as 2-regular, Proposition 2.1 implies that $C_E(S)$ is balanced and hence if S is unbalanced and its common-edge signed graph $C_E(S)$ being balanced can not be switching equivalent to S in accordance with Proposition 1.2. Therefore, S must be balanced.

Conversely, suppose that S balanced 2-regular signed graph. Then, since $C_E(S)$ is balanced as per Proposition 2.1 and since $G \cong C_E(G)$ by Proposition 2.4, the result follows from Proposition 1.2 again. □

Corollary 2.6. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $S \sim C_E^k(S)$ if and only if S is a balanced signed graph which is 2-regular.*

3. **Line Signed Graphs**

The *line graph* $L(G)$ of graph G has the edges of G as the vertices and two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The *line signed graph* of a signed graph $S = (G, \sigma)$ is a signed graph $L(S) = (L(G), \sigma')$, where for any edge ee' in $L(S)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by M. K. Gill [6] (See also E. Sampathkumar et al. [12, 13]).

Proposition 3.1. (**M. Acharya** [2]) *For any signed graph $S = (G, \sigma)$, its line signed graph $L(S)$ is balanced.*

For any positive integer k, the k^{th} iterated line signed graph, $L^k(S)$ of S is defined as follows:

$$L^0(S) = S,\ L^k(S) = L(L^{k-1}(S))$$

Corollary 3.2. (**P. Siva Kota Reddy & M. S. Subramanya** [16]) *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $L^k(S)$ is balanced.*

We now characterize those signed graphs that are switching equivalent to their line signed graphs.

Proposition 3.3. *For any signed graph $S = (G, \sigma)$, $S \sim L(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proof. Suppose $S \sim L(S)$. This implies, $G \cong L(G)$ and hence G is 2-regular. Now, if S is any signed graph with underlying graph as 2-regular, Proposition 3.1 implies that $L(S)$ is balanced and hence if S is unbalanced and its line signed graph $L(S)$ being balanced can not be switching equivalent to S in accordance with Proposition 1.2. Therefore, S must be balanced.

Conversely, suppose that S is balanced 2-regular signed graph. Then, since $L(S)$ is balanced as per Proposition 3.1 and since $G \cong L(G)$, the result follows from Proposition 1.2 again. □

Corollary 3.4. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $S \sim L^k(S)$ if and only if S is a balanced signed graph which is 2-regular.*

Proposition 3.5. (**D. Sinha** [15])
For a connected graph $G = (V, E)$, $L(G) \cong C_E(G)$ if and only if G is cycle or $K_{1,3}$.

Theorem 3.6. *For any graph G, $C_E(G) \cong L^k(G)$ for some $k \geq 3$, if and only if G is either a cycle or $K_{1,3}$.*

Proof. Suppose that $C_E(G) \cong L^k(G)$ for some $k \geq 3$. Since $C_E(G) \cong L^2(G)$, we observe that $L^k(G) = L^{k-2}(L^2(G)) = L^{k-2}(C_E(G))$ and so $C_E(G) \cong L^{k-2}(C_E(G))$. Hence, by Proposition 3.5, $C_E(G)$ must be a cycle. But for any graph G, $L(G)$ is a cycle if and only if G is either cycle or $K_{1,3}$. Since $K_{1,3}$ is a forbidden to line graph and $L(G)$ is a line graph, $G \neq K_{1,3}$. Hence $L(G)$ must be a cycle. Finally $L(G)$ is a cycle if and only if G is either a cycle or $K_{1,3}$.

Conversely, if G is a cycle C_r , of length r, $r \geq 3$ then for any $k \geq 2$, $L^k(G)$ is a cycle and if $G = K_{1,3}$ then for any $k \geq 2$, $L^k(G) = C_3$. Since $C_E(G) = L^2(G)$, $C_E(G) = L^k(G)$ for any $k \geq 3$. This completes the proof. □

We now characterize those line signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 3.7. *For any signed graph $S = (G, \sigma)$, $L(S) \sim C_E(S)$ if and only if G is a cycle or $K_{1,3}$.*

Proof. Suppose $L(S) \sim C_E(S)$. This implies, $L(G) \cong C_E(G)$ and hence by Proposition 3.5, we see that the graph G must be isomorphic to either 2-regular or $K_{1,3}$.

Conversely, suppose that G is a cycle or $K_{1,3}$. Then $L(G) \cong C_E(G)$ by Proposition 3.5. Now, if S any signed graph on any of these graphs, By Propositions 2.1 and 3.1, $C_E(S)$ and $L(S)$ are balanced and hence, the result follows from Proposition 1.2. □

Corollary 3.8. *For any signed graph $S = (G, \sigma)$ and for any integers $k \geq 3$, $C_E(S) \sim L^k(S)$ if and only if G is 2-regular.*

4. Jump Signed Graphs

The *jump graph* $J(G)$ of a graph $G = (V, E)$ is $\overline{L(G)}$, the complement of the line graph $L(G)$ of G (See [5] and [7]).The *jump signed graph* of a signed graph $S = (G, \sigma)$ is a signed graph $J(S) = (J(G), \sigma')$, where for any edge ee' in $J(S)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by M. Acharya and D. Sinha [3] (See also E. Sampathkumar et al. [11]).

Proposition 4.1. (**M. Acharya and D.Sinha** [3])
For any sigraph $S = (G, \sigma)$, its jump sigraph $J(S)$ is balanced.

For any positive integer k, the k^{th} iterated jump signed graph, $J^k(S)$ of S is defined as follows:

$$J^0(S) = S,\ J^k(S) = J(J^{k-1}(S))$$

Corollary 4.2. *For any signed graph $S = (G, \sigma)$ and for any positive integer k, $J^k(S)$ is balanced.*

In the case of graphs the following result is due to Simic [14] (see also [5]) where $H \circ K$ denotes the *corona* of graphs H and K [7].

Proposition 4.3. (**S. K. Simic** [14])
The jump graph $J(G)$ of a graph G is isomorphic with G if and only if G is either C_5 or $K_3 \circ K_1$.

Lemma 4.4. (Kulli [9]**)**
For a graph $G = (V, E)$ with n vertices and m edges, the number of vertices in $L^2(S)$ is $\sum_{u \in V} \binom{deg(v)}{2}$

Lemma 4.5. (D. Sinha [15]**)**
For any simple connected graph $G = (V, E)$ on $n \geq 2$ vertices,

$$|E(G)| = \sum_{v \in V} \binom{deg(v)}{2}$$

if and only if G is a cycle or a 3-spider.

Proposition 4.6. *For a connected graph $G = (V, E)$, $J(G) \cong C_E(G)$ if and only if G is C_5.*

Proof. Suppose that $J(G) \cong C_E(G)$. Then the number of vertices in $J(G)$ must be equal to the number of vertices in $C_E(G)$. By Lemma 4.4, the number of vertices in $C_E(G)$ is $\sum_{u \in V} \binom{deg(v)}{2}$. Now, since both $J(G)$ and $L(G)$ have same number of

vertices whence by Lemma 4.5, G must either be a cycle or a 3-spider.
We note that $L^2(G) \cong C_E(G)$ and $J(G) = \overline{L(G)}$. Hence $\overline{J(L(G))} \cong L(G)$. By Proposition 4.3, it follows that $L(G)$ is either C_5 or $K_3 o K_1$. Now, $L(G) \neq K_{1,3}$, since $K_{1,3}$ is not a line graph. Hence $G \cong C_5$. The converse is obvious. □

We now characterize those jump signed graphs that are switching equivalent to their common-edge signed graphs.

Proposition 4.7. *For any signed graph $S = (G, \sigma)$, $J(S) \sim C_E(S)$ if and only if $G \cong C_5$.*

Proof. Suppose $J(S) \sim C_E(S)$. This implies, $J(G) \cong C_E(G)$ and hence by Proposition 4.6, we see that $G \cong C_5$.

Conversely, suppose $G \cong C_5$. Then $J(G) \cong C_E(G)$ by Proposition 4.6. Now, if S is a signed graph with underlying graph as C_5, by Propositions 2.1 and 4.1, $C_E(S)$ and $J(S)$ are balanced and hence, the result follows from Proposition 1.2. □

The following result is a stronger form of the above result.

Theorem 4.8. *A connected graph satisfies $J(S) \cong C_E(S)$ if and only if G is C_5.*

Proof. Clearly $C_E(C_5) \cong J(C_5)$. Consider the map $f : V(C_E(G)) \rightarrow V(J(G))$ defined by $f(u_1u_2u_3, u_2u_3u_4) = (u_1u_2, u_3u_4)$ is an isomorphism. Let σ be any signing C_5 . Let $e = (v_1v_2v_3, v_2v_3v_4)$ be an edge in $C_E(C_5)$. Then sign of the edge e in $C_E(G)$ is the $\sigma(u_1u_2)\sigma(u_3u_4)$ which is the sign of the edge (u_1u_2, u_3u_4) in $J(C_5)$. Hence the map f is also a signed graph isomorphism between $J(S)$ and $C_E(S)$. □

Acknowledgement. The authors thank to the referee for his valuable suggestions. Also, the first author very much thankful to Sri. B. Premnath Reddy, Chairman, Acharya Institutes, for his constant support and encouragement for R & D.

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