Contents list available at IJRED website
Int. Journal of Renewable Energy Development (IJRED)
Journal homepage: http://ejournal.undip.ac.id/index.php/ijred

Optimum Tilt Angle at Tropical Region

S. Soulayman[a,*] and W. Sabbagh[b]

[a] *Higher Institute for Applied Sciences and Technology, Damascus P. O. Box 31983, Syria*
[b] *National Energy Research Center, Damascus, Syria*

ABSTRACT: One of the important parameters that affect the performance of a solar collector is its tilt angle with the horizon. This is because of the variation of tilt angle changes the amount of solar radiation reaching the collector surface. Meanwhile, is the rule of thumb, which says that solar collector Equator facing position is the best, is valid for tropical region? Thus, it is required to determine the optimum tilt as for Equator facing and for Pole oriented collectors. In addition, the question that may arise: how many times is reasonable for adjusting collector tilt angle for a definite value of surface azimuth angle? A mathematical model was used for estimating the solar radiation on a tilted surface, and to determine the optimum tilt angle and orientation (surface azimuth angle) for the solar collector at any latitude. This model was applied for determining optimum tilt angle and orientation in the tropical zones, on a daily basis, as well as for a specific period. The optimum angle was computed by searching for the values for which the radiation on the collector surface is a maximum for a particular day or a specific period. The results reveal that changing the tilt angle 12 times in a year (i.e. using the monthly optimum tilt angle) maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. This achieves a yearly gain in solar radiation of 11% to 18% more than the case of a solar collector fixed on a horizontal surface.

Keywords: Optimum tilt, orientation, mathematical model, tropical region

Article History : Received October 10,2014; Received in revised form January 6,2015; Accepted February 8, 2015; Available online

How to Cite This Article: Soulayman, S and Sabbagh, W (2015). Optimum Tilt Angle at Tropical Region. *Int. Journal of Renewable Energy Development*, 4(1), 48-54.
http://dx.doi.org/10.14710/ijred.4.1.48-54

1. Introduction

The performance of a solar collector is highly influenced by its orientation (with respect to the Equator) and its angle of tilt with the horizon (with respect to the ground). This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the surface of the collector. Taking into consideration that designing an installation to yield maximum annual energy helps to minimize the necessary installed capacity and reduce the cost of equipment. To achieve this, solar collector must be mounted at right angles to the sun's rays. Ideally this is achieved by mounting the collector on a two-axis tracker that continuously tracks the sun by the hour and through the seasons. In practice, however, the method is quite cumbersome and inconvenient. Thus, the majority of installations are with fixed mountings. Therefore, it is often practicable to orient the solar collector at an optimum tilt angle, Bopt and to correct the tilt from time to time. For this purpose, one should be able to determine the opt[*]imum slope of the collector at any latitude, for any surface azimuth angle, and on any day or any period of the year.

As the goal of this work is treat this question regarding the tropical region it is reasonable to restrict ourselves to main available literature concerning this zone directly or indirectly. In this context, Soulayman (1991) proposed a general algorithm for calculating B_{opt} for south facing collector. Soulayman and Sabbagh (2014) proposed an algorithm for determining B_{opt} at any latitude, *L*, and for any direction (surface azimuth angle, G). This algorithm could be used for treating B_{opt} in tropical region. Stanciu, and Stanciu (2014) proposed a simple formula for determining the optimum tilt of south facing collector at latitudes from 0º to 80º. Nijegorodov, Devan & Jain (1997) presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60° south to 60° north. Qiu and Riffat (2003) found the yearly optimum slope angle Bopt,y of solar collectors as $B_{opt,y}$= L ± 10 at a location with latitude of L where the plus sign is for the northern hemisphere and the minus sign is for the southern hemisphere.

[*] Corresponding author : soulayman1952@gmail.com

Aja, Al-Kayiem &. Karim (2013) proposed an analytical investigation on the optimum tilt angle for solar collectors at low latitude, a case study of PETRONAS (UTP), 4.39°N, Malaysia is presented in this work. Eke (2011) studied the optimum tilt angle in Zaria, 11.13° N, Nigeria. Waziri et al. (2014) determined the collector tilt angle and orientation for Kano, Nigeria (L= 12.1°N) experimentally. Diaz et al. (2014) determined Bopt experimentally in Philippine. Oko and Nnamchi (2012) studied theoretically the optimum tilt angle for low latitudes, 4.86- 13.02°N, spanning the territory of Nigeria and expressions for the optimum tilt angles with respect to the low latitudes were also provided. Ng et al. (2014) found that the optimum angles for seasonal south- and north-facing surfaces are 14.4° and 14.8°, respectively. Kamanga et al. (2014) mentioned that for Zomba district there are only two seasonal adjustments that are needed for PV solar panels and these should be carried out at the end of February and at the end of September. For fixed solar panels with no seasonal adjustments, the optimum tilt angle for the PV solar panels that are north facing has been determined to be 25°.

So, in the previous studies, no definite value or relation is accepted by all researchers for the optimum tilt angle. Therefore, several attempts were made to determine, or at least to estimate, optimum tilt angle Bopt theoretically and experimentally.

The main objective of this study is to develop a simple and easy way for finding daily, monthly, seasonally, half-yearly and fixed optimum tilt angles for any location in the tropical region and to determine the yearly energy gain. The results of this study will be compared with those available in literature.

2. Methodology

2.1 Solar collector orientation

The incidence angle θ of solar rays on a surface of tilt angle B and azimuth angle G, at location of latitude L and on a day of the year where solar declination is at solar hour angle ω could be determined as follows Duffie & Beckman (1991):

$$\cos(\theta) = \sin(\delta)\sin(L)\cos(B) - \sin(\delta)\cos(L)\sin(B)\cos(G) + \cos(\delta)\cos(L)\cos(B)\cos(\omega) + \cos(\delta)\sin(L)\sin(B)\cos(G)\cos(\omega) + \cos(\delta)\sin(B)\sin(G)\sin(\omega) \quad (1)$$

On Northern Hemisphere *L* is positive, *B* is positive when surface is oriented toward equator and negative for the opposite direction. On Southern Hemisphere *L* is negative, *B* is negative when surface is oriented toward equator and positive for the opposite direction.

Eq. (1) could be used to determine the period where solar collector should be oriented to Equator or to opposite direction. This may be done by studying the dependence of L-δ as function of day number in the year. In Northern Hemisphere, the solar collector should be oriented to equator when L-δ is positive and to opposite direction when it is negative. In Southern Hemisphere, the solar collector should be oriented to Equator when L-δ is negative and to opposite direction when it is positive. Table 1 shows the periods for different latitudes at Northern and Southern Hemispheres. Here it should be mentioned that at equator the solar collector should be oriented toward North Pole (NP) during half year and another half year should be oriented toward South Pole (SP).

Table 1 shows that for high latitudes, latitudes $L \geq 23.5°$ and $L \leq -23.5°$, solar collectors should be oriented toward Equator permanently while, for tropical zone, Equator direction and opposite direction should be used depending on the application period. Moreover, for any other latitude at tropical region there are two periods for solar collector orientation: one is laid between 22/9 and 21/3 while the other is laid between 22/3 and 21/9. The duration of each period depends on the latitude value. During one of these periods solar collector should be orientated toward Equator while during the other period solar collector should be orientated oppositely toward Pole.

Tabel 1.
Latitudes, corresponded duration (days) and required orientation.

L (°)	Equator facing	North Pole facing	South pole facing
23.5	[1-365]	-	-
20	[1- 140] + [204-365]	[141- 203]	-
15	[1-121] + [223-365]	[122 - 222]	-
10	[1-106] + [239-365]	[107 - 238]	-
5	[1- 93] + [251-365]	[94 - 250]	-
0	-	[81- 263]	[1- 80] + [264-365]
-5	[69 - 275]	-	[1-68] + [276-365]
-10	[56 - 288]	-	[1-55] + [289-365]
-15	[41 - 304]	-	[1-40] + [305-365]
-20	[22 - 322]	-	[1-21] + [323-365]
-23.5	[1 - 365]	-	-

2.2 Solar collector optimum tilt

Soulayman & Sabbagh (2014) found that the optimum tilt angle of a solar collector of any orientation and for any period of time could be obtained by solving the nonlinear algebraic equation:

$$\sum C(N)\left\{\left(\frac{\partial A2}{\partial B}\right)[\sin(Wss) - \sin(Wsr)] + A2\cos Wss\partial Wss\partial B - \cos Wsr\partial Wsr\partial B + \partial A1\partial BWss - Wsr + A1\partial Wss\partial B - \partial Wsr\partial B - \partial A3\partial B\cos Wss - \cos Wsr + A3\sin Wss\partial Wss\partial B - \sin Wsr\partial Wsr\partial B = 0 \right. \quad (2)$$

in relation to *B* where the summation should be ignored when dealing with daily period. In Eq. (2):

- Wss (rad) is the sunset hour angle on tilted surface:

$$Wss = min\left\{arccos[-tan(\delta)\,tan(L)], arccos\left(-\frac{A1}{A4}\right) + arcsinA3A4\right\} \quad (3)$$

- Wsr (rad) is the sunrise hour angle on tilted surface:

$$Wsr = max\left\{-arccos[-tan(\delta)\,tan(L)], -arccos\left(-\frac{A1}{A4}\right) + arcsinA3A4\right. \quad (4)$$

- C(N) is the daily correction factor for Sun-Earth average distance:

$$C(N) = 1 + 0.034\cos\left(\frac{2\pi N}{365}\right) \quad (5)$$

- δ (°) is the solar declination angle which could calculated as below:

$$\delta = -23.45cos\left[\frac{2\pi(N+10.5)}{365}\right] \quad (6)$$

A1, A2, A3 and A4 are functions of solar and collector angles:

$$A1 = \sin(\delta)\,[\sin(L)\cos(B) - \sin(B)\,cos(L)\cos(G)] \quad (7)$$

$$A2 = cos(\delta)\,[cos(L)\,cos(B) + sin(B)\,sin(L)\,cos(G)] \quad (8)$$

$$A3 = cos(\delta)\,sin(B)\,sin(G)\,; \quad A4 = (A2^2 + A3^2)½ \quad (9)$$

Here it should be mentioned that, for south facing collectors in Northern Hemisphere, W_{ss} = -W_{sr} , G=0 and W_{ss} is B independent only during the period started from September 22nd to March 21st, and the monthly $B_{opt,m}$ solution of Eq. (2) becomes:

$$Bopt,m = L - arctan[\sum C(N)Wss\;sin(\delta\delta)/\sum C(N)\;cos(\delta\delta)sin(Wss)] \quad (10)$$

The daily solution $B_{opt,d}$ of Eq. (2) takes the form:

$$Bopt,d = L - arctan[Wss\;sin(\delta\delta)/cos(\delta\delta)sin(Wss)] \quad (11)$$

So, El-Kassaby (1989), El-Kassaby and Hassab (1994) and Skeiker (2009) made a fatal error when mentioning that Eq. (10) is applicable all over any period of time and Eq. (11) is applicable all over the year. Moreover, all works, based on Eqs (11) & (12) suffer of the same uncertainty (see for example Diaz et al. (2014)).

3. Results and Discussion

3.1 Daily collector optimum tilt

Recently Stanciu & Stanciu (2014) (SS) found that when applying Hottel & Woertz model for estimating the incident solar radiation, for flat plate collector at latitudes from 0° to 80° the daily optimum tilt angle $B_{opt,d}$ should be computed as simples as $B_{opt,d} = L - \delta$ function on the latitude and declination. Therefore, the first step of this work is to shed a light on the real applicability of this proposal. When calculating $B_{opt,d}$ for different latitudes of tropical region using Eq. (2) one obtains the results given in Table 2 with noting that negative $B_{opt,d}$ values for positive L mean that solar collector should be oriented toward the NP while positive $B_{opt,d}$ values for negative L mean that solar collector should be oriented toward the SP during these days.

Tabel 2

Daily optimum tilt angle $B_{opt,d}$ (°) for tropical zone latitudes.

δ(°)	Date	Latitude *L* in degrees (°)				
		0	5	-5	10	-10
23.0	1/1	33.7	38.1	29.3	42.5	25
21.2	15/1	31.4	35.8	27	40.3	22.5
17. 5	1/2	26.6	30.9	21.7	35.5	17.1
13.2	15/2	20.2	25	15.5	29.8	10.7
8.2	1/3	12.8	17.7	7.9	22.8	0
2.7	15/3	4.3	9.3	-0.7	14.2	-5.7
-4.1	1/4	-6.4	0	-11.4	3.5	-16.4
-9.5	15/4	-14.7	-9.9	-19.6	0	-24.5
-15.0	1/5	-22.8	-18.1	-27.5	-13.4	-32.2
-18.8	15/5	-28.2	-23.7	-32.8	-19.2	-37.3
-22.1	1/6	-32.5	-28.1	-36.9	-23.7	-41.4
-23.3	15/6	-34.1	-29.8	-38.5	-25.5	-42.9
-23.1	1/7	-33.8	-29.5	-38.2	-25.1	-42.6
-21.5	15/7	-31.7	-27.3	-36.2	-22.9	-40.6
-17.9	1/8	-26.8	-22.3	-31.4	-17.7	-36.0
-13.7	15/8	-21.0	-16.2	-25.7	-11.5	-30.4
-7. 7	1/9	-11.9	-7.0	-16.8	0	-21.7
-2. 2	15/9	-3.3	1.7	-8.3	6.7	-13.3
4.3	1/10	6.8	11.7	0	16.7	-3.2
9.6	15/10	15.2	19.9	10.2	24.7	0
15.4	1/11	23.5	28.1	18.8	32.8	14.1
19.2	15/11	28.7	33.2	24.2	37.8	19.7
22.1	1/12	32.6	37.0	28.2	41.4	23.8
23.3	15/12	34.1	38.5	29.8	42.9	25.5

Date	Latitude *L* in degrees (°)					
	15	-15	20	-20	23.5	-23.5
1/1	46.9	20.7	51.3	15.5	54.4	0
15/1	44.8	18.2	49.3	13.9	52.4	0
1/2	40.1	12.6	44.7	0	48.0	0
15/2	34.5	0	39.3	0	42.6	-2.1
1/3	27.5	-2.0	32.4	-7.0	35.8	-10.5
15/3	19.2	-10.7	24.2	-15.7	27.7	-19.2
1/4	8.5	-21.4	13.4	-26.3	17.0	-29.8
15/4	0	-29.3	4.8	-34.2	8.4	-37.6
1/5	0	-36.9	0	-41.6	0	-44.9
15/5	-14.7	-41.9	0	-46.5	0	-49.6
1/6	-19.4	-45.8	-15.2	-50.3	0	-53.4
15/6	-21.2	-47.3	-17.0	-51.7	0	-54.7
1/7	-20.9	-47.0	-16.7	-51.4	0	-54.5
15/7	-18.6	-45.1	-14.3	-49.6	0	-52.7
1/8	-13.2	-40.6	0	-45.2	0	-48.4
15/8	0	-35.2	0	-39.9	1.3	-43.3
1/9	2.9	-26.6	7.9	-31.5	11.4	-35.0
15/9	11.7	-18.3	16.7	-23.3	20.2	-26.8
1/10	21.7	-8.2	26.6	-13.2	30.1	-16.7
15/10	29.6	0	34.5	-4.6	37.9	-8.1
1/11	37.5	9.5	42.2	0	45.5	0
15/11	42.3	15.2	46.9	0	50.1	0
1/12	45.9	19.5	50. 3	15.3	53.4	0
15/12	47.3	21.2	51.7	17.1	54.8	0

According to SS, $B_{opt,d} = L+\delta$. For Equator L=0°. Then $B_{opt,d}=\delta$. So, it is seen from Table 2 that, a considerable deviation between the results of this work and those of SS. The same conclusion could be given regarding other latitude values. In relation to results of this work, the formula, proposed by SS, underestimates $B_{opt,d}$ for period started from 22/9 to 21/3 and overestimates $B_{opt,d}$ for period started from 22/3 to 21/9. Moreover, in addition to the low effectiveness of SS in predicting $B_{opt,d}$ the applicability of SS proposal is discountable on latitudes higher than 66.5°.

On the other hand, Bari (2001) proposed a polynomial equation of six order for calculating $B_{opt,d}$ for different latitudes of the Philippines where the constants of this equation are latitude dependent. When applying this equation for L= 5° and L=15° and tracing the obtained results and those of this work one obtains the Fig. 1 from which it is seen that even Bari (2001) equation a little bit overestimates $B_{opt,d}$ during summer and underestimates $B_{opt,d}$ during winter a good agreement is achieved between these applied methods.

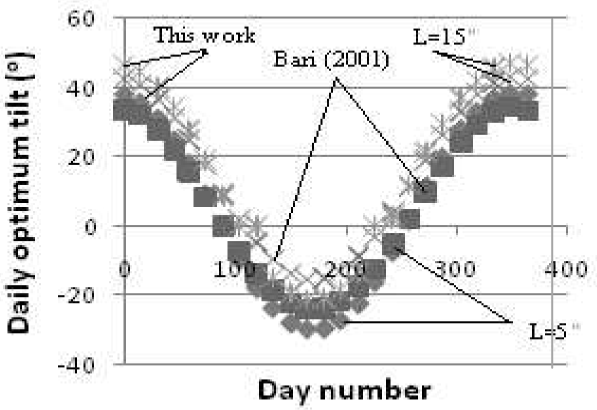

Fig. 1 $B_{opt,d}$ values using **Bari (2001)** and this work methods.

3.2 Collector optimum tilt over a period

The set of equations (2)–(9) is used to obtain the optimum tilt angles B_{opt} over different periods in the tropical zone. At any latitude of this zone, one can distinguish two main periods of solar collector orientation: one corresponds to Equator facing period and the other corresponds to Pole facing period. $B_{opt,p}$ values over these periods are presented in Table 3. Here it should be mentioned that period described as [1-*N1*] + [*N2* – 365] should be read as one period started on day number *N2* and finished on day number *N1* inclusively and B_{opt} values, given in Table 3, are consistent with this understanding. So, these periods could be nominated as characteristic. As tropical region lays on Northern and Southern Hemispheres, it is easy to distinguish for characteristic periods in this region. The calculated optimum tilt angles over these characteristic periods are given on Fig. 2. When trying to divide the characteristic periods regarding calendar months it was found that, parts of the calendar months at the beginning and the end of each characteristic period are included (see Tables 3-5).

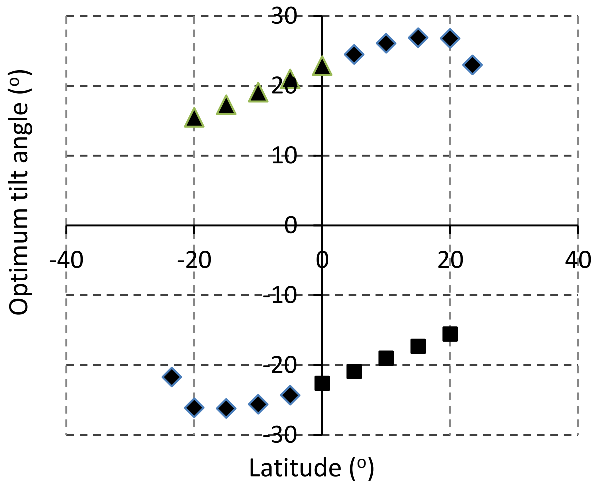

Fig. 2 B_{opt} over characteristic periods (♦ Equator facing, ■NP facing, ▲ SP facing).

So, when calculating the $B_{opt,m}$ for Equator facing, NP facing and SP facing solar collectors, the results, given in Tables 3-5, are found. Values, given in brackets in Tables 3-5, indicate the number of days which corresponds the mentioned orientation. $B_{opt,m}$ for incomplete calendar months represents optimum tilt for the days in brackets.

Tabel 3
Monthly optimum tilt angle for Equator facing collectors.

L (°)	Jan.	Feb.	March	April	May	June
23.5	51.9	42.6	27.0	8.2	0	0
20	48.7	39.3	23.5	4.7	0(20)	-
15	44.3	34.6	18.5	0	-	-
10	39.8	29.8	13.6	0(16)	-	-
5	35.3	25.0	7.7	7(3)	-	-
-5	-	-	-4.2(22)	-19.7	-32.8	-38.2
-10	-	0(4)	-6.4	-24.5	-37.3	-42.6
-15	-	0(19)	-11.4	-29.4	-41.9	-47.0
-20	0(10)	0	-16.3	-34.2	-46.4	-51.4
-23.5	0	-2.0	-19.8	-37.6	-49.6	-54.5

L (°)	July	Aug.	Sept.	Oct	Nov	Dec
23.5	0	2.1	20.5	38.2	49.9	54.5
20	0(9)	0	17.0	34.8	46.8	51.4
15	-	0(21)	12.0	29.9	42.2	47.0
10	-	0(5)	7.0	25.1	37.7	42.6
5	-	-	4.2(23)	20.3	33.1	38.2
-5	-35.6	-25.0	-7.3	- (1)	-	-
-10	-40.1	-29.7	-12.9	0(15)	-	-
-15	-44.6	-34.5	-17.9	0	-	-
-20	-49.1	-39.2	-22.9	-4.1	0(18)	-
-23.5	-52.2	-42.5	-26.3	-7.6	0	0

Tables 3-5 show that the terminology of monthly optimum tilt angle $B_{opt,m}$ in the tropical region is meaningless for incomplete calendar months as parts of these months referred to opposite directions. However, it is possible to distribute each characteristic period into some approximate months with guarding the calendar names.

Tabel 4
Monthly optimum tilt angle -$B_{opt,m}$(°) for NP facing collectors.

L (°)	March	April	May	June	July	Aug.	Sept.
20	-	-	13.7 (11)	16.7	14.9 (22)	-	-
15	-	-	14.9 (30)	20.9	18.0	11.3 (10)	-
10	-	-9.7 (14)	19.2	25.1	22.3	12.1 (26)	-
5	-	10.8 (27)	23.7	29.4	26.7	15.5	5.2 (7)
0	3(10)	14.8	28.2	33.8	31.2	20.2	6.1 (20)

Table 5
Monthly optimum tilt angle $B_{opt,m}$(°) for SP facing collectors.

L (°)	Jan.	Feb.	March	Sept	Oct	Nov	Dec
0	30.8	20.3	6.7 (21)	3.3 (10)	15.4	28.6	33.8
-5	26.4	15.6	5.5(9)	-	11.1	24.0	29.5
-10	22.0	11.9 (24)	-	9.9 (26)	0 (15)	19.5	25.2
-15	17.6	10.8 (9)	-	-	9.9 (26)	15.1	20.9
-20	14.7 (21)	-	-	-	-	13.7 (12)	16.7

3.3 Comparison with published results

Wasiri et al. (2014) stated that, for Kano, Nigeria L=12.1° N, $B_{opt,m}$ for months of November to April were found to be 35°, 45°, 35°, 25°, 15° and 15° respectively. These results, except April, coincide well with the results of this work (39.6°, 44.5°, 41.7°, 31.8°, 15.7° and 0°(22) -9.2° (8)). Eke (2011) found for Zaria, Kaduna State, Nigeria 11.13° N that $B_{opt,m}$ values are 26.5, 24.5, 10.0, 19.5, 26.0, 30.0, 24.0, 21.0, 11.5, 19.5, 27.0 and 30.0°, in the months of January to December, respectively which means that his results are approximate regarding those of Wasiri et al. (2014) and of this work and the sign from May to August should be negative indicating that solar collector is orientated to the North during this period. Uba & Sarsah (2013) found, for WA, Ghana 10.01° N, that collector tilt should be changed three times a year: January – March (26.4°), April – August (29.7°) and September – December (25.9°). When calculating B_{opt} for theses periods using the proposed methodology of this work one obtains: January – March (27.7°), April – August (-17.68°) and September – December (28.1°). So, the results of this work are in a good agreement with those of Uba & Sarsah (2013) for the first and third periods while the result of Uba & Sarsah (2013) should be corrected for the second period because 1) the solar collector should be orientated to the North during April – August period and 2) the proposed value is too high. Ng et al. (2014) found, for Bangi, Malaysia (L = 3° N), that $B_{opt,m}$ = 22, 16, 5, -8, -18, -22, -21, -11, 0, 11, 19, 24 for January to December respectively. When calculating $B_{opt,m}$ for theses months using the proposed methodology of this work one obtains that $B_{opt,m}$ = 33.5°, 23.1°, 7.2°, -11.9°, -25.5°, -31.2°, -28.5°, -17.4°, -5.6°(12) 3.8° (18), 18.3°, 31.3°, 36.5° for January to December respectively. So, even Ng et al. (2014) determined orientation correctly they underestimated $B_{opt,m}$ for October to March and overestimated $B_{opt,m}$ for April to September.

Finally, Bari (2001) proposed a polynomial equation of six order with latitude dependent constants for calculating B_{opt} for any period of time and for different latitudes of the Philippines. As Bari (2001) obtained this equation by integrating the proposed by himself equation for calculating the daily optimum tilt over the required period, the constants of daily optimum tilt equation for a definite latitude should not be changed in contrast to his proposal. Moreover, when applying the proposed by him equation for calculating the $B_{opt,m}$ for different latitudes of the Philippines the obtained results are not adequate. In addition, Bari (2001) gave two examples for calculating B_{opt} for periods (February 10 to March 10 at L=16° and December 10 to January 20 at L=13°). The given values of B_{opt} are 27° and 41° respectively. When comparing these values with those of this work 30.9º and 44.7º, a good agreement is observed.

Tabel 6
Results of different approaches regarding $B_{opt,m}$ calculation.

Month	L=0 (N)	(I)	(O)	This work
Jan.	29	25	3.17	30.8
Feb.	17	15	0.04	20.3
March	4	0	2.86	6.7 (21)-3(10)
April	-10	0	5.10	-14.8
May	- 24	-25	3.20	-28.2
June	- 34	-25	1.36	-33.8
July	- 30	-25	0.80	-31.2
Aug.	- 17	-15	2.47	-20.2
Sep.	- 2	15	8.18	-6.1 (20),3.3(10)
Oct.	12	15	2.29	15.4
Nov.	25	25	0.82	28.6
Dec.	34	25	2.26	33.8
	L=10			
Month	(N)	(I)	(O)	This work
Jan.	37.9	35	17.4	39.8
Feb.	26.7	25	11.2	29.8
March	14	10	12.3	13.6
April	0	10	17.7	0(16),-9.7 (14)
May	-14.7	-15	17.5	-19.2
June	-25.3	-15	13.7	-25.1
July	-21.1	-15	8.9	-22.3
Aug.	-7.3	-5	11.4	-12.1 (26), 0(5)
Sep.	8	25	25.6	7.0
Oct.	22	25	15.7	25.1
Nov.	34.3	35	9.7	37.7
Dec.	42.7	35	11.1	42.6

Nijegorodov,Devan & Jain (1997) (N), Idowu, Olarenwaju & Ifedayo (2013) (I) and Oko & Nnamchi (2012) (O) proposed sets of equations for calculating $B_{opt,m}$. When comparing their results at different latitudes (see Table 6) it was found that the results of (O) set of equations are not compatible with other results and they could not be justified. The results of (I) set of equations are too approximate in relation to those of this work and (N). A relatively very good agreement is observed between the results of this work and those of (N). Here it should be mentioned that, for each of the studied latitudes, this work gives two values of different signs for optimum tilt angle for two months (see Table 6). For L=0° these months are March and September while for L=10° these months are April and August. In addition, each tilt angle value is accompanied by a number in brackets. This should be read as follows: first value relates to the number of days in brackets for which the orientation of solar collector coincides with the before coming period while the second value relates to the number of days in brackets for which the orientation of solar collector coincides with the after coming period.

3.3 Tilt factor

The geometric tilt factor Rb, the ratio of beam radiation on the tilted surface to that on a horizontal surface at any time or period of time, can be calculated exactly in the case of extraterrestrial radiation by appropriate use of solar incidence angle on tilted surface and on horizon. For daily, monthly, seasonally and yearly values of Rb, the following equation can be used:

$$Rb = \sum H(N,L,B,G) / \sum H(N,L,B=0,G) \qquad \textbf{(12)}$$

where solar radiation is given by:

$$H(N,L,B,G) = \left(12 \times \frac{3600Gsc}{\pi}\right) C(N) \times \{A2[\sin(Wss) - \sin Wsr + \ A1 Wss - Wsr - A3 \cos Wss - \cos Wsr \ Jm2 \qquad \textbf{(13)}$$

and summation by N should cover the length of period in consideration. For daily values no summation by N is used.

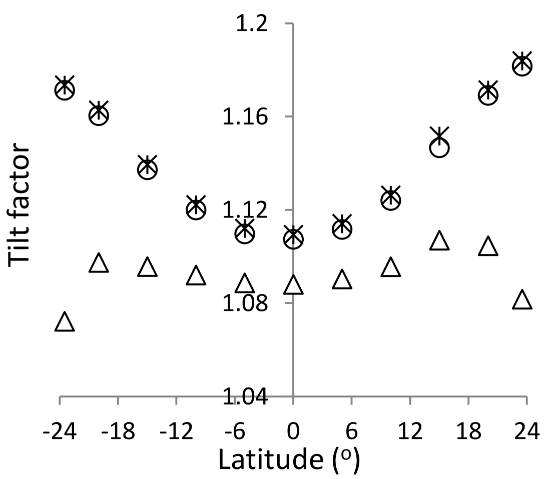

Fig. 3 Solar radiation yearly gain on basis of $B_{opt,d}$ (ж), $B_{opt,m}$(O) and $B_{opt,p}$(Δ).

When calculating the total yearly extraterrestrial solar radiation at B = 0, B = B_{opt} on a daily as well as on a monthly and periodic basis and taking the ratio between the value on a tilted surface to the value on a horizontal one for the same period of time, the corresponded tilt factor is calculated for each case. In the case of yearly evaluation of tilt, the results are plotted in Fig. 3 where it is seen that a yearly gain in solar radiation of 11% (for L=0°) to 18% (for L=23.5°) more than the case of a solar collector fixed on a horizontal surface is achieved.

It is seen form Fig. 3 that from solar radiation gain of view it is sufficient to orientate solar collector monthly as no practical losses between daily and monthly orientation.

4. Conclusions

Finally one can conclude that for tropical region:

- The characteristic periods during which solar collector should be oriented toward Equator, North Pole (NP) or South Pole (SP) are determined precisely.
- $B_{opt,d}$, $B_{opt,m}$, $B_{opt,p}$ are determined precisely on the base of extraterrestrial solar radiation.
- Changing the tilt angle 12 times a year (i.e. using $B_{opt,m}$) maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value.
- Using $B_{opt,m}$ achieves a yearly gain in solar radiation of 11% (for L=0°) to 18% (for L=23.5°) (depending on latitude angle) more than the case of a fixed solar collector at a horizontal surface. For intermediate latitudes the yearly solar radiation gain is between these two values.
- A good agreement is achieved between the results of this work and those of Bari (2001) regarding daily optimum tilt angle.
- A good agreement is achieved between the results of this work and those of Nijegorodov, Devan & Jain (1997) regarding monthly optimum tilt angle.

Acknowledgements

One of the authors (W. S) would like to acknowledge Dr. Y. Ali, the Director of National Energy Research Center, for valuable assistance.

References

Aja, O. C., Al-Kayiem, H. H., Abdul Karim, Z. A. (2013) Analytical investigation of collector optimum tilt angle at low latitude. AIP J. Renewable Sustainable Energy, 5, 063112; http://dx.doi.org/10.1063/1.4829434

Bari, S. (2001) Seasonal orientation of solar collectors for the Philippines. ASEAN Journal on Science and Technology for Development, 18 (1): 45-54.
Diaz, F., Ngo, J., Pascual, M., Planes, A., Chua, A. (2014) Enhanced energy conversion performance of Philippine photovoltaic panels through tilt angle adjustments: A mechatronics approach. Philippine Science Letters, 7(1): 7-12.
Duffie, J.A., Beckman, W.A. (1991) Solar Engineering of Thermal Processes, 2nd edn. Wiley, New York.
Eke, A. B. (2011) Prediction of optimum angle of inclination for flat plate solar collector in Zaria, Nigeria. Agricultural Engineering International: CIGR Journal, 13, Manuscript No. 1840.
El-Kassaby, M.M. (1988) Monthly and daily optimum tilt angle for south facing solar collectors; theoretical model, experimental and empirical correlations. Solar and Wind Technology5(6),589–96. http://dx.doi.org/10.1016/0741-983X(88)90054-9
El-Kassaby, M.M., Hassab, M.H. (1994) Investigation of a variable tilt angle Australian type solar collector. Renewable Energy 4, 327–332. http://dx.doi.org/10.1016/0960-1481(94)90036-1
Idowu, O. S., Olarenwaju, O. M., Ifedayo Odesola, I. (2013) Determination of optimum tilt angles for solar collectors in low-latitude tropical region. International Journal of Energy and Environmental Engineering , 4:29, 1-10.
Kamanga, B., Mlatho, J. S. P., Mikeka C., Kamunda, C. (2014) Optimum Tilt Angle for Photovoltaic Solar Panels in Zomba District, Malawi. Journal of Solar Energy, 2014, Article ID 132950, 9 pages. http://dx.doi.org/10.1155/2014/132950
Ng, Kh. M., Adam, N. M., Inayatullah, O., Ab Kadir, M. Z. A. (2014) Assessment of solar radiation on diversely oriented surfaces and optimum tilts for solar absorbers in Malaysian tropical latitude. International Journal of Energy and Environmental Engineering, 5:5, 1-13.
Nijegorodov,N., Devan, K.R.S., Jain, P.K. (1997) Atmospheric transmittance models and an analytical method to predict the optimum slope on an absorber plate. Renewable Energy, 4, 525–530. http://dx.doi.org/10.1016/0960-1481(94)90215-1
Oko, C.O.C., Nnamchi, S.N. (2012) Optimum Collector Tilt Angles For Low Latitudes, The Open Renewable Energy Journal, 5, 7-14.
Qiu, G., Riffat, S.B. (2003) Optimum tilt angle of solar collectors and its impact on performance. International Journal of Ambient Energy, 24, 13–20. http://dx.doi.org/10.1080/01430750.2003.9674898
Skeiker, K. (2009) Optimum tilt angle and orientation for solar collectors in Syria. Energy Conversion and Management 50, 2439–2448. http://dx.doi.org/10.1016/j.enconman.2009.05.031
Soulayman, S. (1991) On the optimum tilt of solar absorber plates. Renewable Energy, 1(3-4), 551-554. http://dx.doi.org/10.1016/0960-1481(91)90070-6.
Soulayman, S., Sabbagh, W. (2014) Comment on 'Optimum tilt angle and orientation for solar collectors in Syria' by Skeiker, K. Energy Conversion and Management, Available online 1 November 2014. doi:10.1016/j.enconman.2014.10.023
Stanciu, C., Stanciu, D. (2014) Optimum tilt angle for flat plate collectors all over the World – A declination dependence formula and comparisons of three solar radiation models. Energy Conversion and Management 81, 133–143. doi:10.1016/j.enconman.2014.02.016
Uba, F. A., Sarsah, E. A. (2013) Optimization of tilt angle for solar collectors in WA, Ghana. Pelagia Research Library, Advances in Applied Science Research, 4(4):108-114.
Waziri, N. H., Usman, A. M., Enaburekhan, Babakano,J. S. A. (2014) Determination of Optimum Tilt Angle and Orientation of a Flat Plate Solar Collector for Different Periods in Kano. Scirj, 2, 2014 Edition, ISSN: 2201-2796.