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Prospective Teachers Proportional Reasoning and Presumption of Student Work

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Abstract

This study aimed to describe the proportional reasoning of prospective teachers and their predictions about students' answers. Subjects were 4 prospective teachers 7th semester Department of Mathematics Education, Muhammadiyah University of Purworejo. Proportional reasoning task used to obtain research data. Subjects were asked to explain their reasoning and write predictions of student completion. Data was taken on October 15th, 2014. Interviews were conducted after the subjects completed the task and recorded with audio media. The research data were subject written work and interview transcripts. Data is analysed using qualitative analysis techniques. In solving proportional reasoning task, subjects using the cross product. However, they understand the meaning of the cross product. Subject also could predict students' reasoning on the matter.

Keywords*: proportional reasoning, prospective teacher*

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Introduction

Mathematical thinking is important in education and also in their daily lives. This is indicated by Stacey (2007) who states that the ability to think mathematically and using mathematical reasoning to solve problems is an important goal of education. One of mathematical thinking is proportional reasoning. According Sriraman & Lesh (2006) there are three types of basic mathematical thought most useful relevant to the everyday world, namely: 1) proportional thinking; 2) estimation; and 3) mathematical modeling activities in line with the development of the concept of proportional thinking. Seem that third mathematical thinking is also closely related to proportional thinking.

Proportional reasoning is one of the most fundamental topics in mathematics middle class. According to Ellis (2013) students' ability to think proportionally affect their understanding of fractions and measurement in elementary school, and to support their understanding of the functions and algebra in high school and beyond. In the initial learning of mathematics, students use proportional reasoning when they think about the 8 as two four-four or four two-two rather than thinking of it as one of the more than seven. Similarly, when they think about how the speed of 50 km / h is equal to the speed of 25 km / 30 min. Students continue to use proportional reasoning when they think about the slope of the line and the rate of change.

The essence of proportional reasoning is an understanding of numbers in the form of relative instead of absolute form. Students use proportional reasoning when they decided that a group of 3 children grown to 9 children are more significant changes compared to the group of 100 children grown to 150. In the first case, the number of children increases tripled, whereas in the second case only grown 50%, even not up to double. From here reasoner viewed as one, relative reasoner or absolute reasoner. Absolute reasoners view the situation in the understanding of additives. Additive reasoner illustrates the change from 2 to 10 as the addition of 8. Relative reasoner views the situation in multiplicative understanding. They looked at changes from 2 to 10 as multiplying by 5.

Proportional reasoning involves the careful use of multiplication relationships to compare the number and to predict the value of a quantity based on other values. That is, proportional reasoning is about the number reasoning more than formal procedural to solving proportion. In resolving proportion problem, teachers often explain the cross-product procedures. This procedure has the advantage of efficient and widely applicable in all contexts and domains. However, research has shown that students do not easily learn the cross product algorithm, or they refuse to use it when they do (Lamon, 1993; Kaput & West, 1994). This is probably due to the difficulty of connecting cross multiplication algorithm with their previous understanding of the ratio (Smith, 2002). This procedure does not match the mental operations involved in building strategies and less meaningful in certain situations.

Ellis (2013) pointed out, the label on the box of cookies say that calories per serving 210 calories. One serving contains 3 cakes. How many calories in 5 cakes? It is possible to adjust the proportions to solve this problem:

$$\frac{210\ calori}{3\ calori} = \frac{x\ cookies}{5\ cookies}$$

When cross multiplying, we will get the equation 3x = 210 × 5. The unit for 210 × 5 is not calories per cookies, and calorie-cookies is not meaningful in the context of the problem. If the ratio is not yet formed mentally, either as a multiplication ratio or as a unit composed, students may not understand what is represented in the proportion of cross-product procedures. In contrast, a student may potentially interpret the proportion simply as a template for inserting whole numbers into boxes (Lobato & Ellis, 2010). Researchers have found that students often engage in more sophisticated reasoning when not using the algorithm of proportion, and that the algorithm can obscure or even disrupt the students' understanding of proportionality (Lamon, 2007; Singh, 2000).

Why prospective teachers? Prospective teacher knowledge about the content is very necessary when they enter the world of education in primary and secondary schools. An understanding of the reasoning and the content will help them prepare for learning. The literature showed that prospective teacher’s understanding is problematic (Zevenbergen, 2005). This is in line with the findings of Burgess (2000) that prospective teacher understanding same with students understanding on the material probability, where prospective teachers and students showed some common misconceptions. Burgess (2001) stated that the level of understanding of teachers in relation to effective teaching was critical. Livy and Vale (2011) stated that most of the prospective first-year teachers lack knowledge of standard measures and methods of settlement of proportion. They have difficulty interpreting worded multi-step problem, the question of the ratio (scale), the error associated with the knowledge of or conversion ratios and measurements.

From this background, i intend to investigate prospective teachers proportional reasoning as well as how their predictions about students' reasoning.

Method

This is a qualitative exploratory study. The subjects were four 7th semester prospective teacher of Mathematics Education Department, Muhammadiyah University of Purworejo. Subjects consist of 1 male FA and 3 females are namely AF, EM, and ZNH. They are willing to take the time to complete the task and willing to do an interview. Tasks were such as contextual problem of proportion. Subjects were asked to explain their completion by using descriptions, images or table to show their reasoning. Furthermore, subjects were asked to think about and write down the possible completion by the student. In addition, they were asked to solve problems by using the proportion of the table. Data were collected on October 15, 2014. Interviews were conducted after they had completed the task and recorded with audio media. The analysis was performed by examining the written work and the transcript of the interview. Data were analyzed using qualitative analysis techniques.

Results and Discussion

First Task

The first task is the comparison problem of the leaves eaten by caterpillars. The problem is taken from the National Center for Education Statistics, National Assessment of Education Progress (NAEP) http://nces.ed.gov/nationalreportcard/itmrls/startsearch.asp.

> *Fourth grade students require 5 leaves every day to feed their two caterpillars. How many leaves they need every day for 12 caterpillars? Use pictures, words, or numbers to indicate how you obtain your answer*

Subjects AF answer using the comparison by letting unknown elements with a variable. Then he uses the cross multiplication to find the value of the variable. Although AF was using the cross product, he is able to interpret and provide reasoning on such comparisons. This means that he is not just using the cross multiplication but also to understand its meaning. Other subjects answered with the help of pictures and can explain that proportional reasoning they used.

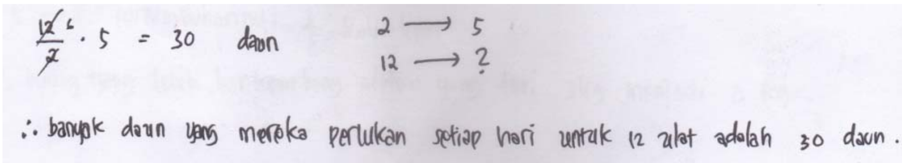


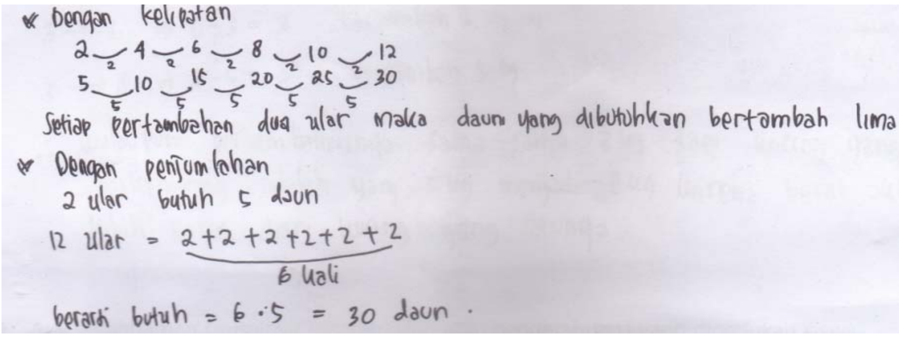
Figure 1. Subject AF Completion for T1

AF outlining its reasoning as follows:

> Because I already know the comparison, then i use the comparison. Here... known ... a ... for 2 caterpillars they need 5 leaves. What asked is for 12 caterpillars. So, the asked later as the numerator, than ... as it is equally caterpillar.. is denominator. So if the numerator was the caterpillar means as the denominator also caterpillar. Twelve caterpillars per 2 multiplied by the number ... if two caterpillar took how many leaves? ... multiplied by 5 to 30 leaves.

Subject AF was using additive reasoning in describing his reasoning about the proportion and his presumption towards students reasoning. AF presume that students answered using multiples and summation.

> We have, two caterpillars doubled-doubled. Suppose 2 to 4 it ... doubl ... plus 2. Later, 4 to 6 (with) plus 2 plus 2 again. Then up to 12 ... 12 caterpillars. Then the leaves ... if 5 ... so later 5 plus ... if 4 caterpillar means 5 plus 5. Then if 15 so ... 5 ... er ... if 6 caterpillar means 5 plus 5 plus 5, and so on until 12 caterpillar. So it is 5 ... uh ... that leaves 5 times 6. The leaves are 30.



- Dengan kelipatan

$2 \underset{2}{\smile} 4 \underset{2}{\smile} 6 \underset{2}{\smile} 8 \underset{2}{\smile} 10 \underset{2}{\smile} 12$

$5 \underset{5}{\smile} 10 \underset{5}{\smile} 15 \underset{5}{\smile} 20 \underset{5}{\smile} 25 \underset{5}{\smile} 30$

Setiap pertambahan dua ular maka daun yang dibutuhkan bertambah lima

- Dengan penjumlahan

2 ular butuh 5 daun

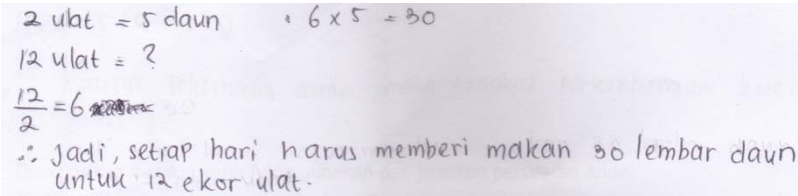
12 ular = $\underbrace{2+2+2+2+2+2}_{6\text{ kali}}$

berarti butuh = $6 \cdot 5 = 30$ daun.

Figure 2. Subject AF Presumption about Student Workk for T1

Subject FA outlines the reasoning for the answers as follows.

> Is known that... suppose 2 caterpillars spend 5 leaves. So we share 12 caterpillars with 2 and we get 6. It means multiple, multiple of 2. Then we find multiples, 6 times of 5, the leaves. So the result is 30.



2 ulat = 5 daun · $6 \times 5 = 30$

12 ulat = ?

$\frac{12}{2} = 6$

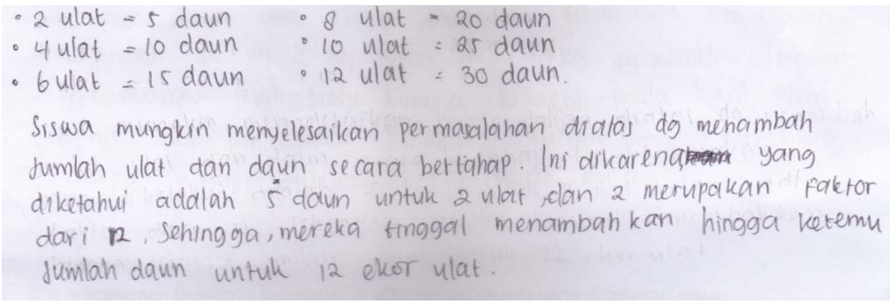
∴ Jadi, setiap hari harus memberi makan 30 lembar daun untuk 12 ekor ulat.

Figure 3. Subjec FA Completion for T1

FA outlines alleged student reasoning as follows.

> I think the students there will be add in stages. So suppose the initial number 2, added 2, add 2 more so that met 12 caterpillars. It makes 30 leaves.

It appears that the FA using the multiplicative reasoning but using additive reasoning when surmise student reasoning.



- 2 ulat = 5 daun
- 4 ulat = 10 daun
- 6 ulat = 15 daun
- 8 ulat = 20 daun
- 10 ulat = 25 daun
- 12 ulat = 30 daun.

Siswa mungkin menyelesaikan permasalahan diatas dg menambah jumlah ulat dan daun secara bertahap. Ini dikarenakan yang diketahui adalah 5 daun untuk 2 ulat, dan 2 merupakan faktor dari 12. Sehingga, mereka tinggal menambahkan hingga ketemu jumlah daun untuk 12 ekor ulat.

Figure 4. Subject FA Presumption about Student Workk for T1

EM using the cross multiplication and outlines the reasoning of the first task as follows.

> The children has 5 caterpillar um ... 5 leaves is for 2 caterpillars. So if you use the principle of equivalent comparison, we obtain ... 12 caterpillars will spend 30 leaves. Now, therefore, the method used is compared with the equivalent ratio. So suppose that caterpillars eat the same portion, if 2 caterpillars eat 5 leaves ... then 12 caterpillars will eat 30 leaves.

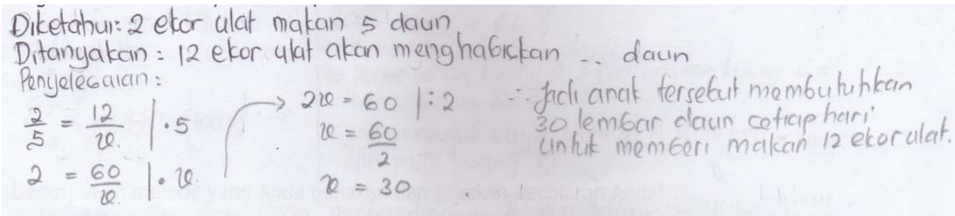


Figure 5. Subject EM Completion for T1

EM surmises that the student uses numerical calculation also use images to make sense of the problem.

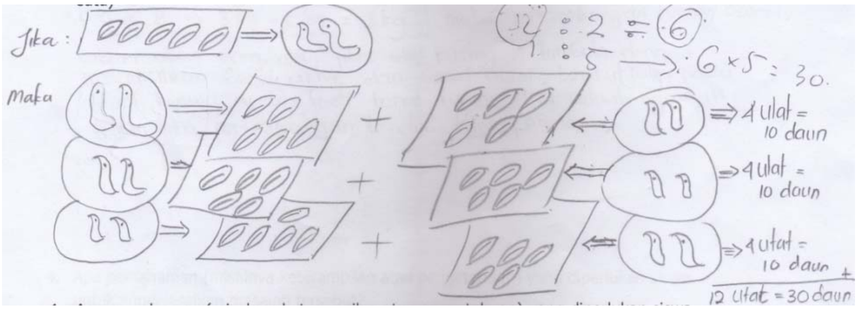


Figure 6. Subject EM Presumption about Student Workk for T1

EM using multiplicative reasoning for herselfs or when she surmises the students answer. The following is EM description on student reasoning:

> There are 2 caterpillars eat 5 leaves, now suppose there are 6 groups of caterpillars. So 2, 2, 2, 2, 2, 2 then 12 caterpillars. And... every two caterpillars eat 5 leaves. Two caterpillars eat leaves 5 then times 6. It makes 30 leaves.

ZNH using the cross multiplication after drawing and using multiplicative reasoning for resolve the problem. Here is a description of her reasoning.

> I already know about the comparison. It for 5 leaves (pointing the picture), five leaves is for 2 caterpillars. How many leaves for 12 caterpillars? I just multiply this (pointing handwriting comparison). That is a kind of the cross multiplication. 12 times 5 are resulting 60. Continuously, for how many leaves this, suppose x, so 2x. So 5 times 12 equals with 2 times x. Sixty is equal to 2 times x. Then x is 30.

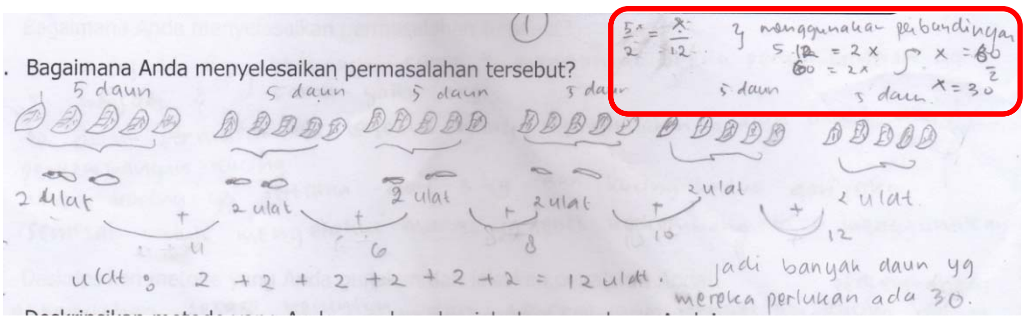


Figure 7. Subject ZNH Completion for T1

ZNH have outlining allegations of student reasoning about the first task as follows.

> Suppose Students do not know about the comparison. Maybe they could use ... could by drawing, it could also use such props. For example using props leaves or anything, it's up. Or

drawn like this. It can also be drawn. There was (pointing 5) for 2 caterpillars. Amount 5. Continuously, 5 again. It will be known for 12 caterpillars. It means that we stay followed 2, 2, 2, 2, 2, 2 till 12. The number of leaves is follow the caterpillar ... so there are as many as 6 .. It was a kind of multiples ... so 5 + 5 + 5 + ... as much as 6 times. So it produced 30 leaves.

ZNH surmise that the student uses adaptive reasoning and continue to move to the multiplicative reasoning when solving the first task.

Being able to describe proportional situations using multiplicative language is an indicator of proportional reasoning (Dole, 2008). So, subject using multiplicative reasoning in solving proportional problem and can make presuption of student work in both adaptive and multiplicative reasoning. This is an asset for them to further develop their understanding of the student's knowledge.

Second Task

One way of assisting students to developmental strategies for solving proportion problems is through the use of ratio tables (Middleton & van den Heuvel-Panhuizen,1995). The value of the ratio table is that it shows the linear nature of the relationship in proportional situations that can be demonstrated through ordering the values (lowest to highest) in a ratio table. The second task associated with the use of tables as a strategy to resolve problem.

Use the table to help resolve this problem. Juice packed in container contains 14 bottles. How many bottles of juice contained in 9 containers?

From these problems, the subject answer is by adding one by one the number of container associated with the number of bottles of juice. However, the reasoning strategies they used vary. Subject AF was not using one-one correspondence between the number of container with the number of bottles, but by filling the cells with the number of bottles per container as follows.

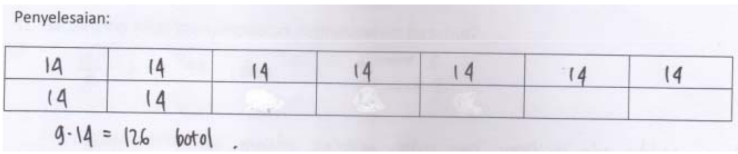


Figure 8. Subject AF Completion for T2

Seem that subject AF using the concept of multiplication in determining the number of bottles of juice. According to AF, the students' answers will be similar to her answer. However, they may use the summation concept.

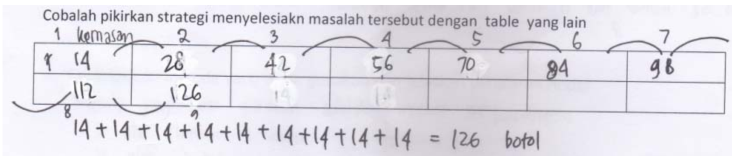


Figure 9. Subject AF Presumption about Student Work for T2

Subject FA was using the concept of one-one correspondence to determine the number of bottles in 9 containers. Because of there are less cells, the subject adding 2 additional cells in order to obtain 9 container. Subject receipts summation concept.

From what is known, one container requires 14 bottles. It means that we can just add. For example, from 2 containers, means 2 times 14 is 28. If 3 container, it means we added 14 again then we get 42. Continued.

FA surmises the completion may students performed. FA surmises that the student uses anothers reasoning strategies in solving these problems.

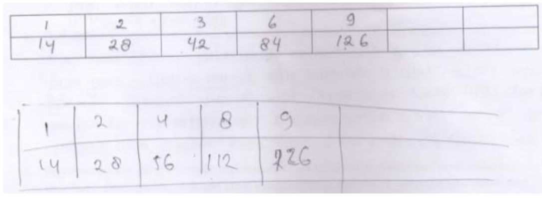
When it is known that for 3 containers is 42, because 9 is a multiple of 3 then we ... just add 3 containers directly. When 3 containers is 42 then 6 containers is doubled that means 84.

Continuously, because of a 6 containers is 84 bottle so for 9 containers we only added 3 (containers) again. It means 84 plus 42 is 126.

In this case the FA uses reasoning strategies: multiply by 3, multiply by 2 and add to 3. When I asked him if there are other tables may be students made, he showed that's there are. Other reasoning strategies alleged by FA is multiply by 2, multiply by 2, multiply by 2, and add to 1. Here is a fragment of his explanation.

Probably still be. So suppose known ... but may be shorter more ... Let's for a containers of 14, continued to 2 containers is 28, doubled then 4 containers we just multiply by 2 is 56, continues multiplied by 2 more (be) 8 to obtain 112. As already 8 packs the just added one pack more so that it becomes 126.

Following is an illustration of alleged reasoning strategies used by students.

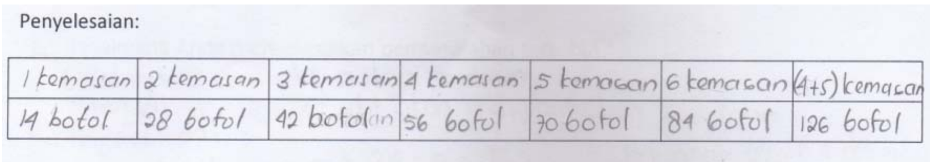


1	2	3	6	9		
14	28	42	84	126		

1	2	4	8	9	
14	28	56	112	126	

Figure 10. Subject FA Presumption about Student Work for T2

Subject EM using one-one correspondence to determine the number of bottles in 9 container. However, because the table is limited to 7 column then he used reasoning strategies that 9 container can be obtained from 4 container plus 5 container.



Penyelesaian:

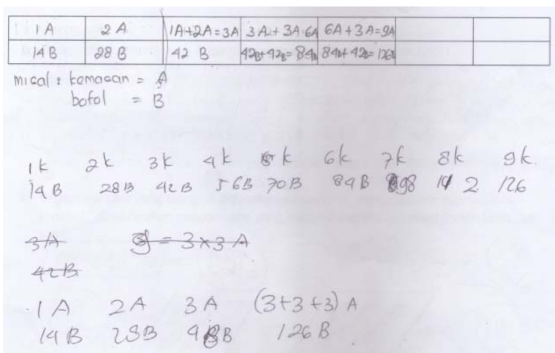
1 kemasan	2 kemasan	3 kemasan	4 kemasan	5 kemasan	6 kemasan	(4+5) kemasan
14 botol	28 botol	42 botol	56 botol	70 botol	84 botol	126 botol

Figure 11. Subject EM Completion for T2

EM gives 3 allegations of completion table that may be made by the students with the concept of summation, namely:

1. By add 1, add 1 and add 1 more to get 3 container, add 3 by 3 to get 6 container, and add by 3 to get 9 container.
2. With one-one correspondence from 1 container to 9 container, still use the concept of summation.
3. By add 1 until get 3 container then add 3 container and 3 container more.

Following is EM allegations about the table students made.



1 A	2 A	1A+2A=3A	3A+3A 6A	6A+3A=9A		
14 B	28 B	42 B	42B+42B=84B	84B+42B=126B		

misal : kemasan = A
botol = B

1k 2k 3k 4k 5k 6k 7k 8k 9k
14B 28B 42B 56B 70B 84B 98 112 126

~~3A~~ ~~9 = 3x3 A~~
~~42B~~

1A 2A 3A (3+3+3) A
14B 28B 42B 126B

Figure 12. Subject EM Presumption about Student Work for T2

Subjects ZNH also use one-one correspondence for determine the number of bottles of juice for 9 containers Reasoning strategy that are used is the summation.

> So, one package was contained 14 bottles. Then for 2 containers, 2 containers can be obtained from 1 container plus 1 container. Because of 1 container contain 14, then plus 1 container more so we get 28 bottles. Lha... for 3 containers it could have 1 package (+) 1 package (+) 1 package, or it could be 2 containers plus 1 container. Two containers, it's been calculated before, so just follow the next pattern.

ZNH describe completion table as follows.

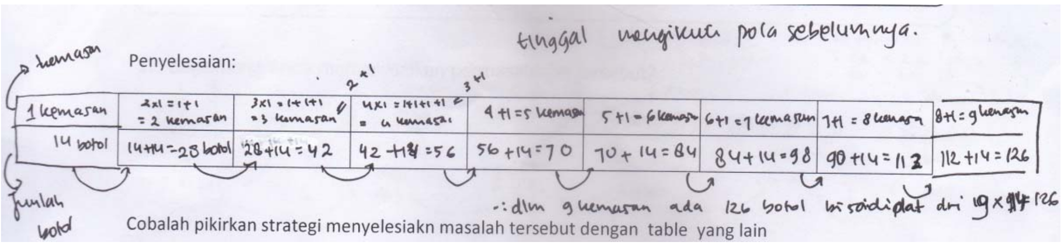


Figure 13. Subject ZNH Completion for T2

ZNH surmise that the student uses multiplicative reasoning in making the completion table but as the concept of summation. Here is alleged ZNH on student completion table for the problem.

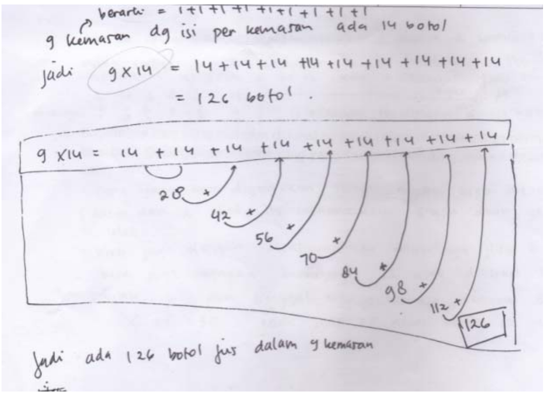


Figure 14. Subject ZNH Presumption about Student Work for T2

According to Dole (2008), discussion of the solution strategies and other possible pathways assists students in seeing that ratio tables are tools for determining proportional situations. For prospective teacher's, by make presumption of students work about using tables ratio, they can see possible pathway to assist their student in advanced task to revised or improved students understanding in determining proportional situations. By giving them the actual students work, we can develop their understanding about stundents proportional reasoning. Their own proportional reasoning is the first order knowledge and their understanding about students proportional reasoning is the second order knowledge. According to *Schiefsky (2012)*, *f*irst-order knowledge is knowledge about the world, whether theoretical or practical in orientation; it may be a knowledge of how things are, or a knowledge of how to do or make things. Second-order knowledge is knowledge that derives from reflection on first-order knowledge: for example, a method for generating new procedures. Second-order knowledge is also an "image of knowledge" insofar as it sets out a conception or norm for what knowledge is in a particular domain.

Conclusions

The results showed that some subjects using cross multiplication to solve the problem of proportion. However they understand proportional reasoning of the problem. They do not just do the multiplication of the information that is known but they can use additive and multiplicative reasoning. The subjects have multiplicative reasoning but can infer student additive reasoning. Students can develop reasoning additive or multiplicative reasoning is shifted toward the right assignment and preparation of learning by teachers to consider allegations of teachers about student reasoning. By understanding the reasoning of students, teachers will be able to plan anticipatory action against any student responses.

For advanced study, we need to examine prospective teacher’s thinking in making presumption of students reasoning. From their mathematical knowledge about proportional reasoning, we also need to know their understanding about students' mathematical knowledge. This can be done by giving them actual examples of student work. Then they were asked to analyze the work of students in a discussion to get an understanding of mathematical knowledge of students. From here we can see how prospective teacher's thinking about proportional reasoning shift from the first orders knowledge to the second order knowledge. By understanding student mathematical knowledge, prospective teachers will be able to make estimates about the student's work better.

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