

# GENERALIZED HEIGHT-DIAMETER MODELS FOR *Acacia mangium* Willd. PLANTATIONS IN SOUTH SUMATRA

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## ABSTRACT

The aim of this study was to develop a generalized height-diameter model for predicting tree height of *Acacia mangium* plantations in South Sumatra that could account for the variability of site and stand conditions. Six commonly used non-linear growth functions (i.e. Gompertz, Chapman-Richards, Lundqvist-Korf, Weibull, modified logistic, and exponential) were selected as candidate base models and were fitted to individual tree's height-diameter data of *A. mangium* plantations. A total of 13,302 trees collected from permanent sample plots with various spacing, stand age, and site quality were available for this study. The data were split into two sets: one set being the majority (75%) was used to estimate model parameters and the remaining data set (25%) was used to validate the models. The results showed that the six base models produced almost identical fits with a relatively high root mean squared error ( $\pm 3.4$  m) and a relatively low proportion of the total variation in observed tree height (52.5 - 53.4%). The Lundqvist-Korf (LK) model performed slightly better than the other models based on the goodness of fit as well as bias and standard errors of the predictions. This LK model can be fitted easily and provided more satisfactory fit when additional variables were included into the model, hence was selected as the base model. Introducing stand variables into the selected base model resulted in a significant improvement of the accuracy for predicting heights. The root mean squared error decreased by the value between 0.5564 and 1.4252 m and the proportion of variation explained by the model increased by the value between 13.88 and 33.21%. The best improvement based on fit and model validation was achieved by the generalized height-diameter model with inclusion of stand age and site index.

Keywords: *Acacia mangium*, generalized model, height-diameter relationship, non-linear growth function

## I. INTRODUCTION

Individual tree's height and diameter at breast height (dbh) are essential inventory measures for estimating tree volume, site index and other important variables in forest growth and yield models. Tree diameter can be measured easily and at little cost. Total

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tree height, however, is more difficult and costly to measure due to several reasons: (1) time required to complete measurements, (2) chance of observer error, and (3) visual obstruction (Colbert *et al.*, 2002). Often, only a subset of trees in the sample plots with measured dbh is also measured for height. For this reason, quantifying the relationship between total tree height and dbh is required for predicting heights of the remaining trees.

Numerous model forms have been used for predicting height-diameter relationships for different species and in different forest regions. The approaches used for developing height-diameter models have varied from being simple to complex, including linear and nonlinear models. Most of these models (e.g. Arabatzis and Burkhart, 1992; Huang *et al.*, 1992; Moore *et al.*, 1996; Zhang, 1997; Huang, 1999) use only dbh as predictor variable for estimating total tree height and this may restrict their use to the stands where the data gathered. The height-diameter relationship, however, may vary from stand to stand, and even within the same stand, as the height-diameter relationship within the same stand does not remain constant over time (Flawelling and de Jong, 1994; Lappi, 1997; Eerikäinen, 2003); the development of tree height may take longer on poor quality sites than that on good quality sites (Mehtätalo, 2004; 2005); and for a particular diameter, trees that grow in very high density stands may be taller than those growing in less dense stands due to the greater competition among trees (Zhang *et al.*, 1997; Zeide and VanderSchaaf, 2002).

Due to the variability of site and stand conditions, a single height-diameter relationship may not be useful in all the possible situations that can be found in different stands. One alternative is to develop a height-diameter model separately for each stand. Although this approach may provide accurate estimates of individual stands, it is time consuming (inefficient) and expensive. A practical alternative is to develop more generalized height-diameter models, which account for the variability of site and stand conditions, by including additional stand variables as well as tree diameter (e.g. Bi *et al.*, 2000; Staudhammer and LeMay, 2000; Sánchez *et al.*, 2003; Sharma and Zhang, 2004).

Despite the importance of height-diameter model in forest growth and yield prediction systems and the long time over which these models have existed for other forest or plantation species and in other regions, relatively little has been published on height-diameter models for *Acacia mangium* plantations, including a base model of height-diameter relationships. Therefore, the objective of this study is to develop a generalized height-diameter model for predicting tree height of *A. mangium* grown in the plantations of South Sumatra that could account for the variability of site and stand conditions. Several models to predict total tree height from diameter were selected from the literature as candidate base models. The base model using diameter as a predictor variable was then modified to include stand variables for improving height predictions.

## II. MATERIALS AND METHODS

### A. Data Description

Data used in this study were obtained from the permanent plots established in unthinned *A. mangium* plantations located in South Sumatra, Indonesia. The plots were subjectively selected to represent the range of stand age, density (spacing), and site quality of *A. mangium* plantations in the region. All plots were established with the same size (0.1 ha), but the stands as sampled varied in planting spacing (i.e. 3x2, 3x3, 3x4 and 2x4 m). They were established between 1993 and 1996, and the first measurements were taken at 2-4 years after planting. Most plots were measured at a 1-year time interval until 8-11 years after planting.

From each plot, all sample trees that had both dbh and height measurement data were extracted (removed) and the combined data were used for developing and evaluating the height-diameter models. A total of 13,302 pairs of individual tree's height-diameter measurements were available for this study. These data were split by trees into two sets. The majority of the data (75%) were used for model fitting, while twenty-five percent of the data selected randomly across the range of the diameter classes was used for model validation (cross-validation procedure). Summary statistics of the tree diameter and height as well as characteristics of the plots from which the sample trees used for model fitting and validation are presented in Table 1. Scatter plots of the tree diameter and height data for both data sets are also illustrated (Figure 1).

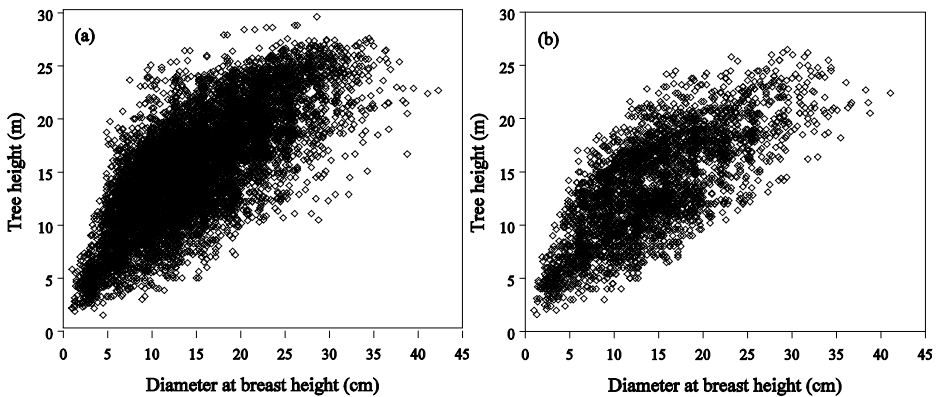


Figure 1. Scatter plots of tree height against diameter at breast height (dbh) of *A. mangium* trees for the fitting (a) and the validation (b) data sets.

Table 1. Summary of statistic descriptions of the sample trees and characteristics of the plots from which the trees used for model fitting and validation

Variable (characteristics)	Descriptions			
	Mean	Min	Max	Standard Deviation
Fitting data (No. of trees = 9977):				
Tree height (m)	15.2	1.5	29.5	5.0
Diameter at breast height (cm)	14.8	1.0	42.3	6.5
Stand age (years)	5.8	2.0	11.0	2.0
Quadratic mean diameter (cm)	14.7	3.2	28.3	4.4
Dominant height (m)	16.9	3.6	28.6	5.0
Site index (m)	18.1	8.7	25.9	2.7
Stand density (stems ha <sup>-1</sup> )	971	290	2020	287
Basal area (m <sup>2</sup> ha <sup>-1</sup> )	16.5	0.7	34.8	7.4
Validation data (No. of trees = 3325):				
Tree height (m)	13.6	1.6	26.5	4.8
Diameter at breast height (cm)	14.4	1.0	41.1	6.6
Stand age (years)	5.5	2.0	10.0	2.0
Quadratic mean diameter (cm)	15.1	3.3	26.8	4.8
Dominant height (m)	16.0	5.0	25.6	4.6
Site index (m)	17.7	11.1	23.6	2.4
Stand density (stems ha <sup>-1</sup> )	855	350	1920	289
Basal area (m <sup>2</sup> ha <sup>-1</sup> )	16.1	1.8	32.4	7.6

## B. Base Height-Diameter Model Selection

To develop a generalized height-diameter relationship of trees for *A. mangium* plantations, the appropriate height-diameter model form for relating tree height to diameter was first analyzed by evaluating a range of alternative model forms. Selection of the candidate height-diameter models was first based on a graphical examination of the height-diameter relationships by plotting total tree height against dbh data, which indicated typical sigmoidal-concave shapes (Figure 1), whereby growth rate increased from a minimum value to a maximum, and then declined towards zero at an upper asymptote. In addition, the following important model properties were considered (Lei and Parresol, 2001; Peng *et al.*, 2001): (1) the desirable mathematical properties (e.g. number of parameters, flexibility), (2) possible biological interpretation of the parameters (e.g. upper asymptote, maximum or minimum growth rate), and (3) satisfactory predictions of the height-diameter relationships.

After evaluating a large number of model forms presented in the literature, six nonlinear models (Table 2) were selected as the candidate base height-diameter models. All these models used three parameters that described various biological interpretations, i.e. an upper asymptote, a rate parameter, and a shape parameter. Four parameters were not included in this study since they were more likely to be over-parameterized thereby resulting in instability of the estimates (Fang and Bailey, 1998). According to the previous studies (Huang *et al.*, 1992; Moore *et al.*, 1996; Zhang, 1997; Huang, 1999; Peng *et al.*, 2001), these six models have been shown to provide more satisfactory fits to tree height-diameter relationships for particular species and stand conditions than many alternative model forms.

All models listed in Table 2 can be written in a more general form:

$$Y_i = f(X_i, \beta) + \varepsilon_i \dots\dots\dots (1)$$

where  $Y_i$  is a vector of observations of the dependent variable (i.e.  $Ht$ ),  $X_i$  is a vector of observations of the independent variable (i.e.  $D$  (dbh)),  $b$  is the vector of parameters to be estimated (i.e.  $b_0$ ,  $b_1$  and  $b_2$ ), and  $\varepsilon_i$  is a vector of random errors. A constant value (1.3) was added to the right hand side of all height-diameter models in order to meet the condition that tree height should be 1.3 m when tree dbh is zero. This assumption may not affect the height-diameter relationship very much because the data corresponding to heights lower than 1.3 m were not present in the fitting data (Table 1). However, it was used to avoid negative height estimates for small trees (e.g. Parresol, 1992; Hökkä, 1997).

Table 2. Nonlinear models selected as the base model for estimating tree height-diameter relationship of *A. mangium* plantations

Model	Expression	References
Gompertz (G)	$Ht = 1.3 + b_0 \exp(-b_1 \exp(-b_2 D))$	Winsor (1932), Huang <i>et al.</i> (1992), Zhang (1997)
Chapman-Richards (CR)	$Ht = 1.3 + b_0 (1 - \exp(-b_1 D))^{b_2}$	Richards (1959), Chapman (1961)
Lundqvist-Korf (LK)	$Ht = 1.3 + b_0 \exp(-b_1 D^{-b_2})$	Stage (1963), Zeide (1993)
Weibull (W)	$Ht = 1.3 + b_0 (1 - \exp(-b_1 D^{b_2}))$	Yang <i>et al.</i> (1978)
Modified logistic (ML)	$Ht = 1.3 + \frac{b_0}{(1 + b_1^{-1} D^{b_2})}$	Ratkowsky and Reedy (1986), Huang <i>et al.</i> (1992)
Exponential (E)	$Ht = 1.3 + b_0 \exp\left(\frac{b_1}{D + b_2}\right)$	Ratkowsky (1990), Zhang (1997)

$Ht$  = tree height (m);  $D$  = diameter at breast height or 1.3 m above ground (cm);  $b_0$ ,  $b_1$ , and  $b_2$  = model parameters to be estimated.

### C. Parameter Estimation and Model Comparison of the Base Models

The six candidate base height-diameter models (Table 2) were fitted to the model fitting data set using ordinary nonlinear least squares regression (NLIN procedure) in the SAS/STAT system (SAS Institute Inc, 2005). Because sample plots were measured several times, it may be expected that the observations of diameter and height of the same tree are temporally correlated, which violates the assumption of independent error terms. However, initial analysis in this study revealed that the serial correlation was weak for all candidate height-diameter models tested. This is possible because tree heights were measured on the sample trees based on the range of diameter classes so the sample trees were not necessarily the same in sequential measurements. Therefore, the weak autocorrelation was ignored in the model fitting as the impact of variance underestimation is likely to be masked by fitting each individual tree as an independent observation (Temesgen and Gadaw, 2004; Dorado *et al.*, 2006).

The validity using the assumptions of ordinary nonlinear least squares regression was also investigated by examining the regression residuals. The studentised residual plots for all the candidate base models showed approximately homogenous variances over the full range of the predicted values and no systematic patterns in the variation of the residuals, as also shown in the other height-diameter modelling studies (e.g. Parresol, 1992; Zhang, 1997; Staudhammer and LeMay, 2000; Peng *et al.*, 2001).

Results of the fitting six candidate base height-diameter models were compared according to the criteria: (1) the asymptotic *t*-statistics of the parameters, (2) the root mean squared error, and (3) the adjusted coefficient of determination. In addition, the six candidate base models were employed to predict tree height using the validation data set. The average bias (i.e. observed - predicted) and standard errors of the height predictions were then calculated. To further evaluate the predictive ability of the models over the entire diameter classes, the bias and standard errors of the predicted heights were computed for each dbh class by dividing the observed data into 1-cm dbh class. The performance of the six fitted models was also examined by simulating tree heights for a range of tree diameters.

### D. Inclusion of Stand Variables

To account for the variation between different stand conditions, the selected base model using only dbh as a predictor variable can be modified by including stand variables that introduce the dynamics and state of each stand into the model. The developed model was called a generalized height-diameter model. According to Staudhammer and LeMay (2000), there are two methods that could be used for incorporating stand variables into base height-diameter models. The first method is the "parameter prediction" (Clutter *et al.*, 1983; Temesgen and Gadaw, 2004), also called the "two-stage" method (West, 1982). In this method, a relationship between tree height and diameter is determined for each plot in the first stage and their parameter estimates are then related to stand

variables in the second stage. The second method is the “direct approach” (Parresol, 1992; Bi *et al.*, 2000; Sharma and Zhang, 2004), in which the stand variables are added directly to the model. Staudhammer and LeMay (2000) reported that there were no clear choices between these two methods in terms of the goodness of the resulting fits. The parameter prediction method may be more attractive since the additional variables are used to predict parameters leading to easier model interpretation (Staudhammer and LeMay, 2000). This method, however, may result in a very complex model with many coefficients, particularly if several additional variables are added to the base model, making it difficult to select the starting values in the model fitting using nonlinear least squares regression. This was also evident in the preliminary analysis of this study. For this reason, the direct method was used and a more parsimonious model can be produced compared to the parameter prediction method.

In this study, the following stand variables characterising the plots (as shown in Table 1) were tested for inclusion in the base height-diameter model (Eq. (1)): stand age, site index, quadratic mean diameter, stand density, and basal area. Several combinations of these variables were also considered. Initial screenings of models were based on the significance of model parameters. Both graphical and statistical analyses of the residuals were also carried out. The adequacy of the models was determined from comparison using the same procedures applied in selecting the base height-diameter models.

### III. RESULTS AND DISCUSSION

#### A. Performance and Comparison of the Candidate Base Models

Results of fitting six candidate base models including nonlinear least squares estimates of the parameters, the standard error ( $SE$ ),  $t$ -statistic,  $p$ -value, the root mean squared error ( $RMSE$ ) and the adjusted coefficient of determination ( $R^2_{adj}$ ) are presented in Table 3.

It is apparent from the fit statistics that all six base height-diameter models produced almost identical fits. This is consistent with the findings reported by Huang *et al.* (1992) for major tree species in Alberta (Canada), Zhang (1997) for ten conifer species in the inland Northwest of the United States, and Peng *et al.* (2001) for nine major tree species in Ontario's boreal forests of Canada. All parameters of these models were statistically significant ( $p$ -value < 0.01). Comparing the  $RMSE$  and  $R^2_{adj}$  for each model, the Lundqvist-Korf model had slightly smaller  $RMSE$  and higher  $R^2_{adj}$  than the other five models (Table 3). However, the differences of the  $RMSE$  and  $R^2_{adj}$  among the six models were trivial. In general, the six models produced relatively high  $RMSE$ s (about 3.4 m) and explained a relatively low proportion of the total variation in observed values of the tree height, accounting for only 52.5 - 53.4%. Nevertheless, these results may not be surprising since the height-diameter relationships found in the data were highly variable or scattered (Figures 1). This is likely to be due to the data having come

from sample plots with a wide range of site productivities, stand ages and stand densities (Table 1). This result indicated that the use of models using dbh alone to predict total tree height for *A. mangium* plantations does not appear to be adequate for explaining variability of tree heights among stands and it would not be satisfactory for plantation management. This type of model may be used when only diameter measurements are available, although the error of prediction can be relatively large as all trees with the same diameter in any one plot will have the same predicted height regardless of the stand in which they are growing. Therefore, it was advisable to include additional stand variables to improve the height predictions.

Table 3. Parameter estimates, standard errors and related fit statistics for the six nonlinear models tested for selecting the best base height-diameter model for *A. mangium* plantations

Model	Parameter	Estimate	SE	<i>t</i>	$p <  t $	RMSE	$R^2_{adj}$
Gompertz (G)	$b_0$	23.2	0.33	69.38	0.0001	3.467	0.525
	$b_1$	1.720	0.0243	70.91	0.0001		
	$b_2$	0.0882	0.00288	30.60	0.0001		
Chapman- Richards (CR)	$b_0$	31.4	2.03	15.47	0.0001	3.437	0.533
	$b_1$	0.027	0.0044	6.03	0.0001		
	$b_2$	0.689	0.0254	27.10	0.0001		
Lundqvist- Korf (LK)	$b_0$	114.39	29.122	3.93	0.0001	3.432	0.534
	$b_1$	4.22	0.165	25.61	0.0001		
	$b_2$	0.27	0.032	8.29	0.0001		
Weibull (W)	$b_0$	34.9	3.23	10.82	0.0001	3.436	0.533
	$b_1$	0.073	0.0042	17.48	0.0001		
	$b_2$	0.740	0.0278	26.65	0.0001		
Modified logistic (ML)	$b_0$	47.708	5.0104	9.52	0.0001	3.435	0.533
	$b_1$	0.051	0.0031	16.18	0.0001		
	$b_2$	0.80	0.037	21.76	0.0001		
Exponential (E)	$b_0$	32.3	0.71	45.72	0.0001	3.445	0.530
	$b_1$	-16.5	0.85	-19.49	0.0001		
	$b_2$	6.1	0.47	12.85	0.0001		

SE = standard error; RMSE: root mean square error;  $R^2_{adj}$ : adjusted coefficient of determination



To select the best base model, the adequacy of the six candidate models was also evaluated on the basis of bias and standard errors of the predictions in the validation data. The mean bias and standard error of the predicted height computed for each dbh class are illustrated in Figure 2. Since the number of trees in diameter classes greater than 38.5 cm was fewer than the number of model parameters, the average biases and standard errors of the predictions for those classes were not computed. In general, all six base models produced similar pattern of bias and overall they tended to overestimate tree height. As expected, the standard errors of the predictions for all models were almost the same, and indicated a low precision of height predictions - almost 4 m in most diameter classes.

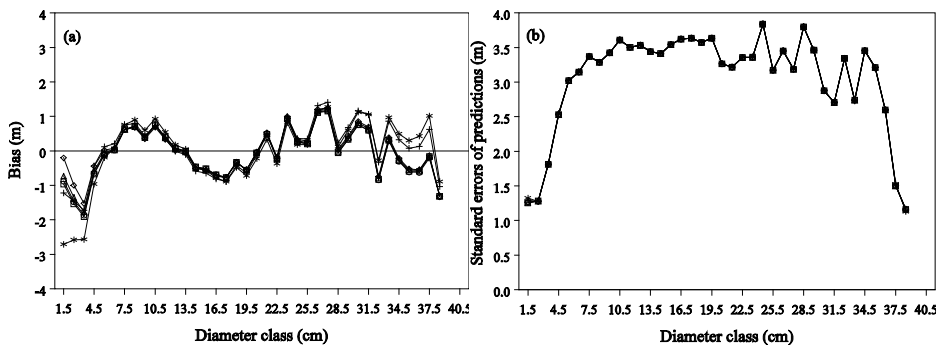


Figure 2. Average bias (a) and standard errors of the predictions (b) of the six base height-diameter models at different dbh classes (*star*: Gompertz, *square*: Chapman-Richards, *diamond*: Lundqvist-Korf, *circle*: Weibull, *plus*: exponential, *triangle*: modified logistic).

The six base models were then applied to predict tree heights with the range of tree diameters as illustrated in Figure 3. In general, all models produced similar predictions of tree height. However, the Gompertz and exponential models predicted smaller tree heights for trees larger than 30 cm dbh than did the other four models. These two models were also found to underestimate the heights of large-sized trees in the studies by Zhang (1997) for ten conifer species for even-aged stands in the inland Northwest of the United States. In addition, heights predicted using both these models did not approach values close to 1.3 m when the diameter at breast height approaches 0. The problem is present only for trees less than 5 cm dbh but could cause significant overestimation of tree height if the models were applied to very young stands.

For all diameters, it seems that any of the other four models, i.e. the Chapman-Richards, Lundqvist-Korf, Weibull and modified logistic, could be used as the base model. However, since the Lundqvist-Korf model performed slightly better than the other models based on the goodness of fit (Table 3) and mean bias and standard errors

of the predictions (Figure 2), the following nonlinear model was chosen as the best base height-diameter model:

$$Ht = 1.3 + b_0 \exp(-b_1 D^{-b_2}) + \varepsilon \dots\dots\dots (2)$$

In addition, further analysis indicated that the Lundqvist-Korf model (Eq. (2)) can be fitted easily and provided more satisfactory fit when additional variables (i.e. stand attributes) were included into the model, compared with the Chapman-Richards, Weibull and modified logistic models.

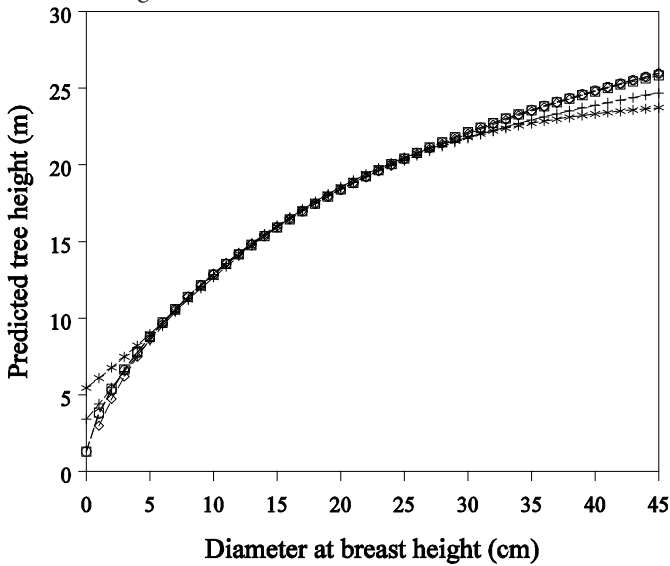


Figure 3. Predicted tree heights using the six base height-diameter models (*star*: Gompertz, *square*: Chapman-Richards, *diamond*: Lundqvist-Korf, *circle*: Weibull, *plus*: exponential, *triangle*: modified logistic).

### B. Performance and Comparison of the Generalized Height-Diameter Models

The selected base height-diameter model (Eq. (2)) was expanded to include additional independent variables (stand variables) to improve height predictions and to adjust for differences between stands. Initially, several models with different combinations of stand variables were fitted. After testing various alternative stand variables, it was found that inclusion of one or two or all of these additional stand variables: stand age ( $A$ ), site index ( $S$ ) and basal area ( $B$ ) improved the resulting fit, compared with that using dbh alone. Other measures of stand density, such as number of stems per ha ( $N$ ) and quadratic mean diameter ( $D_q$ ) were initially included, but these variables were less significant and accounted for only 2.8 - 3.6%

additional variation compared with the base height-diameter model (Eq. (2)). These results may indicate that while the tree height-diameter data come from plots with varying stand densities, differences in the height-diameter relationship were not important. In this case, growth competition may affect the development of diameter significantly more than it does affect height growth. Thus, diameter will vary for trees of a given age and site as competition varies within the stand, but height will not. This result, although it contradicts studies reported by Zhang *et al.* (1997) and Zeide and VanderSchaaf (2002), is consistent with the hypothesis that tree height is relatively insensitive to competition and spacing (Lanner, 1985; Wagner and Radosevich, 1991). According to Smith and Strub (1991) and Smith *et al.* (1997), trees grown in less dense stands will grow faster in diameter, but will be more tapered. If they grow in more dense stands, the trees will grow more slowly in diameter which results in less taper. Thus, variations in stand density cause very large variations in diameter, but remarkably little in height, except at extremely low or high densities (Wagner and Radosevich, 1991; Smith *et al.*, 1997). For this reason, only three stand variables, i.e. stand age ( $A$ ), site index ( $S$ ) and basal area ( $B$ ), were selected for further analysis. The expressions of generalized height-diameter models selected for further examination are given in Table 4 with their parameter estimates and fit statistics are presented in Table 5.

Table 4. Generalized height-diameter LK models selected for comparison

Model	Additional variable	Expression
HD-1	Stand age ( $A$ )	$Ht = 1.3 + b_0 \exp\left(-b_1 D^{-b_2} + \frac{b_3}{A}\right) + \varepsilon$
HD-2	Site index ( $S$ )	$Ht = 1.3 + b_0 \exp(-b_1 D^{-b_2} + b_4 S) + \varepsilon$
HD-3	Basal area ( $B$ )	$Ht = 1.3 + b_0 \exp(-b_1 D^{-b_2} + b_5 B) + \varepsilon$
HD-4	Stand age ( $A$ ) and site index ( $S$ )	$Ht = 1.3 + b_0 \exp\left(-b_1 D^{-b_2} + \frac{b_3}{A} + b_4 S\right) + \varepsilon$
HD-5	Stand age ( $A$ ) and basal area ( $B$ )	$Ht = 1.3 + b_0 \exp\left(-b_1 D^{-b_2} + \frac{b_3}{A} + b_5 B\right) + \varepsilon$
HD-6	Site index ( $S$ ) and basal area ( $B$ )	$Ht = 1.3 + b_0 \exp(-b_1 D^{-b_2} + b_4 S + b_5 B) + \varepsilon$
HD-7	Stand age ( $A$ ), site index ( $S$ ), and basal area ( $B$ )	$Ht = 1.3 + b_0 \exp\left(-b_1 D^{-b_2} + \frac{b_3}{A} + b_4 S + b_5 B\right) + \varepsilon$

Table 5. Parameter estimates, standard errors and related fit statistics of the generalized height-diameter models tested.

Model	Para-meter	Estimate	SE	$t$	$p <  t $	RMSE	$R^2_{adj}$
HD-1	$b_0$	37.1	0.82	45.10	0.0001	2.7482	0.7012
	$b_1$	3.69	0.177	20.87	0.0001		
	$b_2$	0.81	0.038	21.16	0.0001		
	$b_3$	-2.76	0.040	-69.29	0.0001		
HD-2	$b_0$	48.47	14.132	3.43	0.0006	2.8755	0.6729
	$b_1$	4.4	0.22	19.05	0.0001		
	$b_2$	0.168	0.0289	7.89	0.0001		
	$b_4$	0.0517	0.00088	63.85	0.0001		
HD-3	$b_0$	17.8	0.82	21.92	0.0001	2.8567	0.6772
	$b_1$	3.04	0.098	30.99	0.0001		
	$b_2$	0.59	0.037	15.87	0.0001		
	$b_5$	0.0231	0.00035	65.52	0.0001		
HD-4	$b_0$	12.16	0.218	55.89	0.0001	1.9076	0.8560
	$b_1$	3.45	0.1341	25.76	0.0001		
	$b_2$	0.855	0.0299	28.62	0.0001		
	$b_3$	-2.820	0.0273	-103.46	0.0001		
	$b_4$	0.0565	0.00056	100.62	0.0001		
HD-5	$b_0$	23.7	0.55	42.92	0.0001	2.6103	0.7305
	$b_1$	3.57	0.1822	19.58	0.0001		
	$b_2$	0.86	0.0393	21.80	0.0001		
	$b_3$	-1.93	0.0452	-42.69	0.0001		
	$b_5$	0.0126	0.00039	32.15	0.0001		
HD-6	$b_0$	10.7	0.65	16.35	0.0001	2.5141	0.7500
	$b_1$	2.92	0.052	56.62	0.0001		
	$b_2$	0.47	0.032	14.82	0.0001		
	$b_4$	0.0422	0.00081	52.50	0.0001		
	$b_5$	0.0180	0.00033	54.59	0.0001		
HD-7	$b_0$	12.39	0.232	53.40	0.0001	1.9067	0.8562
	$b_1$	3.47	0.134	25.88	0.0001		
	$b_2$	0.855	0.0298	28.71	0.0001		
	$b_3$	-2.89	0.035	-81.72	0.0001		
	$b_4$	0.0574	0.00064	90.32	0.0001		
	$b_5$	-0.0011	0.00033	-3.22	0.0013		

All parameters of the seven generalized height-diameter models were significantly different from zero ( $p$ -value < 0.01). In general, the inclusion of new independent stand variables in the height-diameter models provided significantly better fits of height than those using only dbh (Table 5). The values of  $RMSE$  decreased by the value between 0.5564 and 1.4252 m, while the proportion of variation explained by the model increased by the value between 13.88 and 33.21%. Models that include only one stand variable (i.e. models HD-1, HD-2 and HD-3) resulted in relatively larger  $RMSE$  and smaller  $R^2_{adj}$  than the models that include two or more additional stand variables. In the cases of two stand variables, model HD-4, which included stand age and site index provided a more satisfactory fit than models HD-5 (including stand age and basal area) and HD-6 (including site index and basal area). Model HD-7, which included three stand variables (i.e. stand age, site index and basal area), gave the largest increase in  $R^2_{adj}$  and largest decrease in  $RMSE$  over the base model. However, the improvement was very small relative to model HD-4, as can be seen from similar values of their parameter estimates and related fit statistics of both models (Table 5).

Since models HD-4 and HD-7 provided better fits than the other five models, these two models were further analysed by evaluating their biases and standard errors for predicted height. The mean bias and the standard errors of the predicted heights by 1-cm diameter classes were calculated for both the fitting and validation data sets. As shown in Figure 4, these two generalized height-diameter models produced virtually identical trends for over- or under-estimation and standard errors of the predictions across all diameter classes, when applied to the fitting data set. Overall mean biases were small (<0.005 m in absolute values), with model HD-7 having slightly lower absolute bias. However, when applied to the validation data set, model HD-4 performed much better than model HD-7 (Figure 5). Model HD-4 generally produced small biases over all DBH classes. On the other hand, model HD-7 overestimated height for dbh < 16 cm, but underestimated heights for dbh > 16 cm. The standard errors of the predicted heights of model HD-7 were also consistently higher than that of model HD-4 over DBH classes. Despite significance of the coefficient of basal area, model HD-7 appears to be over-parameterized, which can create problems such as sensitivity to initial values and instability of the estimates when nonlinear least squares algorithm are used to fit it (Draper and Smith, 1998).

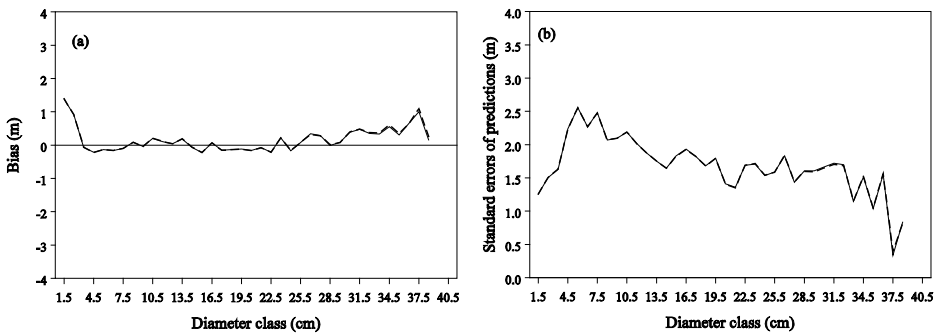


Figure 4. Average bias (a) and standard errors of the predictions (b) of the Models HD-4 and HD-7 at different dbh class based on the fitting data set (*solid line*: Model HD-4, *dashed line*: Model HD-7)

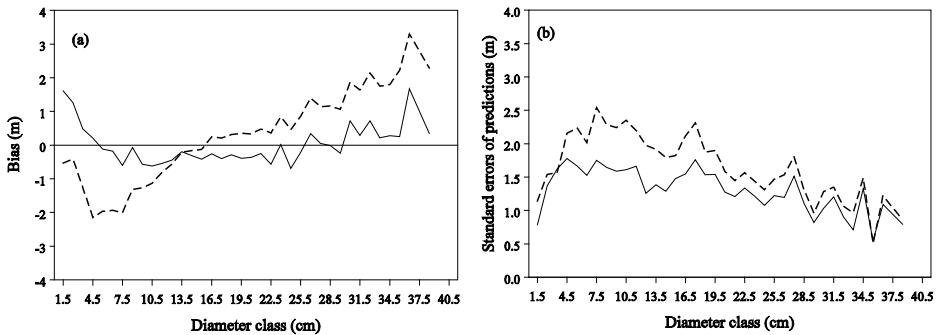


Figure 5. Average bias (a) and standard errors of the predictions (b) of the Models HD-4 and HD-7 at different dbh class based on the validation data set (*solid line*: Model HD-4, *dashed line*: Model HD-7).

Model HD-4, which includes only two stand variables (stand age and site index), performed better than model HD-7, which includes three stand variables (stand age, site index and basal area). The good performance of this model may be due in part to the inclusion of stand age, which is important variable in the consideration for height and diameter relationship of even-aged stands, or plantations. In even-aged stands (or plantations), age is a good indicator of the mean size of the individual trees in the stands (Sánchez *et al.*, 2003; Dorado *et al.*, 2006). This result was also in agreement with the hypothesis that the height-diameter relationship does not remain constant over time (Flawelling and de Jong, 1994; Lappi, 1997; Eerikäinen, 2003).

The importance of site index as an indicator of site quality in the height-diameter model was expected. Mehtätalo (2004; 2005) also noted that the individual tree growth in a stand may take longer on poorer sites than that in high quality sites. The differences

in overall stand growth must be accompanied by a change in tree form. Trees growing on sites with high quality may be taller for a given diameter than trees growing on lower quality sites and the height curves for good quality sites may have steeper slopes than those for poor quality sites.

Although basal area was found to be significant (Table 5), its inclusion did not improve the fit much. This result was not expected, since basal area is the most obvious factor affecting the height-diameter relationship in other studies (Parresol, 1992; Staudhammer and LeMay, 2000; Sharma and Zhang, 2004). However, it should be noted that all these previous studies were applied to complex stands, comprising various tree species (although the species being studied were only the most common present in the stands) with very large variation in basal area and number of stems per ha. Another possible explanation may be the collinearity between the independent variables so inclusion of diameter, stand age and site index in the model may have taken into account most of the observed variability among stands. This result suggested that inclusion of basal area (in addition to stand age and site index) had a minimal effect on the height-diameter relationship for *A. mangium* plantations. As do other measures of stand density discussed earlier (e.g. number of stems), the analysis of height-diameter relationships in this study was consistent with the results by Lanner (1985), Smith and Strub (1991), and Wagner and Radosevich (1991), stated that stand density has little effect of height of trees.

### C. Selected Generalized Height-Diameter Model

Since there was no real gain from inclusion of basal area, the effect of basal area on height will be ignored. Model HD-4 (Table 4), i.e. tree height as a function of three variables (viz. dbh, stand age and site index), was therefore selected for predicting tree height for *A. mangium* plantations. The model may also be applied to thinned stands (although it was developed based on unthinned plots) as stand density did not affect the height-diameter relationship. The final generalized height-diameter model fitted to all data was:

$$\hat{H}t = 1.3 + 12.16 \exp\left(-3.45D^{-0.855} - \frac{2.82}{A} + 0.0565S\right) \dots\dots\dots (3)$$

where  $Ht$  is total tree height (m),  $D$  is diameter at breast height (cm),  $A$  is stand age (years), and  $S$  is site index (m).

To examine differences in height prediction for different stand conditions, tree heights were predicted using Eq. (3) for different stand ages and site index classes (Figures 6a and 6b). Figure 6a shows the development with age of the relationship between tree height and diameter in a stand with site index of 20 m. For a given site index, as stand age increases, the tree height curves are shifted upward but the differences between subsequent age classes tend to decrease. If a certain age has been reached,

height and diameter growth are reduced and changes of the height curves become very small. Similarly, for a given age, the predicted tree height increases with increasing site index, but the differences between the height curves of adjacent site index classes tend to increase (Figure 6b). These predictions follow a biologically consistent progression with increasing dbh, age and site index.

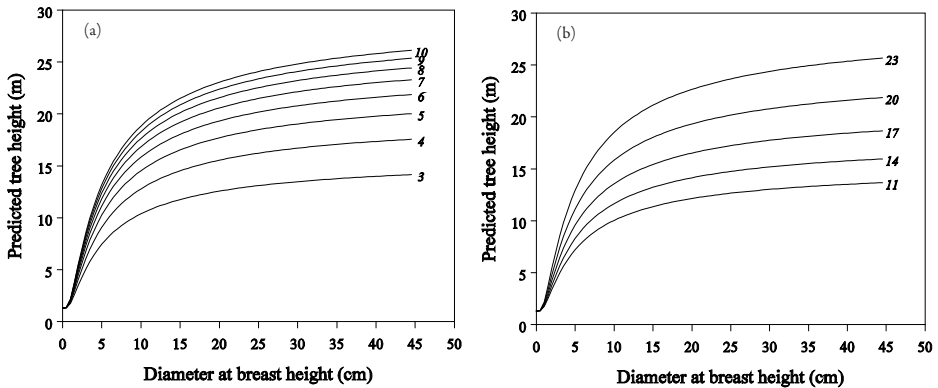


Figure 6. Predicted tree heights for different ages at a given site index of 20 m (a) and predicted tree heights for different site index classes at a given age of 6 years (b) using Model HD-4.

#### IV. CONCLUSION

Six nonlinear models were initially examined to determine the best base height-diameter model for *A. mangium* plantations. The six models provided very similar results in term of the resulting fit and the prediction error (bias) with the Lundqvist-Korf (LK) model being slightly better than the other five (Gompertz, Chapman-Richards, Weibull, modified logistic, and exponential). This model may be used when only diameter measurements are available, although the error of prediction can be relatively large.

Since the model with dbh alone inadequately explained the variability of observed tree heights for different stand and site conditions, additional independent variables were included. Inclusion of stand variables to the selected base model (generalized height-diameter model) improved significantly the precision of height predictions. The generalized height-diameter LK model with stand age and site index, in addition to dbh, provided the best improvement based on fit and model validation.

The proposed model allowed the variability in heights within diameter classes depending on stand age and site index and therefore provided more realistic height predictions across varying stands than models which included dbh only. This feature is considered very important, since the generalized height-diameter model developed in this study may be used to predict tree heights of *A. mangium* not measured in the field.



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