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MATHEMATICAL UNDERSTANDING AND REPRESENTATION ABILITY OF PUBLIC JUNIOR HIGH SCHOOL IN NORTH SUMATRA

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Abstract

This paper is the result of the first phase of the research about the development of students' mathematical understanding and representation ability through Joyful Problem-based Learning (JPBL) at Public Junior High School in North Sumatera, Indonesia. The population is all of the students of public junior high school (PJHS) in North Sumatera. Samples choose based on stratified random sampling. The samples are the students of PJHS 27 Medan, PJHS 1 Percut Sei Tuan, PJHS 1 Tebing Tinggi, and PJHS 2 Pematangsiantar. The techniques used for collecting data is observation, interview, and essay test. The research findings: (1) Based on interview and observation found that conventional approach still uses in all of the class of PJHS, the students engagement in learning activity is very low, and most of the students do not attain minimal mastery achievement. (2) Based on essay test found that performance of the students in mathematical understanding and representation test is categories small.

Keywords: mathematical understanding, mathematical representation, joyful problem-based learning

Abstrak

Makalah ini merupakan hasil penelitian tahap pertama tentang perkembangan kemampuan pemahaman dan representasi matematis siswa melalui *Joyful Problem-based Learning* (JPBL) pada SMP Negeri di Sumatera Utara, Indonesia. Populasi dalam penelitian ini adalah semua siswa SMP Negeri (SMP N) di Sumatera Utara. Sampel penelitian dipilih menggunakan teknik *stratified random sampling*. Sampel penelitian yang terpilih adalah siswa SMP N 27 Medan, SMP N 1 Percut Sei Tuan, SMP N 1 Tebing Tinggi, dan SMP N 2 Pematangsiantar. Teknik pengumpulan data yang digunakan adalah observasi, wawancara, dan tes dalam bentuk esai. Temuan penelitian: (1) Berdasarkan wawancara dan observasi diketahui bahwa pendekatan konvensional masih digunakan di semua kelas SMP N; Keterlibatan siswa dalam kegiatan pembelajaran sangat rendah sebagaimana dengan kemandirian belajar siswa; Sebagian besar siswa tidak mencapai prestasi penguasaan minimal, (2) Berdasarkan hasil tes esai ditemukan bahwa kemampuan pemahaman dan representasi siswa termasuk dalam kategori rendah

Kata Kunci: pemahaman matematis, representasi matematis, joyful problem-based learning

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The objective of teaching and learning mathematics in high school is to understand mathematical concepts, describe the connection between concepts and apply concepts or algorithms in problem solving as flexible, accurate, efficient, and exact, as possible (MoE of Indonesia, 2006). Then it is relevant to Principle Standard of School Mathematics (NCTM, 2000) that the purpose of the student learn mathematics is to develop and deepen understanding of mathematical concepts and relationship as they create, compare, and use various representations. Mathematical understanding as well as mathematical representation is an integral part of mathematical problem solving, while problem solving is the hearth of doing mathematics. It means that the goal of learning various mathematical concepts is in order to perform problem solving, while through problem solving the students developed other kind of mathematical ability such as mathematical comprehension (understanding) and representation.

Anderson (2001) stated that the students is said to understand when they are able to construct meaning from instructional massages, including oral, written, and graphic communication presented to them during lectures, in books, or on computer monitor. The students understand when incoming (new) knowledge connected to their existing knowledge in their cognitive structure. For example, student understands the concepts of fraction addition if the concept connected to the concept of integer addition. According to Hiebert & Carpenter (1992), mathematical ideas, procedures or facts are understood if they are parts of an internal network, exist in cognitive structure of the student.

It seems that there is a connection between mathematical understanding and mathematical representation. On the other hand, mathematical understanding is very important in studying mathematics since it will be ease solving mathematical problem, even it will sharpen problem solving. Now, problem solving is the important thing the student must be mustered (Lee & Tan, 2004; Ronis, 2008) since almost every work field needs this skill. No doubt, teacher should teach the student such that the student has the opportunity in solving mathematical problems as well as mathematical understanding and representation.

In fact, the pilot research shows the achievement of mathematical problem solving of the students in Public Junior High School in Indonesia attain 67% (Minarni, 2013). TIMMS evaluation (Mullis, *et al.*, 2011) reported that Indonesian eight grade student' achievement in problems solving is categorized very low (under 400 from 600). They only get 19% in solving geometry problems, meanwhile the international achievement is 32%; they get 8% in solving algebra problems while the international achievement is 18%. After implementing problem-based learning then the students ability in mathematical understanding increased to 67 % (Minarni, 2013).

Another fact, the writer suspects that student low achievement in such kind of mathematical skills is because of learning approach teacher used, learning material, and rarely engagement of the student in solving mathematical problems. The teacher usually use conventional approach, while Ronis (2008) stated that conventional teaching learning is not bad but not enough to develop high order thinking skill such as problem solving skill. As consequences, the student rarely included in problem solving ability and the teacher give less attention in making learning material.

So, it is emerge that teachers need to know and implement learning approach that can support and facilitate students' mathematical comprehension and mathematical representation. Furthermore, the writer suggest that principle standard for school mathematics (NCTM, 2000) should be grasped by using learning approach which facilitate the students to solve various problems since by working in mathematical problems the students will gain problem solving ability as well as mastering various mathematical representation.

One of teaching learning approach that should be implementing in the school is Problem-based learning (PBL). It can be said that PBL is an instructional (and curricular) learners-centered approach that empowers the learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem (Savery, 2006). PBL is an innovative and

potential approach of teaching learning that can endorse and enable students constructing and reinventing their new knowledge. In PBL, the problem is the central of learning activity. The students are encouraged and facilitated to be actively engaging in solving problems. Using previous knowledge and experience, they try to sharpen their mathematical skills by solving real, challenging, open-ended, and contextual problems. From this kind of learning, it can be hoped that the students be motivated and interested to try to be a problem solver.

An innovation propose by Puri (2014) who stated that what happens inside schools has a deep and lasting effect on the mind-sets that children develop toward lifelong learning. A teacher affects eternity and one can never tell when his influence stops. In relation with that statement, she suggested that it is better that the teacher make classroom as interesting as game, joyful classroom. So, the writer considers that PBL maybe will be more powerful if dress up with the concept of joyful learning.

By considering the problem describe, the writer design the research with the purpose of increasing mathematical understanding and representation ability of public junior high school (PJHS) by developing joyful problem-based learning. This research consists of three phase, each phase take 9 month. The first phase has finished and reported here, and it relates to research questions in each based on two kind of instrumentation:

1. Based on Interview and observation,
   a. Do the teachers still use the conventional learning?
   b. Do the teachers know learning approach such as PBL that can improve the students' mathematical understanding and mathematical representation ability?
   c. Do mathematics teachers make the learning material by themselves?
   d. Do mathematics teachers make the instrument (test) as a part of learning material at the beginning of the semester?
   e. To what category is the students' engagement in learning activity? How is the students' self-regulated learning?
   f. Do the students attain the minimal mastery achievement in mathematics?
2. Based on Essay test,

   Do the students have the ability of mathematical understanding and representation?

Research questions based on two kinds of instrument describe descriptively. Formulation of competences of mathematical understanding and representation best suited for student presented in instrumentation section.

Nickerson (1985) describes that the student understands something if,

1. She/he can see the characteristic of the concepts deeply
2. She/he searches specific information on a situation quickly
3. She/he able representing a situation and view a situation with schema model
4. She/he also underlines the important of knowledge and capability to connect knowledge.

The other consideration of understanding ability suggested by Skemp (1987) who stated that there are three different kinds of understanding, there are instrumental understanding, relational understanding, and formal understanding. Instrumental understanding is the ability to apply appropriate remembered rule to the solve problem without knowing why the rule works. Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationship, and formal understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. So, mathematical understanding that will be developed in this research is relational understanding.

On the other side, *Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000) proposed that all students should grasp not only mathematical understanding but also mathematical representation ability. We can see in Puri (2014) that representation is a configuration that can *represent* something else in some manner. People develop representations in order to interpret and remember their experiences in an effort to understand the world. More specific, Kilpatrick *et.al.*(2001) stated that representation can be used to understand mathematics. Mathematics requires representations because of the abstract nature of mathematics such that people have access to mathematical ideas only through the representation of those ideas"

We can find in NCTM (2000) that mathematical representation will enables students to:

1. Create, and use representation to organize, record, and communicate mathematical ideas
2. Select, apply, and translate among mathematical representation to solve problems
3. Use representation to model and interpret physical, social, and mathematical phenomena

Mathematical representation can be represented into both visual representation and non visual. Visual representation including graph, table, sketch/figure, and diagram; non visual representation including numerical representation, and mathematical equation or mathematical model. The power of representation can be seen clearly whenever visual and numerical representation is used in the problems of ratios, proportions, and percents. The research focus on non-visual mathematical representation.

Students in grade two must familiar with various representation such as drawing physical objects, charts, graphs, symbols, and mathematical models. The students use these representations to organized and record their thinking about mathematical ideas, for example, they use representations to develop or apply their understanding of proportionality when they make or interpret scale drawings or figures or scale models of objects (NCTM, 2000). Without mustering representation, the students will not easy to solve a range of algebra problems, geometric, and linear equation problems due to they cannot easily move from one type of representation to another.

Puri (2014) proposed some ideas to create joyful in learning in classroom, they are let the students: (1) enjoy learning as well as they enjoy games, (2) create songs and rhythm when learning something new, (3)) decide on the topic they want to study for that week and then as "experts" they will teach the next week, (4) create things such as make a newspapers and magazines, brochures, stories, picture books, posters, PowerPoint

presentations, interviews, oral histories, models, diagrams, blueprints and floor plans, plays and role-plays, mock trials, photographs, paintings, songs, surveys, graphs, documentary videos, (5) show off students work, for example hang it at the wall, (6) get outside cause it is delightful for a student to sit under a tree and read or for a class to sit in a circle on the grass and talk, (7) read good book, allow books beyond the texts simply for the sake of student enjoyment.

METHOD

This prior research is conducted in four schools on behave of the population. The population is all of public junior high school students (PJHS) in North Sumatera, Indonesia. Samples chose through cluster sampling techniques are PJHS 27 Medan, PJHS 1 Percut Deli Serdang, PJHS 2 Pematangsiantar, and PJHS 2 Tebing Tinggi.

At each school, one of research team interviews teachers based on a set of interview guidance for the purpose of collecting data in relation with research question number 1-6, while another team observe the available of learning material based on observation guidance. Written test is gave to 40 students.

Data are collected using two sets of non test instrument: the first one is a set of interview guidance and observation. The interview guidance including four aspects, they are:

1. The aspect of learning approach

   Under this aspect, the interviewer must ask whether mathematics teacher have known about the innovative learning that stressed the important of students' engagement in the class, whether they interested in or have implemented the student-centered approach in the class, or they still use the conventional approach, etc.
2. The aspect of material instruction

   For the purpose of this aspect, the teachers are asked whether they have lesson plan and student's activity worksheet, whether they make supplementary book either for themselves or the students?
3. The aspect of instruments

   The questions related to the aspect of instrument including whether the teachers make the instrument based on instructional objectives or other considerations.
4. The aspect of students profile:

   The students profile including achievement in mathematics after the learning process, the students engagement in class discussion, and students self-regulated learning, etc.
5. Related to the student's achievement in mathematics

   The instrument consists of some question about students' mastery achievement in mathematics.

For the purpose to measuring students' mathematical understanding and representation, the researches has design essays test. The first set of test formulation was design based on aspects of mathematical understanding below,

1. Use figure to aid in solving problem
2. Give an example and non example for a concept
3. Classify examples into categories
4. Proposed mathematical equation
5. Understand and use the pattern to solve the problem
6. Apply similarity or differences to solve the problem
7. Explain the solution

Table 1. Holistic Analytic Marking Scheme for Math Understanding Test

Student performance	Mark
a. Understand the problem by represent it in the form (sketch, graph, diagram, or table) that can aid to solve the problem. b. Knowing relevant concept to apply in solving problem c. Showing an aspect of mathematical understanding which is fit to the problem c. Complete operations that lead the correct answer.	4
a. Understand the problem by represent it in the form that can aid to a correct answer if followed by an attempt to solve it. b. Showing an aspect of mathematical understanding which is fit to the problem c. The solution included all of mathematical component in the problem d. Complete operation but not lead to the correct answer	3
a. Understand the problem by represent it in the form of sketch, graph, diagram, or table that can aid to solve the problem. b. Showing an aspect of mathematical understanding which is fit to the problem c. The solution not included all of mathematical component in the problem d. Incorrect answer due to computational error or error in writing mathematical component	2
a. Understand the problem by represent it in the form that lead to correct answer. b. Use incorrect concepts or procedures that lead to incorrect answer.	1

The second set of test formulation was design based on aspects of mathematical representation below,

1. Represent story problem into symbolic form or mathematic equation.
2. Make table to solve the problem
3. Create mathematic equation from information presented in table
4. Using graphic as a tool to solve the problem
5. Make mathematic equation from the graphic

Table 2. Holistic Analytic Marking Scheme for Math Representation Test

Student performance	Mark
a. Represent the problem in the form (sketch, graphic, diagram, table, or mathematical equation that can lead to solve the problem. b. Use relevant concept in solving problem. c. Complete operations that lead to the correct answer.	4
a. Represent the problem in the form that can lead to a correct answer if followed by an attempt to solve it. b. The solution included all of mathematical component in the problem c. complete operation but not attain the correct answer	3
a. Represent the problem in the form (sketch, graph, diagram, or table) that can lead to solve the problem. b. Show incorrect answer due to computational error. c. Show incorrect answer though there is attempt to show some logical steps and operations that follow logically from an inappropriate representation.	2
a. Represent the problem in the form that could not lead to correct answer. b. Use incorrect concepts or procedures.	1

Data obtained is analyses descriptively based on four aspect included in the interview and aspects of observation as well as data of mathematical understanding and mathematical representation test result. If the students did not show any aspect of mathematical understanding or representation in their answers, then it classified as "*No answer*" and mark zero. For students who did not complete or attempt some problems, the works classified as "*No attempt*".

RESULTS AND DISCUSSION

In relation to learning approach, there's a school which is the teachers know learning approach which is based constructivism beside behaviorism, but they have not got training related to such learning yet. The teachers know learning approach that emphasizes on student-centered feature, but they still used conventional approach although they know learning approach such as Problem Based-learning (PBL) that can improve the students' mathematical understanding and mathematical representation ability even mathematical problem solving ability. Some of them have trained to implement Discovery Learning (DL) or Realistic Mathematics Education (RME), but they face obstacles to run it. They argument that it is hard to apply the learning approach in the class because of time consume such that only a little knowledge and information that can be delivered to the students, meanwhile the curriculum put abundant of knowledge that must be acquired by the students. The teachers worry their students could not get high score in National Examination conducted by the government at the end of school program. Success in National Examination is the indicator of the success of the school.

Interview results show that there are two out of ten teachers make material instruction (learning material). Other teachers tend to use ready learning material made by the team of mathematics teachers or buy it at a bookstore, they said it is more practical.

About instrument of the test: Though the experts of mathematics education stated that the instrument of the test must be an integral part of material instruction, the result of the study showed that some teachers not made an instrument before learning activity. Again, only two out of ten teachers made instrument (test) but the type of the test is multiple choices test, not an essay test. It means, learning objective teachers made is not intended for measuring the process of solving problems. It is obvious that multiple choices test could not measure the real mathematical ability of the students. Learning objective teachers made pointed out learning approach teachers used. It is reasonable to conduct the training for the teacher in relation with learning approach that can empowered teachers designing good instrument.

About the profile of the students: The students' engagement in learning activity and students' self-regulated learning are very low, about 25%. Majority of the students only sit down in the class, see their teacher explains the material and demonstrates solving routine problems. Students rarely ask the teacher about the material they did not understand, they just do exercise as explained by the teacher. Luckily, there still students who are willing to write their work on the board and share with friends

Another finding of the research is about students' achievement. Most of the students do not attain minimal mastery achievement in solving mathematical understanding test, while minimal achievement for the learning purpose is 65%. The last one of preliminary results showed that the performance of the students in solving mathematical understanding test belong to low category (the average achievement is only 19% (19 from ideal score 100). Figure 1 showed few students performance for the test. These students contribute great score for average score of the class that consists of 40 students. Some of the students solve the problem as in Figure 2. Most of them exhibit their ability as in Figure 3. It means that the students have the difficulty to represent problem in other manner that can make them understand the problem better.

Problem 1

There is a boat sailed from Port A in the North straight to Port B in the South along 20 km. The boat turn to the East as far as 24 km to reach Port C. From Port C, the traveling forward to Port D in the South along 12 km. Find the distance from Port A to Port D.

The answer of the students for this problem presented in Figure 1-3.

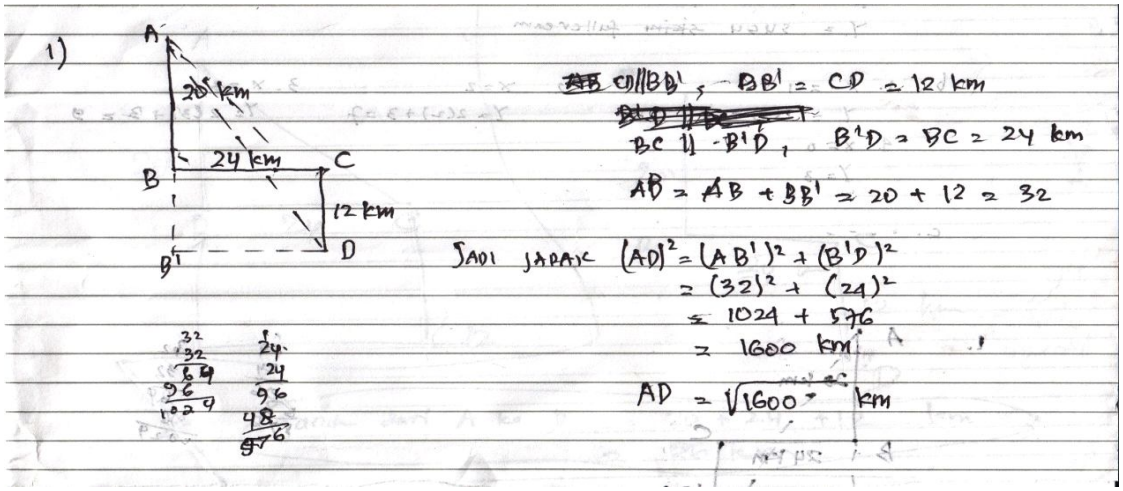

Figure 1. Student's work

It can be concluded from Figure 1 that the students have grasped the concept of Pythagorean completely, but they forget about how to find square root number such that it hard for them to solve the problem.

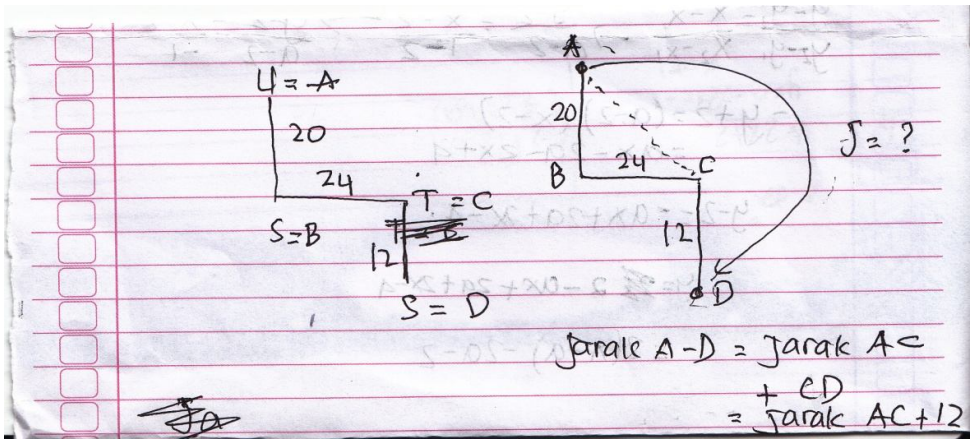

Figure 2. Student's misconception

On the other hand, Figure 2 showed that the students make figure to represent the problem but they forget the concept of *distance* so that they could not attain the solution. Student performance in Figure 3 does not show any representation anymore. While lot of students give solution similar to this performance.

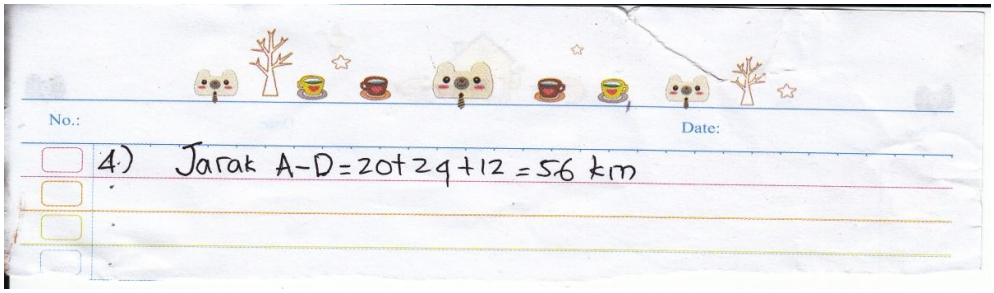

Figure 3. . Student performance

These fallacies are not due to the instrument since five education experts have validated it. The student themselves count for their disability as well as learning approach teachers used. According to Hiebert & Carpenter (1992) and Carpenter & Lehrer (1999), the students will not be easy to retrieve knowledge from the memory if the knowledge is not store firmly in their cognitive structure. Fluency retrieval affects the fluency of applying knowledge to solve the problems. On the other hand, conventional class could not enable students grasp various mathematical representation because the students rarely engage in mathematical problem solving.

Problem 4

Let the line l passes through P(2, 2) and A(1, a) and intersect the line $y = 2x + 2$ perpendicular. Determine the value of a. Give an explanation for each step you used in solving the problem.

Students' solution for this problem can be categories into three type, each type presented in Figure 1 until Figure 3 respectively.

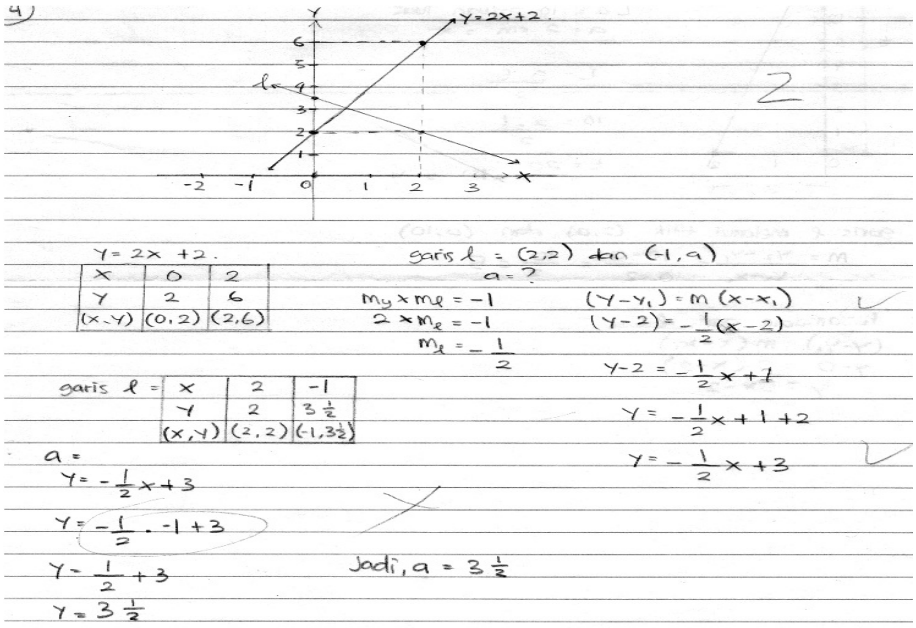

Figure 4. Student's ability to create charts

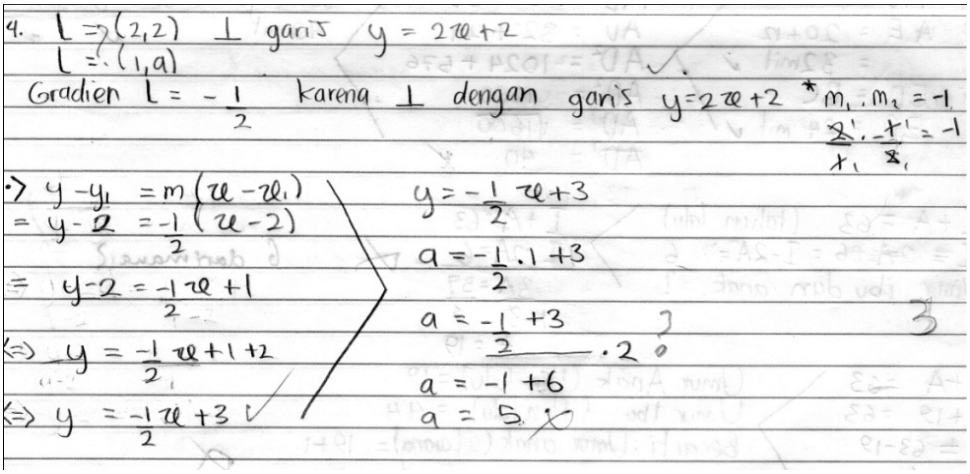

Figure 5. Similar the solution

We can see from Figure 4 that he student's ability to create charts, tables, and mathematical model is quite good. This student has the ability of multiple representations. Unfortunately, he did not read the problem carefully so that he write line *l* passes through the point (-1, *a*). In the problem, line *l* is actually passes through the point (1, *a*). Based on holistic marking, the performance of the student in solving mathematical representation test should be marked 3. Only four students (about 10%) solve the problem as shown in Figure 4. Up to 90% of the students gave the solution for problem 4 similar the solution in Figure 5 and Figure 6.

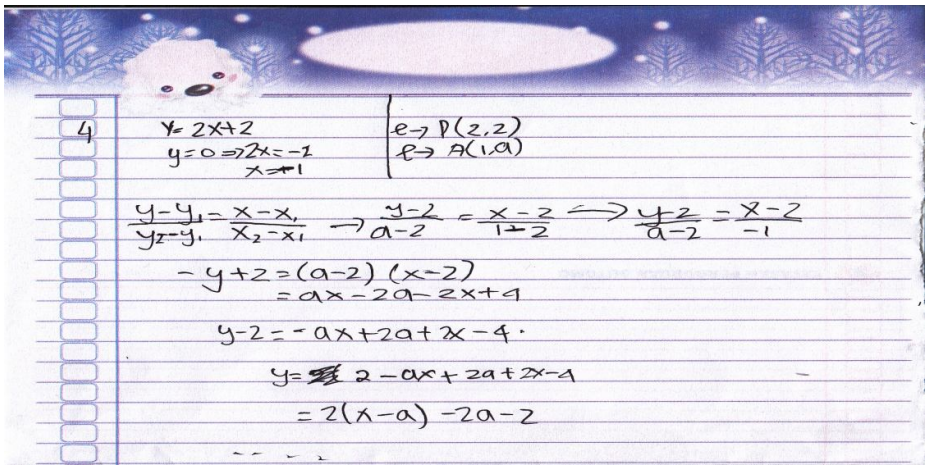

Figure 6. Another student's answer

It is obvious that most of the student have no comprehensive capability in solving mathematical representation problem. There is no visual representation in both figures. Based on interview result, the students think it is not important to create graph for this problem, just use the formulae of a line equation. We see, only knowing the formula of line equation is not enough to attain the correct answer. In general, the students do not like to spend much time in doing mathematics. Probably it is due to the phenomena that the heart of many students tends to play the game on the computer or at the internet. Only few students that still have the spirit

and willingness doing mathematics seriously. It is make sense why Puri (2014) suggested us the teacher to create classroom such that the students enjoy learning as they enjoy game.

It is emerge that there must be massive and intensive training about innovative learning approach for teachers so the teachers are motivated and eager to implement such as *Problem-based Learning* (PBL), Discovery Learning, and other approach as Indonesia Minister of Education suggested. The teachers must also train and provoke to write a book and other material instruction based on PBL approach so that the students engaged in solving mathematical problems continuously. While the students solving the problem they develop the ability of mathematical representation, understanding, reasoning, and mathematical communication. On the other hand, they develop such mathematical ability, then the ability of mathematical problem solving become increasing.

Evaluation system for assessing the student is also the important part to be improved. It is time to promote holistic evaluation system. It means that the teachers should not evaluate or give marking to the students based on paper and pencil test only but also must consider the students performance in the class. The teachers should create classroom conducive enough to make students dare to speak out what they are thinking in connection with mathematical problems they face.

For the purpose of paper test in PBL, items of the test must be designed such that each item is non-routine problem. Such problems are motivated and invested the students in the developing of the solution. If the students always solve non-routine problems, it can be hoped that they will be become a good problem solver.

The students' performance in the answer sheet can become a good indicator of the degree of the success in implementing PBL. The students' answer sheet should be consist of the convey of external representations of their understanding about the problem, their thinking about the strategy and how to execute the problem so the teacher can be realized whether the students show the ability of solving problem or not, especially solving mathematical understanding and representation problems.

CONCLUSION

There are two conclusions drawn from the research. First, based on Interview and Observation, there are conventional approach still use in all of the class of Junior Public School, the teachers have known there is innovative learning approach such as PBL that can improve the students' mathematical understanding and mathematical representation ability, the teachers have not been accustomed to make teaching material, test is not made before learning activity, the students' engagement in learning activity is very low as well as students' self-regulated learning, and most of the students do not attain minimal mastery achievement. Second, base on Essay test, there is the students' ability in mathematical understanding and representation test is categories low. So, the writer (the team of researcher) continue to develop competencies of mathematical understanding and representation along developing material instruction.

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