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SUPPORTING STUDENTS' UNDERSTANDING OF LINEAR EQUATIONS WITH ONE VARIABLE USING ALGEBRA TILES

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Abstract

This research aimed to describe how algebra tiles can support students' understanding of linear equations with one variable. This article is a part of a larger research on learning design of linear equations with one variable using algebra tiles combined with balancing method. Therefore, it will merely discuss one activity focused on how students use the algebra tiles to find a way to solve linear equations with one variable. Design research was used as an approach in this study. It consists of three phases, namely preliminary design, teaching experiment and retrospective analysis. Video registrations, students' written works, pre-test, post-test, field notes, and interview are technic to collect data. The data were analyzed by comparing the hypothetical learning trajectory (HLT) and the actual learning process. The result shows that algebra tiles could support students' understanding to find the formal solution of the linear equation with one variable.

Keywords: linear equation with one variable, algebra tiles, design research, balancing method, HLT

Abstrak

Penelitian ini bertujuan untuk mengetahui bagaimana penggunaan *algebra tiles* dapat mendukung pemahaman siswa dalam mempelajari konsep persamaan linear satu variabel. Artikel ini merupakan bagian dari penelitian besar pada desain pembelajaran persamaan linear satu variabel menggunakan *algebra tiles* dengan metode menyeimbangkan. Oleh karena itu, artikel ini akan membahas satu aktivitas yang difokuskan pada bagaimana peranan *algebra tiles* unuk menemukan metode penyelesaian persamaan linear satu variabel. *Design research* digunakan sebagai metode dalam penelitian ini. *Design research* terdiri dari tiga tahap yaitu pendahuluan, uji coba lapangan dan analisis. Teknik pengumpulan data diperoleh melalui rekaman video, lembar kerja siswa, *pre-test*, *post-test*, catatan lapangan dan wawancara. Teknik analisis data dilakukan dengan cara membandingkan antara HLT yang telah dirancang dan apa yang terjadi selama proses pembelajaran. Hasil penelitian menunjukkan bahwa *algebra tiles* dapat mendukung pemahaman siswa dalam menemukan penyelesaian formal persamaan linear satu variabel.

Kata Kunci: persamaan linear satuvariabel, *algebra tiles*, *design research*, metode balancing, HLT

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Linear equation with one variable is a beginning algebraic topic taught in the 7th grade. Cai, et al., (2005) clarified that "algebra has been characterized as an important 'gatekeeper' in mathematics". Besides, in Al Khawarizmi's book, it is stated that "a motivation for studying algebra was the solution of equations" (Krantz, 2006). It shows that linear equation with one variable is really necessary to support learning of other topics in mathematics. However, learning process that use in Indonesia is not support the students to understand the concept of solving linear equations with one variable. Most of learning processes are only acquainted with the formal strategy in teaching linear equation with one variable (Jupri, 2015).

Solving linear equation is particularly important concepts in algebra and on that causes confusions for students (Magruder, 2012). Then, Magruder (2012) clarified that there are three primary subtopics where students found difficulties when solving equations are; 1) symbolic understanding; 2) the meaning of the equal sign; and 3) a reliance on procedural knowledge without conceptual understanding. Moreover, Jupri, Drijvers and van den Heuvel-Panhuizen (2014) stated that one of common mistakes on understanding the concept of linear equation is applying arithmetic operation. For instance, to find the value of x in the equation $3x = 5$, it has to be 5 divided by 3. However, the students commonly come with $x = 5 - 3$. To learn the topic of solving linear equation with one variables, students struggle to balance conceptual and procedural knowledge (Magruder, 2012). Linear equations are often difficult for students in transition from a concrete mathematics to an abstract concept. So, a learning that bridges the students' thinking from abstract to real is needed.

Pendidikan Matematika Realistik Indonesia (PMRI) is a learning approach which can supports the students' thinking from informal to formal level. Sembiring (2010) stated that PMRI is adopted from Realistic Mathematics Education (RME). The concept of RME is based on Hans Freudenthal's view of mathematics as a human activity which is discovering and organizing between content and form (Freudenthal, 2002). Treffers (1987) clarified that there are five characteristics of RME as described below.

1. Phenomenological exploration

   The first instructional phase takes an extensive phenomenological exploration. Therefore, a concrete context should have taken place in the mathematical activities. Contexts or real situations can be a starting point in teaching (Zulkardi, 2002). Therefore, context of tilling was used in this activity that students are often face in their daily life.

2. The use of models

   The theoretical was build step-by-step from informal to formal level. Gravemeijer (1994) explained that developing models is imagined by changing process from a certain situational (model of) to a formal situational (model for). Further, he elaborated four levels of emergent modelling, namely situational level, referential level, general level, and formal level.

3. The use of students' own constructions and productions

   The steps to reinvent a concept should be done by the students themselves. The biggest contributions on learning process come from students' constructions when they move from informal to formal strategies.

4. Interactivity

   The interactivity means that the students also expected with the production of their fellows, it can stimulate their way in the learning, to help themselves to develop procedures of others. Interactivity between each other, students and teacher help the students to develop their understanding of the lesson.

5. Intertwining

   Intertwining stands means that the students have relation the using of relevant learning theory, interrelated, and integrated with other learning topics. The intertwinement of various mathematical strands can increase students' knowledge.

Fauvel and van Maanen (2002) stated that history of mathematics have a role to supports learning mathematics. History of mathematics can be used to be one of the resources to understand formal process in thinking of mathematics (Fauvel & van Maanen, 2002). Many researchers suggested to use history to learn mathematics (see Fauvel & van Maanen, 2002 and Fachrudin, Putri & Darmawjoyo, 2014). Radford (1996) stated that the historical construction is one of ways to support students' understanding in constructing their knowledge of mathematics. Grugnetti (2000) and Fachrudin, Putri & Darmawjoyo, (2014) clarified that there are three points of history of mathematics that contribute to overcome the problems in learning as mentioned below.

1. By using old problems, students can use their own strategy and compare it with the original strategy.
2. By employing the history for constructing concepts and skills in mathematics, it could help the students to improve the way of thinking on how a concept was developed.
3. An analysis of historical and epistemological allows teachers to develop a didactic approach and to understand why the concepts are so difficult for students.

Based on the perspective history of mathematics, the concept of algebra appears from the mathematical problems related to geometry. Al-Khawarizwi wrote that the algebraic problems appears on many contexts in the human life, for instance in the geometrical computation (Merzbach & Boyer, 2010). The word "*muqabalah*" on Al Khawarizmi's book, titled *Hisob al-jabarwa'lmuqabalah,* refers to "reduction" or "balancing". It is the cancellation of like terms in equation from opposite sides (Merzbach & Boyer, 2010). Moreover, van de Walle, Karp and Williams (2010) stated that one way to work on equation is through balancing. Therefore, the concept of "balancing" or "reduction" become a basic concept to solve linear equation. That is appropriate with five commons senses, which was written in the Elements book of Euclid (Bernardz, Kieran, & Lee, 1996). Three of the senses are mentioned as follows:

1. Things which are equals to the same things are also equal to one another
2. If equals be added to equals, the wholes are equal
3. If equals be subtracted from equals, the remainders are equal

Three of them become the fundamental concept to design the learning of solving linear equations with one variable. Therefore, the researcher applies the concept of addition, subtraction, multiplication and division of the same things on both sides. However, many students have difficulty to understand the concept in solving linear equation with one variable (e.g., Amerom, 2002 and Jupri, 2015). It is caused the way of teaching by memorization of the formulae to understand the concepts (see for instance, Zulkardi, 2002 and Jupri 2015). Moreover, Kemendikbud (2013) clarified that formal way of algebra education in Indonesia start in the first semester of the 7th grade. The lack of understanding in the fundamental concept and the difficulty to bring linear equation into the real life become one of the factors inhibition of learning and teaching algebra.

Delvin in Witzel, Mercer and Miller (2003) stated that one way to simplify students' understanding of abstract concept is by using manipulation and pictorial representations. Therefore, the researcher promotes the use of manipulative in mathematics especially linear equation with one variable in order to help students bridge their thinking from concrete to abstract mathematics. The manipulative in this study refers to a model which stated by Lesh in Tasman, den Hertog, Zulkardi and Hartono

(2011) that "models are the 'the things' that mathematicatians use for interpreting situations mathematically by mathematizing objects, realtions, operations and regularities".

A set of algebra tiles is one of the powerful manipulative for students to explore and express mathematical problems into algebra (Heddens & Speer, 2001). Algebra tiles is a mathematical load area that implicates the acquirer of patterns and models, variables, exponents, graphing, and functions (Heddens & Speer, 2001). Algebra tiles are manipulatives that support students to visualize polynomial operations and solve equations. A set of algebra tiles consists of at least three different shapes as the following dimensions and values.

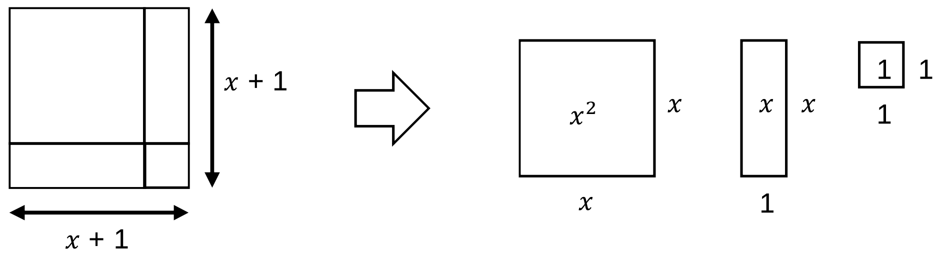


Figure 1. A Set of Algebra Tiles

In this study, we merely use the rectangle and the small square of algebra tiles in learning linear equation with one variable. Heddens and Speer (2001) stated that to advance the students' understanding in linear equation related to subtraction operation, they can use algebra tiles with two different colors, one to represent the positive while the other is negative. The purpose of using different color is to get zero pairs of algebra tiles as illustrated bellow.

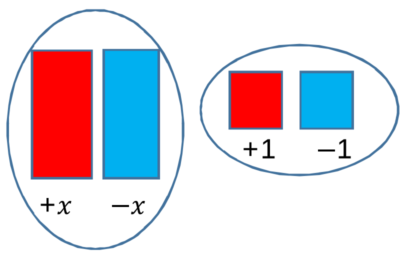


Figure 2. Zero Pairs of Algebra Tiles

It is extremely important that students verbalize using algebra tiles when solve linear equation. The students have to know what they are doing in order to find the value of *x*.

The aim of this study is to explore how the algebra tiles with balancing method supports the students' understanding in solving linear equation with one variable. Therefore, the researcher formulate the general research question as: *how can the algebra tiles using balancing method support students' understanding to solve linear equations with one variable in grade 7 Junior High School?*

METHOD

This research involved 32 students of 7th grade and a regular teacher as the teacher model during the teaching experiment. This research was conducted in SMP Pusri Palembang. Design research was chosen as method in this research. Design research is a systematic study of designing, developing and evaluating educational interventions (such as programs, strategies and instructional materials, products and systems) as a solution to solve complex problems in the practice of education (Plomp & Nieven, 2007). Design research is aimed at developing a local instructional theory (LIT). It based on the existing theories (the theory-driven) and empirical experiments through the cooperation between researchers and teachers to improve the relevance policy and practice of education (Gravemeijer & van Eerde, 2009). There are three main phases in design research, namely preliminary design, teaching experiment and retrospective analysis (Gravemeijer & Cobb, 2006).

In the first phase, researcher designed a hypothetical learning trajectory (HLT) consisting of students' starting points, learning goals, learning activities, and the conjecture of students' thinking during the learning process. In the phase of teaching experiment of this study involved there are two cycles, namely pilot experiment (cycle 1) and teaching experiment (cycle 2). The pilot experiment is aimed to test the HLT that has been designed, then revised if needed and implemented in the next teaching experiment. Data were collected through video registrations, students' written work, pre-test, post-test, interview, and field notes during both cycles. Data were analyzed by comparing the actual learning process and the HLT during the retrospective analysis.

In the learning design, we developed five activities in learning the concept of linear equation with one variable. As a part of large study, in this study, we merely focus on the fourth activity of designed learning activities. This activity is aimed to investigate the students' strategy in finding the solution of linear equation with one variable.

RESULTS AND DISCUSSION

This part will discuss the analysis of students' work on the use of algebra tiles to find the solution of linear equation with one variable. In this activity, students have to know what they should do with the algebra tiles that they arranged. Students are given mathematical problems related to the context of tiling as the representation of algebra tiles. The problems were given to the students can see in the Figure 3 below.

Mr. Arul wants to put square and rectangle tiles in the bathrooms A and B. Given that the area of bathroom A same as bathroom B. This illustrated as shown below.

Bathroom A needs four rectangular and two square of tiles. Bathroom B needs two rectangular and 6 square of tiles. Then, Mr. Arul want to know how many square which equivalent with a rectangle.

Figure 3. The Problems in Activity 4.1

At first, students counted how many tiles they used. They began with using the algebra tiles to represent the quantities of the given problem. Students' arrangement of the algebra tiles of the given problems is shown in Figure 4 below.

Tiles model in left side (bathroom A)	Tiles model in left side (bathroom A)

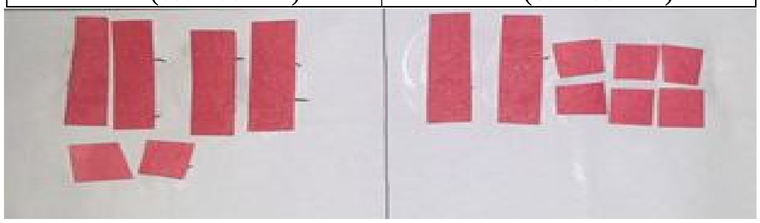


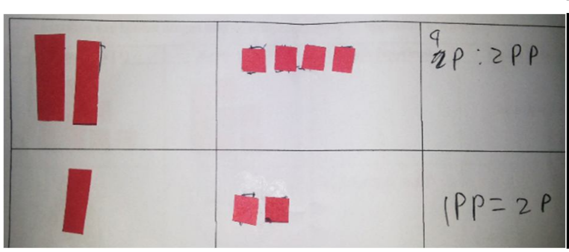
Figure 4. Students' written work on arranging Algebra Tiles

Based on the Figure 4 above, we can see that students were able to represent the problem by using the algebra tiles correctly and accurately. The Students put four red rectangular tiles and two red square tiles on the first column. Then, in the second column, they put two red rectangular tiles and 6 red square tiles.

The next activity, the students have to find how many squares are equal with a rectangle. The students know that both sides are in balance (see Figure 4). In discussion, the students may remove square or rectangle at first. Then, teacher asked them to remind which one is not know the quantity. Most Students said that the unknown is rectangle. It means that the students understand that variable is unknown which is appropriate with the conjectured.

In discussion, some groups, like Hazel's Group, removed the tiles one by one until they found the number of square. We observed that they do that for determine the number of square easily. In other groups, like Syafri's group, removed two red tiles at first. This way is more effectively than Hazel' Group. In the next row, the students remove any tiles that have the same value from each side (see Figure 5).

Remove $2pp$	
Remove $2p$	



$4p = 2pp$	
$1pp = 2p$	

Figure 5. Syafri's Group Work in Exploring the Problem Using Algebra

From the Figure 5 above, we can see that the students were able to find a simple form of algebra tiles they formed. In a very concrete level, the students were able to remove each column with two red rectangular tiles. Then, they reduced two red square tiles from each sides, leaving one rectangle on the left side and two squares on the right. Therefore, the "x" tile represents 2 tiles such that $x = 2$ is the solution of equation. Here are the Fragment that shows how the students used the algebra tiles to solve the given problem.

Fragment 1

Syafri	: there are 4 red rectangular tiles and 2 red square tiles
Researcher	: yes
Redho	: this is bathroom B (refers to the second column)
Syafri & Redho	: the bathroom B consist of 2 red rectangular tiles and 6 red rectangular tiles
Stafri	: then…cancelling
Researcher	: what do you mean by cancelling?
Redho	: cancelling 2 red rectangular tiles (refers to 1 and 2 columns)
Syafri	: subtracted by 2
Researcher	: each of it?
Syafri & Redho	: subtracted by 2 rectangles
Researcher	: 2 rectangles…if each of it, I subtracted by 2 rectangles, will the both of right side and left side have the same value?
Syafri & Redho	: same
Researcher	: still the same? Then, what did you do after that?
Syafri	: this is bathroom A, so there are 2 rectangles and 2 squares, it is still have 1, 2, 3…
Redho	: 6 squares
Syafri	: yes, 6 squares
Researcher	: then?
Syafri	: subtracted by 2 squares. So, this is subtracted by….
Researcher	: what? What did you subtract?
Syafri&Redho	: 2 squares
Syafri	: have 2 rectangles, this 4 rectangles
Researcher	: yes
Syafri	: this subtracted by 2 rectangles
Researcher	: why did you subtract the rectangles? Is there any rectangles here?
Redho	: there is not…
Syafri	: divided
Researcher	: how did you divide it?
Syafri	: each of them is divided by 2
Researcher	: each of them is divided by 2. So, 1 rectangle is equals to?
Syafri&Redho	: 1 rectangle is equals to 2 squares

Based on the Fragment 1, we can see that the students did reducing and balancing method to find the simple form of algebra tiles. They balanced the both side as they did in the previous activity. The balancing method becomes a main idea to find the solution of linear equation with one variable. The purpose of reducing or balancing of the both sides is to get how many square tiles that are equal to 1 red rectangular tile.

The next activity was asking the students to analyze every step that they do. In discussion, the students not know how to represent removed step in mathematics model. At the first, Syafri's group does not understand that "removed or cancel step" is means "subtract". The teacher asked them to replay what they do to find the number of squares, then asked them to understand that activity in mathematics model.

In the next discussion, the students have to formulae the steps in algebraic form. Based on the Figure 5, then Syafri's group represented the tiles which they arrange in x, where x is equals to red rectangles tiles (see Figure 6).

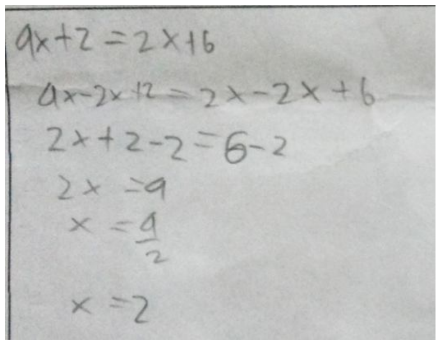


Figure 6. Representation of Algebra Tiles on Formal Strategy

Based on the Figure 5, we can analyze that students were able to find the solution of linear equations with one variable as shown in Fragment 2.

Fragment 2

Researcher	: how did you find the solution?
Rizki	: I try… $4x + 2 = 2x + 6$
Researcher	: what steps are you did?
Rizki	: what?
Researcher	: your steps
Rizki	: both of them were subtracted by 2
Researcher	: what did you means by 2?
Rizki	: 2 rectangles
Researcher	: 2 rectangles was equals to?
Rizki	: x
Researcher	: so, subtracted by what?
Rizki	: subtracted by $2x$

The transcript above shows that the students were able to analyze every step in their arrangement of algebra tiles into the steps of solving linear equation with one variable. The students found the formal way to get solution of linear equation with one variable.

Further, the students solved the problems related to the subtraction form in the linear equation with one variable. As for an illustration of the problem can be seen in Figure 7.

> " A builder said that the area of two bathrooms will same if bathroom A prepared of four rectangular tiles subtracted seven square tiles while bathroom B prepared of 2 rectangular tiles and 3 square tiles"

In this activity, the teacher informed the students that the blue tiles is negative form in algebraic. So, the student could arrange the problem in the algebra tiles form which involves the pair of algebra tiles. This can be seen in the following student answers (see Figure 7).

Figure 7. Syafri's Group and Talitha's Group Answer in Composition of Algebra Tiles Related in Negative Linear Expression

In this activity, the students discuss what would they do to remove the blue tiles. Some groups, like Syafri's group and Talitha's group try to remove 7 blue squares in left side, but they have confuses because in right side there are no blue squares. We observed that they try to solve it with the same way in the problem before. So, both of groups employed the strategy of subtracting each column with 2 red rectangle tiles (see Figure 7).

When the students found the blue tiles in left side, they get difficult to determine the next step. The teacher remind them that the pair of blue tiles is red tiles. The purpose of this form is to get zero value so that the blue tiles removed from the side. Most students have difficult to understand this step. So, the teacher asked Syafri's group to show their step (see Figure 8).

Figure 8. Student Presentation

From the presentation above (see Figure 8), the student understand that to remove blue tile in one side, they can use the pair of it to cancel the tiles. Then, students added each side with 7 blue square tiles in order to get zero from both sides. They got 2 red rectangles tiles in the left side and 10 red square tiles in right side. We observed that student understand that to remove negative form in algebraic is by added the number of it. Although, in this activity is not up to the mathematical modelling. Further, they divided both of sides by 2 such that they found that one red rectangle tile is equals to 5 red square tiles. It shows that students were able to solve subtracting problems related to linier equation with one variable. They used the pairs of algebra tiles to find the zero value. It is appropriate with the HLT which we designed.

CONCLUSION

Based on the learning process that has been conducted, we can conclude that students can use algebra tiles solve linear equation with one variable in a formal way. Moreover, the use of algebra tiles can minimize the common mistakes that happened when solving linear equation with one variable. Besides that, students' understanding is developed from informal to formal level. The researcher suggests that to support students' understanding of solving linear equation with one variable, teachers can use algebra tiles assisted with contextual problems that close to students' daily life. Moreover, PMRI approach can make the learning process become meaningful to students in understanding linear equation with one variable.

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