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Helping Students Acquainted with Multiplication in Rectangular Model

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Abstract

Usually, multiplication is introduced to students to represent quantities that come in group. However there is also rectangular array model which is also related to multiplication. Barmby et al (2009) has shown that the rectangular model such as array representations encourage students to develop their thinking about multiplication as a binary operation with row and column representing two inputs. Considering that finding, this study focusses on a design research that was conducted in Indonesia in which I investigate second grade students' (between 7 and 8 years old) in Madrasah Ibtidaiyah Negeri (MIN) 2 Palembang, Indonesia, ability to structuring the situation and their ability to represent rectangular model into multiplication sentence. The results shows us that students activity to structuring the situation, looking the number of objects in row or in column, lead them to repeated addition and transform it into multiplication sentence.

Keywords: Rectangular pattern, Multiplication

Abstrak

Biasanya, perkalian diperkenalkan kepada siswa untuk menyatakan jumlah total objek dalam kelompok. Namun ada juga model persegi panjang yang juga terkait dengan perkalian. Barmby et al (2009) telah menunjukkan bahwa model persegi panjang seperti sebagai representasi dari susunan-susunan objek dapat mendorong siswa untuk mengembangkan pemikiran mereka tentang perkalian sebagai operasi biner dengan baris dan kolom mewakili dua input. Menimbang temuan tersebut, penelitian ini fokus pada desain research yang dilakukan di Indonesia pada siswa kelas dua (usia antara 7 dan 8 tahun) di Madrasah Ibtidaiyah Negeri (MIN) 2 Palembang. Dimana siswa kelas dua tersebut di selidiki kemampuan mereka untuk penataan situasi dan kemampuan mereka untuk menyatakan jumlah objek pada model persegi panjang ke kalimat perkalian dengan menggunakan pendekatan Matematika Realistik. Hasil dari penelitian menunjukkan bahwa dengan penataan situasi, mencari jumlah objek dalam baris atau kolom, menggiring siswa kepada penjumlahan berulang dan mendorong mereka untuk mengubahnya menjadi kalimat perkalian.

Kata Kunci: Model Persegi Panjang, Perkalian, Desain Research, Matematika Realistik

Introduction

In Indonesia, learning multiplication usually focus on group model. However, multiplication can take not only group model (bags, boxes,…) appereances in context situations, but also line model (chain, strip, number line), and rectangular model (starts, grids, …) (Van den Hauvel Panhuizen, 2001). These contexts problems and the models are so important because they reveals important variants of the underlying basic structure of multiplication and offer insight into the properties to basic operation which is important for calculation.

Barmby et al (2009) has shown that the rectangular model such as the array representation is a key representation for multiplication in elementary school students. The array representation encourages students to develop their thinking about multiplication as a binary operation with row and column representing two inputs. Initially, students structure array as one dimensional path, where they can see the structure in one dimension (row or column) but not both (Dolk and Fosnot, 2001).

Considering the important contexts situation and the models, I designed a study to examine second grade students' ability to represent the total number of objects in rectangular model into multiplication sentence. I used contexts 'the tiles' in my design because the students usually see tiles in the floor of their house or the floor of their school. This report discusses experimental study in which I aimed to better understanding of multiplication for students in grade 2 elementary school.

Theoritical Framework

I used some theoretical framework to underpin this research, those were: multiplication and rectangular model – as concept behind my research goal; Realistic Mathematics Education (RME) as an approach of mathematical lesson; and Emergent Modeling as a bridge for students to reach the mathematical goal.

1. Multiplication and Rectangular Model

   None of the mathematical operations, not even addition and subtraction, is understood as spontaneously as multiplication (Freudenthal 1983). Multiplicative term such as "times" precede multiplication as arithmetical operation. The term "times" is related to the language that students usually hear in daily life. The term "times" means iterating the unit, for example, 3 km is 3 times as long as 1 km if the unit is 1 km, 6 apples is 3 times as many as 2 apples if the unit is 2 apples. Eventually it serves as a tool for thought as starting point to learn multiplication.

   To come to multiplication, the term "times" first connects to the idea "add so many times" (Van Hauvel-Panhuizen et al, 2001). When students add so many times, this situation represents familiar procedure which is students are able to perform multiplication (Coney et al, 1988). When students are counting using repeated addition with long strings of repeated addition, this can be tedious and difficult for students. The students often combine a group to make addition easier (Van Galen and Fosnot, 2007). For example 8 groups of 4, students might make 4 groups of 8, transform these into 2 groups of 16. This idea is called by Van Galen and Fosnot as regrouped repeated addition and they determine this idea as one of the big ideas when students learn multiplication.

When students learn multiplication by mathematizing – the human activity for organizing and interpreting reality mathematically – their reality, mathematical models become important. Models are the “things” that mathematicians use for interpreting situations mathematically by mathematizing objects, relations, operations and regularities (Lesh et al, 2003). Sometimes students need to modify or extend them by integrating, differentiating, revising, or reorganizing their initial interpretation. According to Dolk and Fosnot (2001), they interpret models as tools for thought. It often begins simply as representations of situation or problems by the students.

The rectangular model is important for mathematics learning because of its use to model multiplication (Outhered. L, 2004). Students might be not seeing the structure of array as contiguous squares thus they might not connect an array of squares of multiplication. To link the rectangular model to multiplication, students need to perceive that the number of rows or columns is equal and correspond to equivalent group.

Only gradually do students learn that the number of units in a rectangular model can be calculated from the number of units in each row and column (Battista, Clements, Arnoff, Battista, & Borrow,1998 in Outhred L,2004). According to Dolk and Fosnot (2001), understanding array is a big idea in itself. Like as students understand unitizing – how numbers of objects can simultaneously be one group – they struggle to understand how one square can be simultaneously be part of a row and a column.

When students learn arithmetic, it is essential that they not only learn number facts (such as multiplication tables) and algorithms but also develop a conceptual understanding of relevant underlying mathematical principles (Squire et al, 2004).

The students need a greater understanding of the process of multiplication as well as when and how to use the multiplication facts (Caron, 2007). In order to make a greater understanding of the process of multiplication, a real and meaningful instructional activity is needed. Therefore we used Realistic Mathematics Education approach in our design.

2. Realistic Mathematics Education

According to Freudenthal, in his book Revisiting Mathematics Education ; China Lecture (1991).

> Mathematics has arisen and arises through mathematising. Mathematising is mathematising something – something non-mathematical or something not yet mathematical enough, which need more, better, more refined, more perspicuous mathematising. Mathematising is mathematising reality, pieces of reality. Mathematising is didactically translated into reinventing, the reality to be mathematised is that of the learner, the reality into which the learner has been guided, and mathematising is the learner's own activity. (P.66)

To help students mathematize reality, Realistic Mathematics Education (RME) has five tenets or principles (Treffers, (1987) in Gravemeijer, K. Van den Hauvel, M & Streefland 1990) that were also applied in this research.

The tenets and application in this research are described below;

1. Constructions stimulated by concreteness.

   The instructional activities that we designed does not starts in the formal level but starts with a situation that is experientially real for students with purpose that it will make meaningful for the students because the students can explore and construct the mathematical idea with it. Therefore, we used contexts 'the tiles' that the students usually see in the floor or wall of their house or school.

2. Developing mathematical tools to move from concreteness to abstraction.

   This tenet of RME is bridging from a concrete level to a more formal level by using models and symbols. Students' informal knowledge as the result of their experience needs to be developed into formal knowledge. The teacher helps the students by guiding them while students mathematizing their reality. In our instructional activity, counting tiles, we ask students to make their representation of complete tiles that arrange in rectangular pattern. Consequently, the class discussion will be held to encourage the students making their model-of situation and move to model-for for their mathematical reasoning. Therefore the rectangular model presents as the students model of situation for the students. When students ask to represent the situation in multiplication sentence, repeated addition are expected arose as model for their mathematical reasoning. After that the multiplication sentence introduces to them with connected with the idea “add so many times”.

3. Stimulating free production and reflection

   The idea of this tenet is to raise the levels must be promoted by reflection, which means thinking about one’s own thinking. Students’ own construction or production assumed will be meaningful for them. During the activities and class discussion the students’ construction are used to guide them to the next level, or more formal level. The students’ strategies in each activity were discussed in the following class discussion to supports students’ acquisition of multiplication.

4. Stimulating the social activity of learning by interaction

   Because the learning process takes place in the social school environment, this situation makes the students have interaction between each other. This interaction is a kind of social process. The understanding of the lesson can be come from students’ interaction with each other, when they communicate their work and thought in the social interaction in the classroom. In our instructional activities, the students do the activities in the group of three or four. After they discuss in group, the class discussion are held to make they share their idea with other students.

5. Intertwining learning strands in order to get mathematical material structured

   This principle of instruction concerns intertwining learning strands. Intertwining learning strands means that the topic that the students learn should have relation with other topics. This tenet suggests that to integrate various mathematics topics in activity. The relationship between multiplication in the domain of counting and the area in the domain of measurement relies on this rectangular array model.

3. Emergent Modeling

The implementation of the second tenet of RME produced a sequence of models that supported students' acquisition of the basic concept of multiplication. Emergent modelling asks for the best way to represent situation that the students can reinvent or develop their idea about the concept of mathematics (Gravemeijer, 2004[*]). That situation makes emergent modelling is one of the heuristics for realistic mathematics education in which Gravemeijer (1994) describes how *model-of* a situation can become *model-for* for more formal reasoning. There are four levels of emergent modelling. The levels of emergent modelling are shown in figure 1 below:

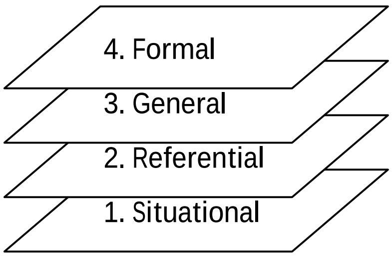


Figure 1. Levels of emergent modelling from situational to formal

The implementation of the four levels of emergent modelling in this paper is described as follows;

1. Situational level

   Situational level is the basic level of emergent modelling. In this level domain specific, situational knowledge and strategies are used within the context of situation. In this research, we give the students situation for counting. We expect that the students could find efficient strategy to count such as counting in groups by using the structure of objects.

2. Referential level

   In this level models and strategy refers to situation that sketched on problems. This level also called model-of. A class discussion encourages students to shift from situational level to referential when students need to make representation (drawings) as the model-of their strategies to count the objects.

3. General level

   In this level, a mathematical focus on strategies that dominates the reference of the context, this is also called models-for. We expect students could see the structure of objects that supports their strategy to determine the total objects by repeated addition.

4. Formal level

   In this level, students work with conventional procedures and notations. In this level the focus of discussion moves to more specifics of models related to the multiplication concept, the students can know why they can represent the total number of tile in 6 rows of 4 tiles as 6x4 is for example.

Goal of The Research

The main goal of this research is to help students in grade two to elementary school able to represent the total number of objects in rectangular pattern into multiplication sentence and using the structure of objects to do efficient counting.

Hypothesis and Research Question

My own hypothesis, which underlies this study, is completing the objects in rectangular pattern can give students clue that the number of objects in each row or column is same that provoke them to count in group and lead them to repeated addition which is transformed into multiplication sentence.

Related to my hypothesis, I formulated research question for this study that is: “How can the activity ‘counting tiles’ could promote students to multiplication in rectangular model?”

Hypothetical Learning Trajectory

In this research a learning trajectory is defined as a description of the path of learning activities that the students can follow to construct their understanding of multiplication, where in that path considers the learning goal, the learning activities and the conjecture of learning process such as students’ strategy to solve the problem. The learning trajectory is hypothetical because until we apply our design or until students really work in the problem, we can never be sure what they do or whether and how they construct new interpretations, ideas and strategies.

In this research instructional activities for multiplication in rectangular model were design. Table 1 shows the general overview of hypothetical learning trajectory (HLT) of multiplication in rectangular model for grade 2 students’ elementary school.

Name of Activity	Students activity	Learning Goal	Math Idea	Strategy
Counting the tiles.	▪ Completing the picture ▪ Counting objects in rectangular pattern ▪ Writing their strategy to count ▪ Representing the number of objects in multiplication sentence	▪ Students are able to represent the number of objects in rectangular pattern into multiplication sentence	▪ Structuring (viewing pattern and regularity)	▪ Counting tiles in column ▪ Counting tiles in row

Table 1. Overview of Learning Trajectory.

Methods

Participants

Twenty eight students in 2d class of MIN 2 Palembang were participated in this research. They were separated in seven groups namely: Apel group, Anggur group, Mangga group, Jeruk group, Leci group, Nanas group, and Durian group.

Material and Procedure

Description of Activity:

In this activity, the teacher gives instruction sheet to the students. Instructional sheet provides picture of a handyman tiles as shown in figure 1 below. The teacher tells to the students that the handyman tile was working to install the tiles.

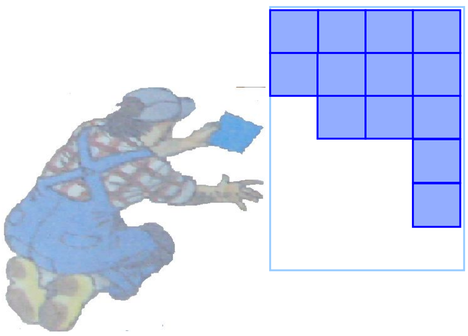


Figure 2. The handyman tiles was working to install the tiles

The students are asked to draw the complete installation of the tiles that the handyman tiles installs, to write their strategy to determine the total number of tiles that they draw and to represent the total number of tiles in multiplication sentence if they are able to do it. The students work in the group of 3 or 4. The teacher gives them a poster to make representation of complete tiles if Pak Toni finished his work. After they finish doing the tasks, the class discussion is held. Focus in the class discussion is representing the number of tiles into multiplication sentence.

Conjecture of students thinking and discussion:

The students are asked to draw the complete installation of the tiles. When students make their drawing they might come up with;

- o Some students in their group might imitate the picture in the instruction sheets, and started to complete the tiles in row, or in column.
- o Some students in their group might be completed the tiles in their instruction sheet as a model for them. After know how the complete installation looks like they might draw in their picture in their poster.
- o Some students in their groups might draw directly 4 columns which is 6 tiles in each column or six rows which is 4 tiles in each row because they had mental image of complete installation of the tiles in their head.

When students wrote their strategy to count the total number of tiles, the students might come up with:

- o Some students might count the tiles one by one. These students do not use the structure of objects to do efficient count. They might also have difficulties to keep track of their counting.
- o Some students might count the complete tiles by repeated addition because they know the number of tiles in each row/column is same. They might add
  - o 4+4+4+4+4+4, when they counted in row, they might determine the total number of tiles by adding the 4 one by one or by regrouped the repeated addition that they made into 8+8+8.
  - o 6+6+6+6, when they counted in column, they might determine the total number of tiles by adding the 6 one by one, or by regrouped the repeated addition that they made into 12+12.

When students represent the total number of tiles into multiplication from repeated addition that they made, they might come up with:

- o Some students in their groups might add 4+4+4+4+4+4 and transform it into 6x4. These students know that there were 6 times of the 4 that they add.
- o Some students in their groups might add 4+4+4+4+4+4 and transform it into 4x6. These students have difficulties to determine where they have to put the number of multiplier and the number of multiplicand in multiplication sentence.

If some students count in column and others count in row, it would be two multiplication sentences that they get, 6 x 4 and 4 x 6. Those two multiplication sentences give them same product, 24. This is might be a conflict for the students, why this can be happen. This situation provides wonderful starting point to learn commutative property of multiplication.

Result and Analysis

The lesson started by giving a picture of a handyman tile who was working to install the tiles as shown in figure 1. Students were asked to draw completed installation of the tiles. How students made complete installation of the tiles, how they determined the total number of tiles, and how they can represent it into multiplication sentence were observed.

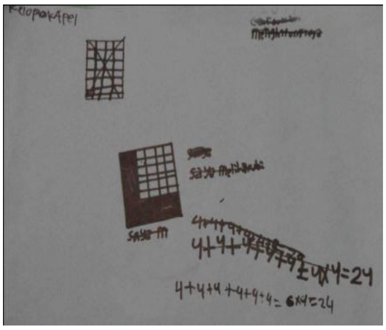
One of the groups, Nanas group, imitated the picture in the instruction sheet as shown in the figure 3 below.

Figure 3. Nanas group imitated the picture in the instruction sheet

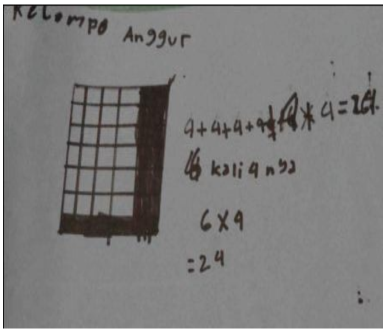
They said to the teacher that they finished making the complete installation of the tiles. The teacher asked them to read the instruction sheet. We observed that they read the instruction and realized that they have to draw the complete installation of the tiles not to draw the same picture as they thought. They continued to draw their picture, as our conjectured they completed their drawing, row by row till they finished. Finally this group succeeded to draw the complete tiles well.

Some groups, like Jeruk group, completed the drawing in the instruction sheet first. They made it as model for them. They realized themselves that it would be easier if they had image of complete installation of the tiles before started to draw in their poster. When they finished with their model, they counted the number of tiles in the first column, and realized that there were 4 columns that consist of 6 tiles in each column. After that, they started to move the picture to their poster.

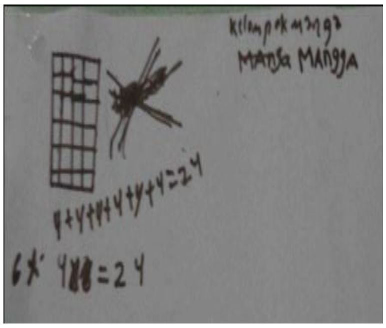
We analyzed that 4 out of 7 groups counted the number of tiles in row, 2 out of 7 groups counted the number of tiles in column, it showed from their strategy to count the total number of tiles by using repeated addition that they made. But one group, Durian group, did not make their strategy to count the total number of tiles, and it made us did not know how this group counted the number of tiles from their poster. Students' posters showed in figure 4 below.



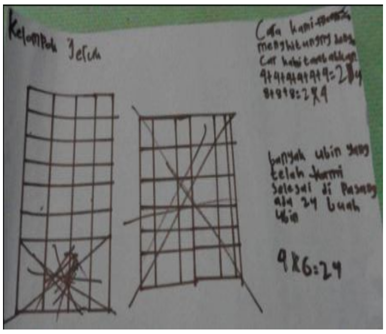
Apel's works



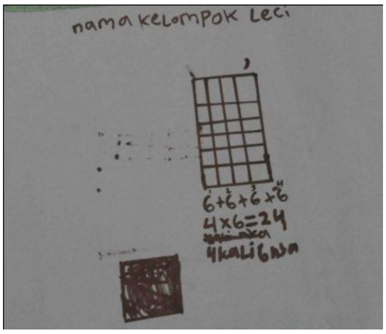
Anggur's works



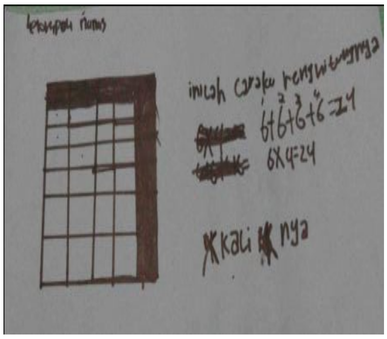
Mangga's works



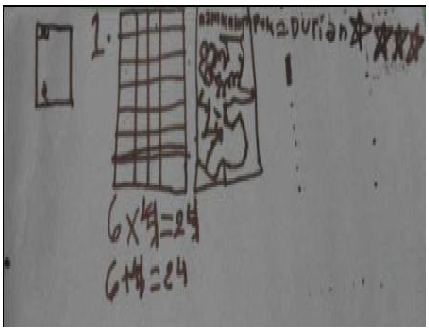
Jeruk's works



Leci's works



Nanas's works



Durian's works

Figure 4. Students' posters of activity counting tiles

Some groups, Jeruk and Nanas, had difficulties to represent the repeated addition into multiplication sentence. It showed from their poster. Jeruk group, counted the number of tiles in row, they know the number of tiles in row consist of 4 tiles, therefore to determine the total number of tiles they did repeated addition, 4+4+4+4+4+4. To count the total they regrouped the repeated addition that they made into 8+8+8. But when they have to represent the number of tiles from repeated addition into multiplication sentence, they made 4 × 6. They still influenced from the language “4nya ada 6 kali” that made them to transform it into 4 × 6. Nanas group also had difficulties to represent the repeated addition that they had into multiplication sentence like Jeruk group. They counted the number of tiles in column, they knew that there were six column where in each column consist of 6 tiles, therefore to determine the total number of tiles they did repeated addition 6+6+6+6, they counted how many of the six that they had. They tried to put in word in the same way that discussed in lesson two, “4 kali 6nya” but they had doubt and decided to erase it and wrote it into 6 × 4.

To start a fruitful discussion, the teacher asked the students to hang their work in the whiteboard and let them to observe what their friends made and give comment if they have comment on it as shown in figure 5 below.

Figure 5. Students observed their friends’ work

As a result from their observation, two of the students from Nanas group, complained with Durian’s works. Durian group made their drawing by seven rows, therefore nanas group said to the class it was wrong because the row of the complete tiles must be six rows. Their argumentation was accepted by the whole class, but Nanas group did not said about the multiplication sentence that Durian group made, and the teacher also let it till another groups complained. But none of the students paid attention on that in that moment.

After had complained from Nanas group, one of the students from Durian group, Shella, complained with Nanas’ works as shown in figure 3. The following is a segment from our video recording about student argumentation.

Teacher : Ok class, Shella found mistake from Nanas groups. Ok Shella please!

Shella : What Nanas group did was not correct. It must be 4x6 because there were 4 times of the 6.

Teacher : So it must be?

Shella : four times six (4×6)

From the segment above we can see that Shella knew that it was 4 times of the 6. She understood well about the meaning of 'times'. She knew that there were 4 times of 6 and her knowledge about the word 'times' symbolized as '×' in mathematics made her able to symbolized it as 4×6.

Conclusion

In summary, the result of this research show that completing the tiles made the students to have information that the number of tiles in each row or column was same. Therefore lead them to count by group as our hypothesis. They counted the tiles in row or in column and used the repeated addition as their strategy to determine the total. Throughout this research, we found that some of students had difficulties to transform the repeated addition into multiplication sentence. The difficulty is much influenced by the language. Most of the students tend to said in word "7nya 4 kali" for example, that provokes them to transform in multiplication sentence 7×4. The teacher could help them by remaining them to the term 'times' that they usually hear in their daily life. Afterward the teacher could write in word "4 kali 7nya". By using the word "4 kali 7nya" and stress to the students that in mathematics 'kali' are symbolized by '×', it helps the students to transform the repeated addition into multiplication sentence correctly.

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