# SET A STRUCTURE OF OBJECTS WITH A HELP OF GROUPING TO TEN STRATEGY TO UNDERSTAND THE IDEA OF UNITIZING

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## *Abstract*

The intention of the present study is to know how the pupils can learn to make a group of ten to understand the idea of unitizing. The pupils were given a contextual problem "Counting the Beads" in order to promote their understanding about the idea of unitizing. The process of designing the problem was based on the 5 tenets of Indonesian Realistic Mathematics Education (IRME). The methods of this study was a design research. The researcher designed the Hypothetical Learning Trajectory (HLT) before conducting the lesson in the classroom. The result of this study showed that the pupils learned to make a group of any number then moved to make a group of twenty before using the group of ten as a strategy in finding the amount of the beads. The pupils set a structure of objects with a help of grouping to ten strategy to understand the idea of unitizing.

**Keywords**: Grouping to ten, IRME, unitizing

## Abstrak

Tujuan dari dilaksanakannya penelitian ini adalah untuk mengetahui bagiamana siswa membuat kelompok sepuluhan untuk memahami ide unitizing. Siswa diberikan sebuah soal kontekstual "Menghitung Manik-manik" untuk membantu mereka memahami ide tersebut. Proses perancangan soal tersebut didasarkan pada 5 karakteristik Pendidikan Matematika Realistik Indonesia (PMRI). Metode penelitian yang digunakan adalah design research. Hasil penelitian menunjukkan bahwa siswa awalnya membuat kelompok sembarang lalu membuat kelompok duapuluhan sebelum akhirnya mereka mampu membuat kelompok sepuluhan untuk menghitung banyaknya manik-manik. Strategi yang digunakan ini dapat membantu mereka memahami ide unitizing.

**Kata Kunci**: Pengelompokan sepuluhan, PMRI, unitizing

Place value is extremely significant in mathematical learning, yet the pupils tend to neither acquire an adequate understanding of place value nor apply their understanding of place value when working with computational algorithms (Fuson, 1990). The understanding about place value is very important in mathematics (Kilpatrick, Swafford, and Findell, 2001). The research of Thompson (2000) suggests that pupils are still able to work successfully with two-digit numbers, including the teens, without being explicitly aware that the first digit stands for the number of tens. Many pupils have no adequate understanding of place value.

It is common in Indonesia that the teacher emphasizes the teaching of procedures instead of considering the development of students' own strategies (Rumiati & Wright, 2010). Freudenthal (1991) stated that in guiding the pupils to grasp the mathematical concept, the delicate balance between the force of teaching and the freedom of learning is needed. Therefore, I designed Counting

the Beads activity given to the pupils in order to promote them to structure a set of objects with help of grouping so that they are able to understand the idea of tens and units in multi-digit numbers. In the present study, the pupils was asked to count the beads to find the total amount. I gave this activity to them because counting is the first verbalized mathematics (Freudenthal, 1991).

The design process of the activity was guided by five tenets of RME defined by Treffers (Bakker, 2004). The description is as in the following:

1. *Phenomenological exploration;* The pupils explore the strategy in finding the amount until they re-invent the meaning of place value in three digit numbers. The context is chosen to promote them to do this. Counting the beads is meaningful for the pupils since they are familiar with the beads and knew how to find the amount of something. The pupils are able to count something, in this case, counting the beads.

2. *Using models and symbols for progressive mathematization;* The pupils are free to decide their own strategy whether they like to count the beads one by one, or set the beads into several groups. They are also free to choose the amount of the beads in every group they set.

3. *Using pupils' own construction and productions;* The pupils contribute their own strategy to the learning process that will lead them from the informal level to the more formal standard methods of mathematics. They can develop their way to move to the formal level and their strategy can be used as a tool for the next learning process. The teacher shall be a good guide to promote the pupils finding the best strategy.

4. *Interactivity;* The discussion in the classroom is not only between the teacher and the pupils, but also between the pupils themselves. The pupils is asked to count a large amount of beads. In this situation, they need their friends to help them count. The class discussion is also designed as a moment for the pupils to share their ideas and strategies. They can share their ideas to others and by doing this, they can choose the best strategy than can be used.

5. *Intertwinement;* The range of a mathematical idea shall be in connection with the long-term learning process (Freudenthal, 1991). The idea of unitizing is closely related to the addition and subtraction. Once the pupils are able to understand the meaning of unitizing in place value, they can easily solve the addition and subtraction problems later on.

For the second tenet of IRME, there are 4 levels of emergent modeling. The adaptation of the emergent modeling in the present study is described as follows:

1. *Situational level*; the pupils work with a contextual situation in which they can apply their informal knowledge and any strategy they like to use. They can count the beads one-by-one or use any strategy they like in order to get the idea of unitizing.

2. *Referential level*; the *model-of* situation, the models and the strategies the pupils use to refer to the situation, occurs in this level. The pupils can explore the grouping strategy in order to get the understanding of unitizing idea in place value. The representation they make is also being

considered. The representation can be a pictorial representation or the other representation like a symbolic one. The pictorial representation can be used before they are able to use mathematical symbols.

3. *General level*; in this level, the pupils develop a model or strategy that is applicable in different situations. They can use this strategy in solving almost every problem related to the place value. In the present study, structuring a set of objects with help of making a group of ten can be generalized to different situation in multi-digit numbers.

4. *Formal level*; the pupils are already able to understand the idea of unitizing with the help of making a group of ten. They can work with conventional procedure and the notation is independent from the use of *model-for* mathematical activity.

The socio- and socio-mathematical norms are also important to be considered in interpreting the classroom discourse and communication (Gravemeijer & Cobb, 2006). The approach used in the classroom will be different with what the present study has. It is important to negotiate with the teacher about the difference between the approach the school has and the one in the present study.

The socio-mathematical norms are about the numbers and the place value. The pupils is given as many opportunities as possible to share their ideas about the strategy they can use in finding the amount of the beads. The teacher orchestrate the discussion in the most meaningful way to help the pupils get the idea of unitizing with the help of making a group of ten strategy.

It is important to start the learning activity by giving a context to the pupils to make them more engaged with mathematics. Mathematics is much more better to be started and stay connected within common sense (Freudenthal, 1991; Gravemeijer & Doorman, 1999). The activity is genuinely open and allow pupils to move in the directions appropriate to their perception of the problems (Boaler, 1993). The counting activity is appropriate for the pupils since they are free to choose their own strategy. They can use counting one-by-one strategy, counting from any number strategy, and making a group of ten strategy. These strategic is discussed at the end of the lesson so that the pupils can compare their strategy with the other to find the best one that can be used (Freudenthal, in Gravemeijer & Terwel, 2011). The guidance from the teacher is needed because different pupils respond to the same circumstances somewhat differently (Planas & Civil, 2002; Beswick, 2011).

The problem here is different from the problem the pupils usually find in textbooks. Indonesian textbooks commonly consist of the tasks full of mathematics role and algorithms (Zulkardi, 2002; Pramudiani, Zulkardi, Hartono, van Amerom, 2011). In solving the beads problem, the pupils try to find the best strategy to find the amount. This leads them to the idea of unitizing by structuring the beads with the help of making a group of ten.

Related to the framework of the present study, the research question is "*how can pupils learn to make a group of ten to understand the idea of unitizing?*"

**METHODS**

The type of the present study was a design research to develop theory about both the process of learning and the means that are designed to support learning (Gravemeijer & Cobb, 2006). There are 3 phases in the present study described as follows:

1. Preliminary research; The Hypothetical Learning Trajectory (HLT) was designed in this phase. This is the HLT in the present study:

**Tabel 1. The Hypothetical Learning Trajectory**

| Activity | Learning goal | Mathematics idea | Strategy |
|----------|---------------|------------------|----------|
| Sequencing numbers | The pupils are able to say the number sequence correctly. | Number sequence | Sequencing the numbers up to 500 Counting the beads with their own strategy |
| Counting the beads | The pupils are able to construct their understanding about place value in three digit numbers | Place value in three digit numbers | Grouping the beads Grouping the beads with same amount for each group Counting the beads into tens |

The hypothesis was that the activities above could promote pupils' understanding in the idea of unitizing with the help of making a group of ten.

2. Teaching experiment

After it was designed, the HLT was then being tested in the classroom. There were several mini-cycles in the teaching experiment that was intended to revise the HLT after every meeting. The HLT was re-designed after each meeting was done (Gravemeijer, 1991). There were 6 meetings that was conducted in this present study. In conducting these meetings, the pupils was facilitated with colorful beads and plastic bowls.

3. Retrospective analysis

The data was then analyzed by comparing the HLT and the evidence in the field. The HLT was the guide for the researcher to see what is the main point that has to be analyzed.

The present study was conducted in Sekolah Dasar Negeri 21 Palembang. The subject of this study was the second graders pupils.

**RESULTS AND DISCUSSION**

**Counting Activity**

At the first meeting, the pupils were asked to sequencing the number up to 500 (adapt from Menne, 2001). This activity was conducted to make sure that the pupils were able to say the correct number sequence since they had to count the beads later on. From the evidence in the classroom, it was found that some pupils were still unable to sequence the number at the beginning of the meeting. After doing this activity, most of the pupils were able to say the correct number sequence up to 500.
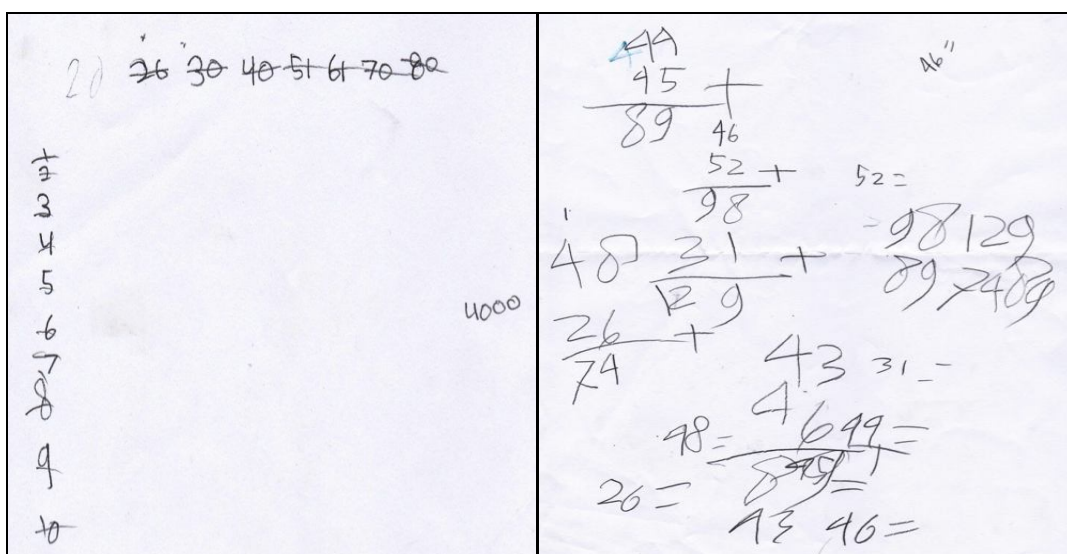
**Counting One-by-one**

The pupils were given a large amount of colorful beads put on a bowl then they were asked to find the amount of the beads. The pupils were work in pairs in order to support them in sharing the strategy and ideas. They could discuss their small-group strategy to the other groups.

Almost all the pupils were able to count one-by-one since the first meeting. When they were given the colorful beads, they directly used this strategy to find the amount of all beads they had. The pupils weren't able to find the other strategy at the first time.

The pupils then found some difficulties in counting since the amount of the colorful beads was large. They often lost their tracks because of the interruptions from the environment, such as the questions from their friends, the noise outside the class, and they themselves were forget the sequence of the number they just saying.

To deal with the difficulties they faced, some group decided to make a representation of what they did. They wrote down the amount of the beads several times at a piece of paper. Below is the figure of the pupils' representation.



**Figure 1. Pupils' Representation of the Amount of The Beads**

**Set a Structure of Objects by Making a Group of Any Number**

Since the pupils found difficulties in using counting one-by-one strategy when were faced with a large amount of the beads, they started to change their strategy into making a group. The teacher facilitated the pupils with several bowls, in order to promote them to use the grouping strategy. The help of the bowls was useful for the pupils. They put several beads into the bowl with different amount in it.

At the first time, they put as many beads as possible in every bowl. Some of the pupils put the beads with the same color in one bowl, some other didn't pay attention on the color of the beads.

When they tried to find the total amount of all beads, the pupils found the other difficulties. It was hard for them to remember the amount of the beads on every bowl. They re-counted the beads on every bowl one-by-one until they found the total amount. Here is the figure of the way the pupils put the beads into the bowl.
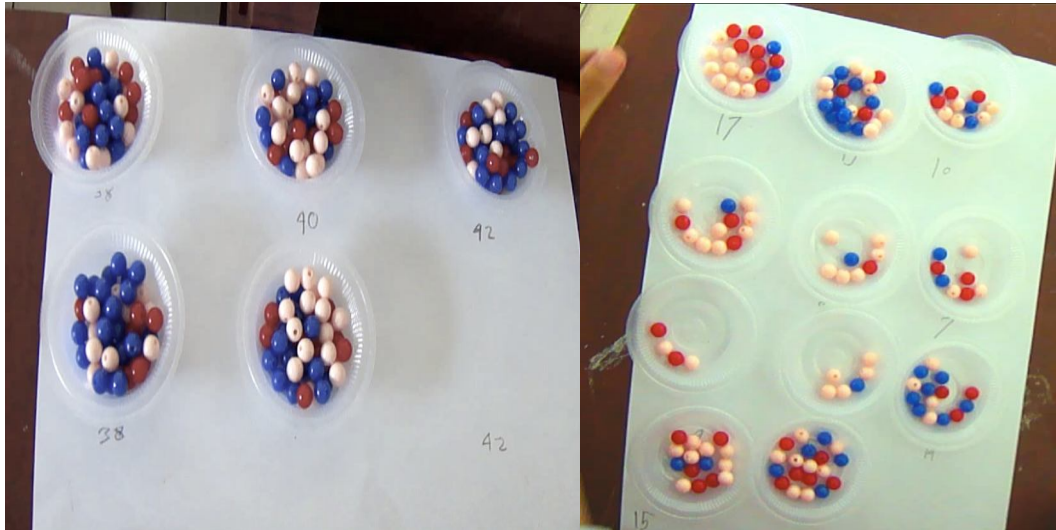


**Figure 2. The Different Amount of The Beads in Each Bowl**

There were some students who already made a group of twenty when counting the beads. They thought 20 was good enough to help them count the beads efficiently. They put 20 beads on each bowl and counted the beads by 20 when they were asked about the total amount of the beads.

**Set a Structure of Objects by Making a Group of Twenty**

Aforementioned, some pupils already used the grouping to twenty strategy. They shared the differences between the strategy with the classmates. From the discussion, some pupils changed their old strategy to what they learned from others. Some other pupils still used their own strategy in counting the beads.

By making a group of twenty, the pupils felt comfortable because they wouldn't lost their track when counting. They knew that they only had to count by twenty if they forget the total amount of the beads. The pupils knew the early idea of unitizing by doing this. They could see that there were 20 beads in one bowl.

**Set a Structure of Objects by Making a Group of Ten**

There was one group of pupils who already posed the idea of making a group of ten at the second meeting. He argued that above all the strategies his friends had, grouping to ten was the most efficient strategy since he only had to count by ten. He showed the strategy to the teacher, about how he counted his beads with the help of this strategy.

This pupil told to the teacher that it would be very easy to count by ten. He then explained how to count all the beads while tagging the bowl synchronize. He said, "this is how I did the counting, 1, 2, 3, 4, 5, 6, 70!" He had 7 bowls on the table at that time. He counted the bowl one-by-one by saying 1, 2, 3, … at the beginning instead of 10, 20, 30, …. When he tagged the last bowl, he said 70 loudly.

After making a group of all the beads, the pupil knew that he had 24 bowls consist of 10 beads on each bowl. To find the total amount of the beads he had, he counted the bowls by setting them into a group of ten bowls. He put 10 bowls together, and put the other 10 together until he got 2 groups of 10 bowls. Then he said loudly when tagging the group of ten bowls as "one… two hundred". He realized that he still had the other 4 bowls left. He continued counting the bowl and said "so here I have two hundreds, and one, two, three, forty. Two hundreds and forty!" At this point, it could be concluded that this pupil was able to understand the idea of unitizing that would lead him to the meaning of place value.

The teacher then asked the pupils to shared their strategy with other in the front of the classroom. Some pupils with different strategies were chosen to explained the strategy their group had to the others. All the pupils paid attention to the explanation given by their friends and started thinking which one was the best. At the end of the discussion, most of the pupils agreed that set a structure of objects with the help of making a group of ten was the most efficient strategy.

## SUMMARY AND SUGGESTION

In conclusion, the present study has shown that the pupils has been able to understand the idea of unitizing by solving the contextual problem "counting the beads" with the help of making a group of ten. The pupils tried to explore the best strategy to find the amount of the beads by discussing the ideas with others. They moved from counting one-by-one strategy to making a group of ten. The pupils could grasp the idea that there were 10 beads in one bowl, and 10 bowls in one group of hundred.

The suggestion for the next study is for the researcher to pay attention on the preliminary design phase. The researcher has to make sure that the teacher knows how to work with IRME and how to orchestrate the learning process in the classroom. Counting the beads activity can also be given to the first grade pupils to promote their understanding about place value in two-digit numbers. It will be better if the pupils can grasp the understanding os place value in their earlier grade.

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