This paper proposes a new variational RVR-method for calculating the three-dimensional stressed-strained state of statically loaded shell elements of structures with holes of arbitrary shapes and sizes. The scientifically substantiated RVR method is based on the use of the Reissner variational principle, the Vekua method, the theory of R-functions by Rvachev, and the general equations of the spatial theory of elasticity. The use of mixed Reissner variational principle leads to an increase in the accuracy of solving boundary value problems due to the independent variation of the displacement vector and the stress tensor. The Vekua method makes it possible to replace the solution to a three-dimensional problem with a regular sequence of solutions to two-dimensional problems. The theory of R-functions at the analytical level takes into account the geometric information of boundary value problems, which is necessary for the construction of solution structures that accurately satisfy all boundary conditions. At the same time, the developed algorithm for bilateral integral accuracy assessment makes it possible to automate the search for such a number of approximations in which the process of convergence of solutions becomes stable. The possibilities of the RVR method are shown in numerous examples of solving boundary problems of calculating cylindrical shells with an elliptical hole when setting centrifugal loads according to a deformed scheme. Calculations according to the specified load scheme of the anisotropic cylinder lead (at certain values of the angular velocity of rotation) to a significant increase in stresses. Therefore, to obtain reliable results, it is necessary to set a centrifugal load that takes into account the change in the size of the body in the process of its deformation. The characteristic features of the proposed RVR-method, which can be used effectively in the manufacture of shell elements of structures in various branches of technology, are discussed

Keywords: rotating orthotropic shell with hole, concentration of stresses, Reissner principle, theory of R-functions

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CALCULATION OF THE STRESSED-STRAINED STATE OF ROTATING ANISOTROPIC CYLINDRICAL SHELLS WITH A HOLE BASED ON VARIATIONAL RVR-METHOD

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1. Introduction

In engineering practice, anisotropic shells weakened by holes are widely used as responsible structural elements whose strength characteristics affect the reliability of the structure as a whole. When performing specific numerical calculations to assess the concentration of stresses near the holes in the shells, it is necessary to study their stressed-strained state (SSS) based on the correct statement of the corresponding three-dimensional boundary value problems. Solving them in spatial statement is associated, as a rule, with overcoming significant mathematical and computational difficulties. Therefore, the actual scientific task in the mechanics of a deformable solid is the development of efficient and easy-to-implement methods for calculating multiconnected elastic shells of arbitrary thickness.

2. Literature review and problem statement

The problems of calculating rotating elastic shells are of not only theoretical but also practical interest in various branches of modern technology [1–5]. In particular, in work [1], based on the theory of thin shells, a numerical calculation of the damping characteristics of rotating multilayer composite cylindrical shells is performed. In article [2], based on the improved Ritz least-square method (IMLS-Ritz), a significant influence on the dynamics of the rotor exerted by the vibration characteristics of rotating composite cylinders has been studied.

Paper [3] reports an analytical study in which a low-speed reaction of a rotating cylindrical shell is predicted, which is subjected to shock, axial, and thermal loads. Article [4] considers the nonlinear parametric resonance behavior of a rotating multilayer cylinder made of composite material under the influence of periodic axial loads and hygrothermal environment. Finally, the authors of work [5] built a model for three-dimensional analysis of the vibration of rotating pre-twisted isotropic cylindrical shells and conducted research to study the effect of rotational speed on the dynamic characteristics of the shell. At the same time, numerical comparisons of the results obtained with the three-dimensional finite-element method (FEM) show the capabilities of the developed model for accurate prediction of frequency results and oscillation forms of rotating elastic cylinders. However, in works [1–5], the principle of invariability of initial dimensions adopted in the resistance of materials is used to determine the SSS of rotating shells. In the strength calculation of a rotating elastic body, this approach of setting centrifugal loads according to an undeformed scheme seems inconsistent since elastic radial movements recognized at the end of the solution are not taken into account at the beginning.

When studying the strength and stiffness of shell elements of structures, various variational methods are used, which are thoroughly described in [6]. These methods are widely used for the construction of direct approximate algorithms for solving boundary problems of mechanics of a deformable solid. Article [7] presents an energy approach for the dynamic analysis of composite gently sloping shells with arbitrary boundary conditions, when the Ritz method is used for the solution procedure. And in work [8], based on the same method, the free oscillations of spherical and cylindrical shells with inhomogeneous thickness are investigated. To derive a semi-analytical solution, the energy method and the theory of first-order shear deformation were used. At the same time, as evidence of the reliability of the presented results, they were compared with the results obtained using the finite-element method (FEM) and experiments.

The presence of holes and cutouts (stress concentrators) significantly affects the bearing capacity of shell structures [9]. At the same time, it follows from the results of work [10] that it is the three-dimensional statement of the SSS study that is important for assessing the structural integrity of through cracks that come from the hole in the elastic shell. The solution to spatial boundary problems for statically loaded anisotropic shells with holes is the subject of monograph [11], which reports a numerical-analytical variational method (called the RVR method) that is theoretically substantiated in [12].

The developed method is based on the Reissner principle, the Vekua method, the mathematical apparatus of the theory of R-functions by Rvachev, and the algorithm [13] of the two-way integral assessment of the convergence of solutions to variational problems. Independent variation of the components of the displacement vector and the stress tensor in the case of using the mixed Reissner variational principle [14] increases the accuracy of solving problems. In turn, the Vekua method [15] uses decomposition of the desired functions into Fourier series according to orthogonal Legendre polynomials with respect to the coordinate along the thickness of the shell. This method of replacing the solution to the three-dimensional boundary value problem of linear elasticity theory [16] with a regular sequence of solutions to two-dimensional problems makes it possible to approach the solution to the three-dimensional theory with the required accuracy. Finally, with the help of the theory of R-functions [17], it is possible to create analytical structures of such solutions that accurately satisfy all given variants of edge conditions on the boundary surfaces of the elastic shells under study.

The capabilities of the proposed RVR-method made it possible to successfully solve in a three-dimensional statement a number of complex problems of an applied nature for critical shell elements of structures under the influence of various types of static load. Thus, in work [9], calculations of an elastic cylinder with an elliptical or rectangular hole under the action of axial forces are presented, and in [18] – calculations of an orthotropic spherical structure loaded with internal pressure. At the same time, a satisfactory coincidence of the numerical values of the stress concentration coefficient obtained based on the RVR method with the experimental data known from the scientific literature is shown.

Thus, the rejection of the principle of immutability of the initial dimensions and the assignment of centrifugal loads according to a deformed scheme is an unsolved scientific problem in the three-dimensional statement of the SSS study of rotating elastic shells with holes. This circumstance confirms the feasibility of conducting a study based on the use of the variational RVR method for obtaining refined analytical and numerical solutions at arbitrary values of the angular velocity of rotation of multiconnected shells.

3. The aim and objectives of the study

The purpose of this study is to calculate the SSS of an orthotropic cylindrical shell with a hole rotating with a constant angular velocity when setting centrifugal loads according to a deformed scheme. This will make it possible to mathematically correctly formulate the edge problem under consideration and achieve reliable results.

To accomplish the aim, the following tasks have been set:

– to construct analytical structures of solutions that accurately satisfy the formulated boundary conditions of the studied elastic region of the shell with an elliptical opening;

– to obtain, using the variational RVR method, numerical results of an applied nature to assess the impact exerted by loading according to the deformed scheme on the SSS of the fiberglass cylindrical shell.

4. The study materials and methods

4. 1. Mathematical statement of the boundary problem of calculating the strength and rigidity of a rotating elastic cylinder with a hole

The object of this study is a hole-weakened orthotropic cylinder rotating around the axis of symmetry with a constant angular velocity ω (Fig. 1) of length $2L$ and thickness h with load-free end surfaces.

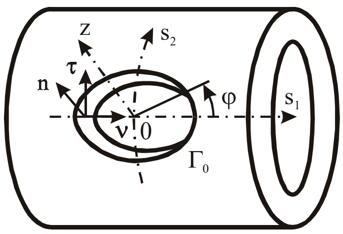

Fig. 1. Cylinder with elliptical hole

Let's introduce in the median surface of the Ω_s radius R of the cylindrical shell a curvilinear coordinate system $\{s_1, s_2, z\}$ with the origin in the center of the elliptical hole with semi-axes r_1 and r_2 along the lines s_1 and s_2. These coordinate lines coincide with the elastic-equivalent directions of the orthotropy of the material (density ρ) of the cylinder (the z coordinate line is perpendicular to Ω_s).

The dimensions of the hole are considered arbitrary, so in the proposed RVR-method [11] there are no restrictions on the value of $\mu = r_0/\sqrt{Rh}$, where r_0 is the smallest hole size, which in the considered numerical calculation is derived from the formula $r_0=(r_1+r_2)/2$.

Let the boundary surface Γ_0 of the hole, the end surfaces Γ_1, and the face surfaces of the cylinder be free of external forces and moments. The boundary conditions formulated through the components of the displacement vector **u** and the stress tensor **σ** edge conditions are:

$$\sigma_{\nu\nu}=0,\ \ \sigma_{\nu\tau}=0,\ \ \sigma_{\nu n}=0 \ \text{ on } \Gamma_0;$$

$$\sigma_{11}=0,\ \ \sigma_{12}=0,\ \ \sigma_{13}=0 \ \text{ on } \Gamma_1 \ \left(|x|=L\right);$$

$$\sigma_{13}=0,\ \ \sigma_{23}=0,\ \ \sigma_{33}=0 \ \text{ at } |z|=h/2. \tag{1}$$

In addition, on the boundary surface Γ_2 defined on the coordinate line s_2 by a distance of $\pi R/N_0$ (N_0 is the number of holes along the cylinder guide), the conditions of periodicity shall be met:

$$u_2=0,\ \ \sigma_{12}=0,\ \ \sigma_{23}=0 \ \text{ on } \Gamma_2. \tag{2}$$

As is known, to determine the SSS of rotating bodies, the principle of invariability of initial dimensions adopted in the resistance of materials is usually used. In this case, the intensity of the inertial forces Q is considered to be equal to:

$$Q=\rho\omega^2 r = qr/R;\ \ q=\rho\omega^2 R. \tag{3}$$

According to [19], the calculation of the strength of the elastic body when it is loaded in the form of (3) seems inconsistent since the elastic radial displacements u_3, recognized at the end of the solution, are not taken into account at the beginning. Therefore, for a refined solution, it is necessary to set the actual value of the centrifugal load, taking into account the change in the size of the body during its deformation, and use the initial ratio in the calculations:

$$Q=q\left(r+u_3\right)/R. \tag{4}$$

Directly from formula (4) it follows that the value of the centrifugal load depends on the value of the elastic radial displacement u_3, which is the desired value in the study boundary problem.

4. 2. Obtaining an analytical expression of the Reissner variational equation for a rotating orthotropic cylindrical shell

To improve the accuracy of solving boundary value problems, it is advisable to determine independently the parameters of the stress and deformed states, which can be carried out using the Reissner variational principle [11, 20, 21]. The numerical implementation of this principle was hampered by difficulties caused by the lack of an extremum at the point of stationarity of the mixed functionality of Reissner $\mathbf{I}_R$. However, this problem is solved by the theorem proved in work [12]: "the sequences of the Ritz method coincide with the exact solution to the boundary problem formulated on the Reissner principle if the solution structures accurately satisfy all boundary conditions." The Reissner equation used for the rotating cylindrical shell in question (the condition of stationarity of the functional $\mathbf{I}_R$) is:

$$\iiint\limits_{\Omega}\left\{\begin{gathered}
-\left[\frac{\partial\sigma_{11}}{\partial s_1}+\frac{1}{\chi}\frac{\partial\sigma_{12}}{\partial s_2}+\frac{\partial\sigma_{13}}{\partial z}+\frac{1}{R\chi}\sigma_{13}\right]\delta u_1-\\
-\left[\frac{\partial\sigma_{12}}{\partial s_1}+\frac{1}{\chi}\frac{\partial\sigma_{22}}{\partial s_2}+\frac{\partial\sigma_{23}}{\partial z}+\frac{2}{R\chi}\sigma_{23}\right]\delta u_2-\\
-\left[\frac{\partial\sigma_{13}}{\partial s_1}+\frac{1}{\chi}\frac{\partial\sigma_{23}}{\partial s_2}+\frac{\partial\sigma_{33}}{\partial z}+\frac{\sigma_{33}-\sigma_{22}}{R\chi}+Q\right]\delta u_3+\\
+\left[\frac{\partial u_1}{\partial s_1}-\frac{1}{E_1}\left(\sigma_{11}-\nu_{21}\sigma_{22}-\nu_{31}\sigma_{33}\right)\right]\delta\sigma_{11}+\\
+\left[\frac{1}{\chi}\frac{\partial u_2}{\partial s_2}+\frac{1}{R\chi}u_3-\frac{1}{E_2}\left(\sigma_{22}-\nu_{12}\sigma_{11}-\nu_{32}\sigma_{33}\right)\right]\delta\sigma_{22}+\\
+\left[\frac{\partial u_3}{\partial z}-\frac{1}{E_3}\left(\sigma_{33}-\nu_{13}\sigma_{11}-\nu_{23}\sigma_{22}\right)\right]\delta\sigma_{33}+\\
+\left[\frac{\partial u_2}{\partial s_1}+\frac{1}{\chi}\frac{\partial u_1}{\partial s_2}-\frac{\sigma_{12}}{G_{12}}\right]\delta\sigma_{12}+\left[\frac{\partial u_3}{\partial s_1}+\frac{\partial u_1}{\partial z}-\frac{\sigma_{13}}{G_{13}}\right]\delta\sigma_{13}+\\
+\left[\frac{1}{\chi}\left(\frac{\partial u_3}{\partial s_2}-\frac{1}{R}u_2\right)+\frac{\partial u_2}{\partial z}-\frac{\sigma_{23}}{G_{23}}\right]\delta\sigma_{23}
\end{gathered}\right\}\chi\,\mathrm{d}s_1\mathrm{d}s_2\mathrm{d}z=0. \tag{5}$$

Here, E_1, E_2, E_3 are Young's moduli (modulus of elasticity) in the principal directions of the orthotropy of the shell; ν_{12}, ν_{21}, ν_{13}, ν_{31}, ν_{23}, ν_{32} are the corresponding Poisson coefficients; G_{12}, G_{13}, G_{23} – shear modules; $|z|\le h/2$. The following ratios are valid:

$$\chi=1+z/R;\ \ \nu_{12}/E_{11}=\nu_{21}/E_{22};$$

$$\nu_{13}/E_{11}=\nu_{31}/E_{33};\ \ \nu_{23}/E_{22}=\nu_{32}/E_{33}. \tag{6}$$

Contour integrals containing static and geometric boundary conditions (1) are absent in equation (5) due to the use of R-functions in the proposed RVR method [11, 22, 23]. The mathematical apparatus of these functions at the analytical level takes into account the geometric information of boundary value problems for multiconnected elastic regions and makes it possible to build solution structures that accurately satisfy all the boundary conditions of the studied region Ω. At the same time, expression (5) differs from the Reissner variational equation presented in work [9] by the presence of the intensity of inertial forces Q, the value of which, when taking into account options (3) and (4), takes the form:

$$Q = q\left(r+\alpha u_3\right)/R = = q\left(R+z+\alpha u_3\right)/R = q\left(1+\frac{z+\alpha u_3}{R}\right), \qquad (7)$$

wherein the values α=0 and α=1 correspond to cases of undeformed (3) and deformed (4) cylinder loading schemes.

5. Results of studies of the stated boundary value problem

5. 1. Construction of structures of solutions to the studied problems

Varied in the functionality of Reisner $\mathbf{I}_R$ and precisely satisfying the boundary conditions (1) and (2) the desired components u_i of the displacement vector **u** and the components σ_{ij} of the stress tensor **σ** are represented in the following series:

$$\left.\begin{aligned}
u_1 &= \sum_{i=0}^{m_1}\sum_{j=0}^{n_1}\sum_{k=0}^{l_1-1} u_1^{ijk} S_i(s_1) C_j(s_2) P_k(\zeta);\\
u_2 &= \sum_{i=0}^{m_2}\sum_{j=0}^{n_2}\sum_{k=0}^{l_2-1} u_2^{ijk} C_i(s_1) S_j(s_2) P_k(\zeta);\\
u_3 &= \sum_{i=0}^{m_3}\sum_{j=0}^{n_3}\sum_{k=0}^{l_3-1} u_3^{ijk} C_i(s_1) C_j(s_2) P_k(\zeta);\\
\sigma_{11} &= \chi^{-1}\left[\begin{aligned}& f_2^2\omega_4 T_1 + \\ &+\omega_0\omega_1 \sum_{i=0}^{m_{11}}\sum_{j=0}^{n_{11}}\sum_{k=0}^{l_{11}-1} \sigma_{11}^{ijk} C_i(s_1) C_j(s_2) P_k(\zeta)\end{aligned}\right];\\
\sigma_{22} &= f_1^2\omega_4 T_1 + \omega_0 \sum_{i=0}^{m_{22}}\sum_{j=0}^{n_{22}}\sum_{k=0}^{l_{22}-1} \sigma_{22}^{ijk} C_i(s_1) C_j(s_2) P_k(\zeta);\\
\sigma_{12} &= \chi^{-1}\left[\begin{aligned}& -f_1 f_2\omega_4 T_1 + \\ &+\omega_0\omega_1 \sum_{i=0}^{m_{12}}\sum_{j=0}^{n_{12}}\sum_{k=0}^{l_{12}-1} \sigma_{12}^{ijk} S_i(s_1) S_j(s_2) P_k(\zeta)\end{aligned}\right];\\
\sigma_{13} &= \chi^{-1}\left[\begin{aligned}& f_2\omega_4 T_2 + \\ &+\omega_0\omega_1\omega_3 \sum_{i=0}^{m_{13}}\sum_{j=0}^{n_{13}}\sum_{k=0}^{l_{13}-1} \sigma_{13}^{ijk} S_i(s_1) C_j(s_2) P_k(\zeta)\end{aligned}\right];\\
\sigma_{23} &= -f_1\omega_4 T_2 + \\ &+\omega_0\omega_3 \sum_{i=0}^{m_{23}}\sum_{j=0}^{n_{23}}\sum_{k=0}^{l_{23}-1} \sigma_{23}^{ijk} C_i(s_1) S_j(s_2) P_k(\zeta);\\
\sigma_{33} &= \chi^{-1}\omega_3 \sum_{i=0}^{m_{33}}\sum_{j=0}^{n_{33}}\sum_{k=0}^{l_{33}-1} \sigma_{33}^{ijk} C_i(s_1) C_j(s_2) P_k(\zeta);\\
T_g &= \sum_{i=0}^{t_1}\sum_{j=0}^{t_2}\sum_{k=0}^{2-g} T_g^{ijk} C_i(s_1) C_j(s_2) P_k(\zeta) \quad (g=1,2),
\end{aligned}\right\} \qquad (8)$$

where u_p^{ijk}, σ_{ps}^{ijk}, T_g^{ijk} (at p, s=1, 2, 3; g=1, 2) – the desired constants; $C_i(s_1)$, $C_j(s_2)$ and $S_i(s_1)$, $S_j(s_2)$ – even and odd functions of coordinates s_1 and s_2; $P_k(\zeta)$ – Legendre polynomials; $\zeta=2z/h$ ($|\zeta|\leq 1$); $\chi=1+h\zeta/2R$;

$$\omega_0 = \left(\frac{s_1}{r_1}\right)^2 + \left(\frac{s_2}{r_2}\right)^2 - 1;\quad \omega_1 = 1-\left(\frac{s_1}{L}\right)^2;$$

$$\omega_2 = 1-\left(\frac{s_2 N_0}{\pi R}\right)^2;$$

$$\omega_3 = 1-\zeta^2;\quad \omega_4 = \frac{\omega_1\omega_2}{\omega_0+\omega_1\omega_2}. \qquad (9)$$

In the resulting solution structures (8) of the number l_i, l_{ii} (i, j=1, 2, 3) of the approximations of the components of the vector **u** and the tensor **σ** by the thickness of the cylinder determine the shift model of the shell. Its choice corresponds (at i, j=1, 2) to the setting of the combination of values (l_i, l_3, l_{ii}, l_{i3}, l_{33}). Here l_i is the number of held terms in expansion along the coordinate ζ of the tangent movements u_i; l_3 – normal movement u_3; l_{ii} is the tangential stresses σ_{ij}; l_{i3} – tangential stresses σ_{i3} and l_{33} – normal stress σ_{33}. Thus, variant (2, 1, 2, 1, 0) corresponds to the theory of shells of the Timoshenko type [16], and variant (4, 3, 4, 1, 2) – to the applied theory [24]. Guide cosines f_1 and f_2 of the normal **ν** (Fig. 1) are calculated according to the theory of R-functions [17] from the following formulas (i=1, 2):

$$f_i = \frac{\partial\omega_0}{\partial s_i}\left[\left(\frac{\partial\omega_0}{\partial s_1}\right)^2 + \left(\frac{\partial\omega_0}{\partial s_2}\right)^2\right]^{-\frac{1}{2}}. \qquad (10)$$

Functions f_1 and f_2 are necessary to satisfy conditions (1) on the curvilinear contour Γ_0 of the hole, while the following equalities [11] are valid:

$$\left.\begin{aligned}
\sigma_{\nu\nu} &= f_1^2\sigma_{11} + 2 f_1 f_2 \sigma_{12} + f_2^2 \sigma_{22};\\
\sigma_{\nu n} &= -f_1\sigma_{13} - f_2\sigma_{23};\\
\sigma_{\nu\tau} &= f_1 f_2\left(\sigma_{22}-\sigma_{11}\right) + \left(f_1^2 - f_2^2\right)\sigma_{12}.
\end{aligned}\right\} \qquad (11)$$

After substituting (8) into the condition of stationarity of the Reissner functional $\mathbf{I}_R$ (5), the stated boundary value problem is reduced to solving the system of linear algebraic equations of the ribbon structure with respect to the desired constants u_p^{ijk}, σ_{ps}^{ijk} and T_g^{ijk}. In turn, all the characteristics of SSS of the studied elastic region Ω are determined by their values.

5. 2. Obtaining numerical results to assess the impact of anisotropic shell parameters on its SSS

In this case, E_0 is the normalizing value of the dimensionality of the modulus of elasticity, and the dimensionless value $\tilde{\omega}^2$ was calculated from the formula:

$$\tilde{\omega}^2 = \rho\,\omega^2 R_0^2/E_0;\quad R_0 = R + h/2. \qquad (12)$$

Fig. 2 shows distribution plots of the concentration coefficients of membrane k_1 and the maximum thickness of bending k_2 stresses along the contour of the free hole:

$$k_1 = \sigma_{\tau\tau}^{k=0}/qR;\quad k_2 = \sigma_{\tau\tau}^{k=1}/qR. \qquad (13)$$

For plots in Fig. 2, solid lines of variants 1, 2, 3 correspond to the values of r_1/r_2=0.75, 1, 1.5; dashed lines – the results of work [25] at r_1/r_2=1. Table 1 gives the values of dimensionless circumferential stresses on the inner surface of the cylinder according to the deformed at α=1 and undeformed at α=0 schemes. Numerical results in Table 1 characterize (depending on the thickness and degree of anisotropy of the material) the concentration of stresses in the cylinder using the shear model $l_i=l_{ii}=4$ of the shells of the fourth approximation.

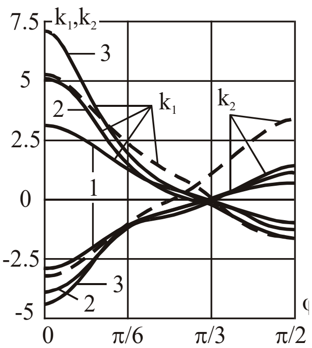

Fig. 2. Plots of coefficients k_1 and k_2 on the section $0\leq\varphi\leq\pi/2$ of the hole contour

Table 1

Values of dimensionless circumferential stresses

h/R		0.013	1/5			1/3		
E_1/E_2		0.72	0.4	0.72	4	0.4	0.72	4
$k_1+\zeta k_2$	α=1	9.174	11.24	10.78	8.257	12.37	12.24	10.26
	α=0	9.112	11.08	10.59	7.730	11.98	11.78	9.079
$\sigma_{\tau\tau}^{-}/qR$	α=1	9.277	12.17	11.63	8.758	13.45	13.39	11.28
	α=0	9.215	11.99	11.41	8.154	13.02	12.87	9.976
σ_{22}^{-}/qR	α=1	1.009	1.128	1.128	1.131	1.217	1.221	1.228
	α=0	1.008	1.127	1.127	1.126	1.216	1.220	1.224

The first four lines of Table 1 contain the results for the most loaded point $\phi=0$ of the contour of the circular (r_1/r_2=1) hole; the last two lines (for stress σ_{22}^{-}/qR) – at the same point of the cylinder without a hole. Calculation according to the specified scheme at $\tilde{\omega}^2=0.2$ leads to an increase in stress values to 10 % for a cylinder with a hole and less than 1 % – without a hole.

In the construction of the theory of thin shells, as a result of reducing the three-dimensional problems of shell theory to two-dimensional ones, the components of the stress tensor **σ** in the shell are replaced by a statically equivalent system of generalized forces:

$$\left.\begin{array}{l}\begin{Bmatrix}N_i\\M_i\\Q_i\end{Bmatrix}=\int\limits_{\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{ii}\\z\sigma_{ii}\\\sigma_{i3}\end{Bmatrix}\left(1+\dfrac{z}{R_{3-i}}\right)\mathrm{d}z;\\ \begin{Bmatrix}S_{ij}\\H_{ij}\end{Bmatrix}=\int\limits_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{ij}\\z\sigma_{ij}\end{Bmatrix}\left(1+\dfrac{z}{R_j}\right)\mathrm{d}z;\ i,j=1,2;i\neq j,\end{array}\right\} \quad (14)$$

where N_i and S_{ij} are normal and shear forces, M_i and H_{ij} are bending and torsion moments, Q_i are cutting forces, R_i are the main radii of shell curvature.

In the resulting solution structures (8), we shall represent the desired components of the tensor **σ** in the form of expansions at coordinate ζ. Then the first two terms of tangential σ_{ij} and the first term of the transverse tangent stresses σ_{i3} statically correspond to their integral characteristics (14) – the generalized forces N_i, S_{ij}, M_i, H_{ij} and Q_i (i,j=1, 2). The remaining terms are (due to the orthogonality of Legendre polynomials) the self-balanced stresses in terms of shell thickness, the magnitude of which is influenced by various factors (the presence of holes, thickness, type of loading of the shell, the degree of anisotropy of the material).

For the study boundary problem, an assessment of the contribution of the self-balanced part of the stresses to the general stressed state can be performed by comparing the stresses $\sigma_{\tau\tau}^{\zeta}/qR$ with the same stresses $k_1+\zeta k_2=\left(\sigma_{\tau\tau}^{k=0}+\zeta\sigma_{\tau\tau}^{k=1}\right)/qR$ without taking into account the self-balanced part. This comparison is made in Table 1 on the inner (most loaded) surface of the shell. As can be seen from Table 1, this contribution reaches 10 % at $\tilde{\omega}^2=0.2$ for a relatively thick (at »h/R=1/3) cylinder.

We shall investigate the effect of the value $\tilde{\omega}^2$ on the SSS of a rotating single-connected cylinder. The boundary problem in this case is axisymmetric, so the displacement u_2 and the stresses σ_{12}, σ_{23} are identically zero; the boundary conditions will be:

$$\sigma_{13}=0,\ \ \sigma_{33}=0\ \text{ at }\ |z|=h/2;$$

$$\sigma_{11}=0,\ \sigma_{13}=0\ \text{ at }\ |x|=L. \quad (15)$$

Exactly satisfying the boundary conditions (15) of the structures of the solutions of the problem under study will automatically follow from the previously used structures (8) when the equations ω_0=1 and ω_4=0 hold. Therefore, in the absence of approximating functions at the circumferential coordinate s_2 in the decompositions of the desired components of the displacement vector **u** and the stress tensor **σ** take the following form:

$$\left.\begin{array}{l}u_1=\sum\limits_{i=0}^{m_1}\sum\limits_{k=0}^{l_1-1}u_1^{\mathrm{ik}}S_i(s_1)P_k(\zeta);\\ u_3=\sum\limits_{i=0}^{m_3}\sum\limits_{k=0}^{l_3-1}u_3^{\mathrm{ik}}C_i(s_1)P_k(\zeta);\\ \sigma_{11}=\chi^{-1}\omega_1\sum\limits_{i=0}^{m_{11}}\sum\limits_{k=0}^{l_{11}-1}\sigma_{11}^{\mathrm{ik}}C_i(s_1)P_k(\zeta);\\ \sigma_{22}=\sum\limits_{i=0}^{m_{22}}\sum\limits_{k=0}^{l_{22}-1}\sigma_{22}^{\mathrm{ik}}C_i(s_1)P_k(\zeta);\\ \sigma_{13}=\chi^{-1}\omega_1\omega_3\sum\limits_{i=0}^{m_{13}}\sum\limits_{k=0}^{l_{13}-1}\sigma_{13}^{\mathrm{ik}}S_i(s_1)P_k(\zeta);\\ \sigma_{33}=\chi^{-1}\omega_3\sum\limits_{i=0}^{m_{33}}\sum\limits_{k=0}^{l_{33}-1}\sigma_{33}^{\mathrm{ik}}C_i(s_1)P_k(\zeta).\end{array}\right\} \quad (16)$$

The numerical implementation of the problem is performed for an isotropic single-linked cylinder with the parameters: $E_i=E_0$; ν_{ij}=0.3; $R=0.7R_0$; $h=0.6R_0$; $L=R_0$. Fig. 3 shows plots of the distribution of some components of the displacement vector **u** and the stress tensor **σ** by thickness ($R_0-h\leq r\leq R_0$) of the cylindrical shell in its cross-section s_1=0.

The values of the ratios of Lagrange $\mathbf{I}_L$ and Reissner $\mathbf{I}_R$ functionals, as well as displacements and stresses (the upper indices correspond to ζ=–1, ζ=0, and ζ=1) in the cross-section s_1=0 are given in Table 2. At the same time, the value of the Castigliano functional $\mathbf{I}_C$ is almost the same as the value of $\mathbf{I}_R$.

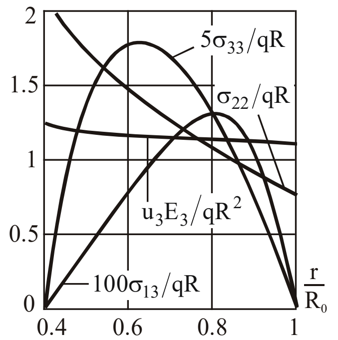

Fig. 3. Distribution of the desired values by cylinder thickness

shell according to deformed and undeformed schemes. Thus, for the thick-walled cylinder, the values of the desired values for the two loading schemes under consideration, given in Table 2 (at $\tilde{\omega}^2=0.2$), differ from each other by about 20 %. This difference with an increase in the angular velocity of rotation (and, at the same time, the value of $\tilde{\omega}^2$) of anisotropic shells can become significant when assessing the results of solving the boundary problem according to an undeformed scheme. Therefore, for a refined solution and for obtaining reliable results, it is necessary to set the actual value of the centrifugal load, taking into account the change in the size of the rotating elastic body in the process of its deformation. Due to this feature of the RVR method proposed in our work, the advantage of this study is provided in comparison with similar ones known from the scientific literature. According to [19, 25], as well as the numerical results presented in Table 1, the use of the principle of immutability of initial dimensions is justified only for rotating bodies made of high-modulus materials. However, such an approach in engineering calculations can lead to significant errors in the study of SSS of rotating bodies made of low-modulus plastics.

Table 2

Values of the desired values depending on the selected shell model

Desired quantity	Tymoshenko theory (2, 1, 2, 1, 0)	Applied theory (4, 3, 4, 3, 2)		Refined theory		
				α=1		α=0
		χ=1	χ=1+$h\zeta/2R$	$l_i=l_{ii}$=4	$l_i=l_{ii}$=10	$l_i=l_{ii}$=10
$\mathbf{I}_L/\mathbf{I}_R$	0.875540	0.797990	0.985981	0.999596	0.999999	0.999999
$u_3^- E_3/qR^2$	1.2460	1.3487	1.2168	1.2398	1.2438	1.0226
$u_3^+ E_3/qR^2$	1.2460	1.1054	1.1251	1.1070	1.1087	0.9194
σ_{33}^0/qR	–	0.1130	0.3404	0.3405	0.3417	0.2980
σ_{22}^-/qR	1.9037	1.3500	2.1291	2.1780	2.1780	1.7904
σ_{22}^+/qR	0.7527	1.1042	0.7761	0.7312	0.7595	0.6275

As follows from the results given in Table 1, the use of applied theory [24] for the considered thick-walled (h/R=6/7) cylinder gives acceptable results only if the terms of order $h/2R$ are preserved in this theory compared to unity. In the last two columns of Table 2, the values of the desired values obtained at $\tilde{\omega}^2=0.2$ and $l_i=l_{ii}$=10 for the two cylinder loading schemes under consideration differ from each other by about 20 %. This difference with an increase in $\tilde{\omega}^2$ can become (especially for anisotropic bodies) significant for assessing the reliability of the results of solving the problem according to an undeformed scheme (α=0).

6. Discussion of results of studying the SSS of a rotating cylindrical shell with a hole

With the help of the developed scientifically based variational RVR method, a numerical implementation of the boundary value problem, mathematically stated in this work for a rotating multiconnected cylinder made of orthotropic material, is carried out. Plots of the distribution of the concentration coefficients of membrane and maximum thickness of bending stresses along the contour of a free elliptical hole are given. In particular, for a circular hole, we have visually compared the results obtained with results known from the scientific literature.

A characteristic feature of the RVR method is the fact that the condition of stationarity of the Reissner functional presented in our work takes into account the intensity of inertia forces in cases of loading of the elastic cylindrical shell according to deformed and undeformed schemes.

From the analysis of the effect of the magnitude of the angular velocity of rotation on the SSS of the cylinder, it follows that the values of the desired stresses and displacements significantly depend on the selected shear model of the shell. The contribution of the self-balanced part of the stresses to SSS is greater the more the difference between the law of stress distribution by thickness and the linear law for normal stresses and the quadratic parabola law for transverse tangential stress. With an increase in the value of the thickness of the shell, as well as the angular velocity of its rotation, ignoring the self-balanced parts of the stresses in the total SSS of the studied elastic region can lead to large errors.

It is important to note that the RVR method employs an effective algorithm of a posteriori integral dual estimation of the accuracy of approximate solutions to mixed variational problems [13]. This algorithm can become a reliable means of verifying the reliability of results, making it possible to carry out an automated search in the constructed analytical structures of such a number of approximations in which the process of convergence of solutions is stable.

The limitation of this study is due to the consideration as the object of strength calculation only of a rotating orthotropic cylinder, weakened by a considerable hole. Therefore, the next stage in the development of the RVR method implies studying the SSS of multiconnected anisotropic shells of arbitrary Gaussian curvature, which are widely used in various branches of modern engineering as critical structural elements.

7. Conclusions

1. To achieve the goal set in the paper, a mathematical expression for a Reissner variational equation is proposed,

which is necessary when calculating the SSS of an orthotropic cylindrical shell rotating with a constant angular velocity. At the same time, the desired components of the displacement vector and the stress tensor, varying in the Reissner functional, are represented in the form of solution structures that accurately satisfy all boundary conditions formulated on boundary surfaces of the multi-connected area under study.

2. Based on the variational RVR method, numerical results of the applied nature when setting centrifugal loads according to a deformed scheme were obtained. Quantitative estimates of the influence of the degree of anisotropy of the material, as well as the values of the angular velocity of rotation and the thickness of the cylinder on its SSS, were carried out. According to our results, the use in engineering calculations of the principle of immutability of initial dimensions is justified for rotating bodies made of high-modulus materials; but for bodies made of low-modulus plastics, it can lead to significant errors. The characteristic features of the proposed RVR-method, which can be used effectively in the design of shell structures, are described.

Conflict of interests

The authors declare that they have no conflict of interests in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

Manuscript has no associated data.

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