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Spectra of extended neighborhood corona and extended corona of two graphs

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Abstract

In this paper we define extended corona and extended neighborhood corona of two graphs G_1 and G_2, which are denoted by $G_1 \bullet G_2$ and $G_1 * G_2$ respectively. We compute their adjacency spectrum, Laplacian spectrum and signless Laplacian spectrum. As applications, we give methods to construct infinite families of integral graphs, Laplacian integral graphs and expander graphs from known ones.

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1. Introduction

Throughout this paper, we consider only *simple graphs*, i.e, an undirected graph with no loops and no multiple edges. Let G be a graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The *adjacency* matrix of G, denoted by $A(G)$, is defined as $A(G) = (a_{ij})_{n\times n}$, where

$$a_{ij} = \begin{cases} 1, & \text{if } v_iv_j \text{ is an edge in } G, \\ 0, & \text{otherwise.} \end{cases}$$

The *degree* of a vertex v_i in G, denoted by $deg(v_i)$ is the number of vertices that are adjacent to v_i in G. The *Laplacian* matrix $L(G)$ of G is defined as $L(G) = D(G) - A(G)$ and

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the *signless Laplacian* matrix $Q(G)$ of G is given by $Q(G) = D(G) + A(G)$, where $D(G) = diag(deg(v_1), \dots, deg(v_n))$. The *adjacency* spectrum $\sigma(G)$, *Laplacian* spectrum $\mu(G)$, and *signless Laplacian* spectrum $\gamma(G)$ of a graph G are defined as follows:

$$\sigma(G) = (\lambda_1(G), \lambda_2(G), \dots, \lambda_n(G)),$$
$$\mu(G) = (\mu_1(G), \mu_2(G), \dots, \mu_n(G)),$$
$$\gamma(G) = (\gamma_1(G), \gamma_2(G), \dots, \gamma_n(G)),$$

where $\lambda_i(G),\ \mu_i(G)$ and $\gamma_i(G)$ are the *eigenvalues* of $A(G)$, $L(G)$ and $Q(G)$, respectively. Also

$$\lambda_1(G) \geq \lambda_2(G) \geq \dots \geq \lambda_n(G),$$
$$\mu_1(G) = 0 \leq \mu_2(G) \leq \dots \leq \mu_n(G),$$

and

$$\gamma_1(G) \geq \gamma_2(G) \geq \dots \geq \gamma_n(G).$$

For the properties of spectrum, Laplacian and signless Laplacian spectrum the reader may refer to [5, 6, 8, 13, 21, 23] and the references therein.

The sum $\varepsilon(G) := \sum_{i=1}^{n} |\lambda_i(G)|$ is known as the *energy* of the graph G. The concept of the energy of a graph was introduced by Gutman [14] and was recently generalized to oriented graphs as skew energy by Adiga, Balakrishnan and So in [1]. If $\lambda_i(G)$ $(i = 1, 2, \dots, n)$ $(\mu_i(G)$, $\gamma_i(G)$, respectively) are all integers, then G is said to be an *integral (Laplacian integral, signless Laplacian integral*, respectively) graph. The notion of integral graphs was first introduced by Harary and Schwenk in 1974 [16]. In general, the problem of characterizing integral graphs seems to be very difficult. More details about integral graphs can be found in [2, 11, 15, 16, 19] and references therein.

Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices, respectively. The *corona* $G_1 \circ G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The corona of two graphs was first introduced by Frucht and Harary in [12]. Barik et al. [3] provided a complete description of the spectrum (and the Laplacian spectrum) of $G_1 \circ G_2$ using the spectrum (and the Laplacian spectrum, respectively) of G_1 and G_2. More about the spectrum, Laplacian and signless Laplacian spectrum of corona can be found in [3, 4, 12, 20]. The *neighborhood corona* of G_1 and G_2, denoted by $G_1 \star G_2$, is the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and joining every neighbour of the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The neighborhood corona was introduced in [18]. Complete description of the spectrum (respectively, Laplacian, signless Laplacian spectrum) of neighborhood corona of two graphs are given in [18, 22].

Motivated by the works carried out on the spectrum of corona of two graphs, in this paper we define two new types of corona namely, extended corona and extended neighborhood corona of two graphs. We compute their adjacency spectrum, Laplacian spectrum and signless Laplacian spectrum. As applications, using the results on adjacency spectra of extended coronaand extended neighborhood corona, we give a method to construct infinite families of integral graphs starting with an integral graph and also using the results on Laplacian spectra of extended corona and extended neighborhood corona, we give a method to construct new families of expander graphs from known ones. Moreover, we prove that if G_1 is an integral regular graph and G_2 is a Laplacian integral graph, then $G_1 * G_2$ is a Laplacian integral graph.

2. Preliminaries

In this section, we introduce extended corona and extended neighborhood corona of two graphs. Also we state a lemma which is useful to prove our main results.

Let G_1, G_2 be two graphs and $V(G_1) = \{v_1, v_2, \dots, v_n\}$ be the vertex set of G_1. We define extended corona and extended neighborhood corona of two graphs G_1 and G_2 as follows:

Definition 2.1. *The extended neighborhood corona $G_1 * G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the neighborhood corona $G_1 \star G_2$ and joining each vertex of i^{th} copy of G_2 to every vertex of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.*

Definition 2.2. *The extended corona $G_1 \bullet G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the corona $G_1 \circ G_2$ and joining each vertex of i^{th} copy of G_2 to every vertex of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.*

Example 2.3.

Fig.1 Graphs $P_2 \bullet K_1$ and $P_3 * K_1$.

Let $A = (a_{ij})$ be a $n \times m$ matrix, $B = (b_{ij})$ be a $p \times q$ matrix then the *Kronecker product* $A \otimes B$ [6] of A and B is the np by mq matrix obtained by replacing each entry a_{ij} of A by $a_{ij}B$.

Lemma 2.1. *[6] If M, N, P, Q are matrices with M being a non-singular matrix, then*

$$\begin{vmatrix} M & N \\ P & Q \end{vmatrix} = |M||Q - PM^{-1}N|.$$

3. Spectrum of the extended neighborhood corona

In this section, we determine the adjacency spectrum, Laplacian and signless Laplacian spectrum of the extended neighborhood corona of two graphs in some cases.

Theorem 3.1. *Let G_1 be a graph on n vertices and G_2 be a r-regular graph on m vertices. Then the adjacency spectrum of $G = G_1 * G_2$ is given by:*

a. $\lambda_i(G_2)$ *with multiplicity n, for $i = 2, 3, \dots, m$.*

b. $\left(\lambda_i(G_1)(m+1) + r \pm \sqrt{(\lambda_i(G_1)(m+1)+r)^2 - 4r\lambda_i(G_1)}\right)/2$, *for $i = 1, 2, \dots, n$.*

Proof. With suitable labelling of the vertices of G, the adjacency matrix $A(G)$ can be formulated as follows:

$$A(G) = \begin{pmatrix} I_n \otimes A(G_2) + A(G_1) \otimes J & A(G_1) \otimes e \\ A(G_1) \otimes e^T & A(G_1) \end{pmatrix},$$

where e is the column vector of size m with all its entries are 1, I_n is the identity matrix of order n and J is the $m \times m$ matrix with all its entries are 1.

Since $A(G_2)$ is a real symmetric matrix, $A(G_2)$ is orthogonally diagonalizable and as G_2 is a r-regular graph, we have $A(G_2) = PD(G_2)P^T$, where P is a square matrix of order n with its first column vector as $1/\sqrt{m}(1, 1, \dots, 1)$, $PP^T = I_m$ and $D(G_2) = diag(\lambda_1(G_2), \lambda_2(G_2), \dots, \lambda_m(G_2))$. So

$$\begin{aligned} A(G) &= \begin{pmatrix} I_n \otimes PD(G_2)P^T + A(G_1) \otimes J & A(G_1) \otimes e \\ A(G_1) \otimes e^T & A(G_1) \end{pmatrix} \\ &= \begin{pmatrix} I_n \otimes P & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I_n \otimes D(G_2) + A(G_1) \otimes P^T J P & A(G_1) \otimes P^T e \\ A(G_1) \otimes e^T P & A(G_1) \end{pmatrix} \begin{pmatrix} I_n \otimes P^T & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} I_n \otimes P & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I_n \otimes D(G_2) + A(G_1) \otimes mJ' & A(G_1) \otimes \sqrt{m}e_1 \\ A(G_1) \otimes \sqrt{m}e_1^T & A(G_1) \end{pmatrix} \begin{pmatrix} I_n \otimes P^T & 0 \\ 0 & 1 \end{pmatrix}, \end{aligned}$$

where $e_1^T = (1, 0 \dots, 0)$ and J' is the $m \times m$ matrix obtained by replacing every entry of J by 0 except the first diagonal entry.

Let $B = \begin{pmatrix} I_n \otimes D(G_2) + A(G_1) \otimes mJ' & A(G_1) \otimes \sqrt{m}e_1 \\ A(G_1) \otimes \sqrt{m}e_1^T & A(G_1) \end{pmatrix}$. Then by the above equation we have

$$|xI - A(G)| = |xI - B|. \tag{1}$$

Expanding $|xI - B|$ by Laplace's method [9] along $(mi+2), (mi+3), \ldots, (mi+m)^{th}$ columns, for $i = 0, 1, \ldots, n-1$, we see that the only non zero $(m-1)n \times (m-1)n$ minor is

$$M = |I_n \otimes diag(x - \lambda_2(G_2), \ldots, x - \lambda_m(G_2))|. \tag{2}$$

The complementary minor of M is $M_1 = \begin{vmatrix} (x-r)I_n - mA(G_1) & \sqrt{m}A(G_1) \\ \sqrt{m}A(G_1) & xI_n - A(G_1) \end{vmatrix}$.

Again as $A(G_1)$ is orthogonally diagonalizable, one can easily see that the M_1 is same as

$$M_1' = \begin{pmatrix} (x-r)I_n - mD(G_1) & \sqrt{m}D(G_1) \\ \\ \sqrt{m}D(G_1) & xI_n - D(G_1) \end{pmatrix}, \tag{3}$$

where $D(G_1) = diag(\lambda_1(G_1), \ldots, \lambda_n(G_1))$.

Now by Lemma 2.1, we have

$$\begin{aligned} M_1 &= |xI_n - D(G_1)| \left|(x-r)I_n - m\, D(G_1) - m\, D^2(G_1)[xI_n - D(G_1)]^{-1}\right| \\ &= [(x^2 - (\lambda_1(G_1)(m+1) + r)x + r\lambda_1(G_1))][(x^2 - (\lambda_2(G_1)(m+1) + r)x + r\lambda_2(G_1))] \\ &\quad \ldots [(x^2 - (\lambda_n(G_1)(m+1) + r)x + r\lambda_n(G_1))]. \end{aligned}$$

And so by (1), (2), (3) and from above equation the result follows. □

In the following corollary, we give a method to construct infinite family of integral graphs starting with an integral graph.

Corollary 3.1. *Let G be an integral graph and m be a positive integer. Suppose $G_0 = G$ and $G_n = G_{n-1} * mK_1$, for $n \geq 1$. Then $\{G_n\}$ is an infinite sequence of integral graphs.*

Corollary 3.2. *Let G be a graph and m be a positive integer. Suppose $G_0 = G$ and $G_n = G_{n-1} * mK_1$, for $n \geq 1$. Then*

a. $\varepsilon(G * mK_1) = (m+1)\varepsilon(G)$.
b. $\{\varepsilon(G_n)\}$ *is a monotonically increasing sequence.*

As the proof of the Theorem 3.2 and Theorem 3.3 are similar to that of above theorem, we omit the details.

Theorem 3.2. *Let G_1 be a r_1-regular graph on n vertices and G_2 be an arbitrary graph on m vertices. Then the Laplacian spectrum of $G_1 * G_2$ is given by:*

a. $(m+1)r_1 + \mu_i(G_2)$ *with multiplicity n, for* $i = 1, 2, \ldots, m$.
b. $\mu_i(G_1)(m+1)$*, for* $i = 1, 2, \ldots, n$.

Corollary 3.3. *Let G_1 be an integral regular graph and G_2 be a Laplacian integral graph. Then $G_1 * G_2$ is a Laplacian integral graph.*

Let $t(G)$ denote the number of spanning trees of G. It is well known [6] that for a connected graph G on n vertices, $t(G)$ is given by

$$t(G) = \frac{\mu_2(G)\dots\mu_n(G)}{n}. \tag{4}$$

Corollary 3.4. *Let G_1 be a r_1-regular graph on n vertices and G_2 be an arbitrary graph on m vertices. Then the number of spanning trees of $G_1 * G_2$ is given by*

$$t(G_1 * G_2) = t(G_1) r_1^n (m+1)^{2(n-1)} \prod_{i=2}^{m} ((m+1)r_1 + \mu_i(G_2))^n.$$

Proof. Proof follows from the above theorem and (4). □

Let G be a graph. It is well known that $a(G) = \mu_2(G)$ is called the *algebraic connectivity* [7, 10] of G, and $a(G)$ is greater than 0 if and only if G is a connected graph. Moreover if v_i and v_j are two non-adjacent vertices of a graph G, then

$$a(G) \leq \frac{deg(v_i) + deg(v_j)}{2}. \tag{5}$$

An infinite family of graphs $\{G_i\}_{i=1}^{\infty}$, is called a family of ϵ-*expander* graphs [17], where $\epsilon > 0$ is a fixed constant, if

a. all these graphs are k-regular for a fixed integer $k \geq 3$,
b. $a(G_i) \geq \epsilon$ for $i = 1, 2, 3, \dots,$

   and
c. $n_i = |V(G_i)| \to \infty$ as $i \to \infty$.

In the following corollary we will use the extended neighbourhood corona to construct new families of expander graphs from known ones.

Corollary 3.5. *Suppose $\{G_i\}_{i=1}^{\infty}$ is a family of r-regular ϵ-expander graphs, then $\{G_i * mK_1\}_{i=1}^{\infty}$ is a family of $r(m+1)$-regular $(m+1)\epsilon$-expander graphs.*

Proof. It is easy to check that $G_i * mK_1$ is a $r(m+1)$-regular graph. Now since $f(x) := x(m+1)$ is an increasing function of x, from the above theorem and (5), we see that $a(G_i * mK_1) = a(G_i)(m+1)$ and $a(G_i * mK_1) \geq (m+1)\epsilon$. This completes the proof. □

Theorem 3.3. *Let G_1 be a r_1-regular graph on n vertices and G_2 be a r_2-regular graph on m vertices. Then the signless Laplacian spectrum of $G = G_1 * G_2$ is given by:*

a. $(m+1)r_1 + \gamma_i(G_2)$ *with multiplicity n, for $i = 2, 3, \dots, m$.*
b. $\left((\gamma_i(G_1) + r_1)(m+1) + r_2 \pm \sqrt{(m\gamma_i(G_1) - mr_1 + \gamma_i(G_1) - 2r_2 - r_1)^2 + 4mr_2(\gamma_i(G_1) - r_1)}\right)/2,$ *for $i = 1, 2, \dots, n$.*

From the above theorem we have the following corollary.

Corollary 3.6. *Let G be a signless Laplacian integral regular graph. Suppose $G_0 = G$ and $G_n = G_{n-1} * K_1$, for $n \geq 1$. Then $\{G_n\}$ is an infinite sequence of signless Laplacian integral graphs.*

4. Spectrum of the extended corona

In this section, we determine the adjacency spectrum, Laplacian and signless Laplacian spectrum of the extended corona of two graphs in some cases.

Theorem 4.1. *Let G_1 be a graph on n vertices and G_2 be a r-regular graphs on m vertices. Then the adjacency spectrum of $G = G_1 \bullet G_2$ is given by:*

a. $\lambda_i(G_2)$ *with multiplicity n, for $i = 2, 3, \ldots, m$.*

b. $\left(\lambda_i(G_1)(m+1) + r \pm \sqrt{(\lambda_i(G_1)(m-1)+r)^2 + 4m}\right)/2$*, for $i = 1, 2, \ldots, n$.*

Proof. With suitable labelling of the vertices of G, the adjacency matrix $A(G)$ can be formulated as follows:

$$A(G) = \begin{pmatrix} I_n \otimes A(G_2) + A(G_1) \otimes J & I_n \otimes e \\ I_n \otimes e^T & A(G_1) \end{pmatrix},$$

where e is a column vector of size m with all its entries are 1, I_n is the identity matrix of order n and J is the $m \times m$ matrix with all its entries are 1.

Using the fact that $A(G_2)$ is orthogonally diagonalizable and G_2 is a r-regular graph , one can easily see that $A(G)$ is similar to

$$B = \begin{pmatrix} I_n \otimes D(G_2) + A(G_1) \otimes mJ' & I_n \otimes \sqrt{m} e_1 \\ I_n \otimes \sqrt{m} e_1^T & A(G_1) \end{pmatrix},$$

where $D(G_2) = diag(\lambda_1(G_2), \lambda_2(G_2), \ldots, \lambda_m(G_2))$, $e_1^T = (1, 0 \ldots, 0)$ and J' is the $m \times m$ matrix obtained by replacing every entry of J by 0 except the first diagonal entry.

So,

$$|xI - A(G)| = |xI - B|. \tag{6}$$

Expanding $|xI - B|$ by Laplace's method [9] along $(mi+2), (mi+3), \ldots, (mi+m)^{th}$ columns, for $i = 0, 1, \ldots, n-1$, we see that the only non zero $(m-1)n \times (m-1)n$ minor is

$$M = |I_n \otimes diag(x - \lambda_2(G_2), \ldots, x - \lambda_m(G_2))|. \tag{7}$$

The complementary minor of M is

$$M_1 = \begin{vmatrix} (x-r)I_n - mA(G_1) & \sqrt{m} I_n \\ \sqrt{m} I_n & xI_n - A(G_1) \end{vmatrix}.$$

Again as $A(G_1)$ is orthogonally diagonalizable, one can easily see that the M_1 is same as

$$M_1' = \begin{vmatrix} (x-r)I_n - mD(G_1) & \sqrt{m} I_n \\ \sqrt{m} I_n & xI_n - D(G_1) \end{vmatrix}, \tag{8}$$

where $D(G_1) = diag(\lambda_1(G_1), \lambda_2(G_2), \ldots, \lambda_n(G_2))$.

Now by Lemma 2.1, we have

$$M_1' = |xI_n - D(G_1)| \left|[(x-r)I_n - mD(G_1)] - \left[m(xI_n - D(G_1))^{-1}\right]\right|$$

$$= [(x^2 - (\lambda_1(G_1)(m+1) + r)x + m\lambda_1^2(G_1) + r\lambda_1(G_1) - m)]$$

$$\ldots [(x^2 - (\lambda_n(G_1)(m+1) + r)x + m\lambda_n^2(G_1) + r\lambda_n(G_1) - m)].$$

And so by (6), (7), (8) and from above equation the result follows. □

As the proof of the Theorem 4.2 and Theorem 4.3 are similar to that of the above theorem, we omit the details.

Theorem 4.2. *Let G_1 be a r_1-regular graph on n vertices and G_2 be an arbitrary graph on m vertices. Then the Laplacian spectrum of $G = G_1 \bullet G_2$ is given by:*

a. $mr_1 + \mu_i(G_2) + 1$ *with multiplicity n, for $i = 2, 3, \ldots, m$.*
b. $\left((\mu_i(G_1) + 1)(m+1) \pm \sqrt{(m\mu_i(G_1) - m - \mu_i(G_1) + 1)^2 + 4m}\right)/2$, *for $i = 1, 2, \ldots, n$.*

From above theorem and by (4), we have the following corollary:

Corollary 4.1. *Let G_1 be a r_1-regular graph on n vertices and G_2 be an arbitrary graph on m vertices. Then the number of spanning trees of $G_1 \bullet G_2$ is given by:*

$$t(G_1 * G_2) = t(G_1) \prod_{i=2}^{m} (mr_1 + \mu_i(G_2) + 1)^n \prod_{i=2}^{n} (m^2 + \mu_i(G_1)m + 1).$$

Theorem 4.3. *Let G_1 be a r_1-regular graph on n vertices and G_2 be a r_2-regular graph on m vertices. Then the signless Laplacian spectrum of $G = G_1 \bullet G_2$ is given by:*

a. $mr_1 + \gamma_i(G_2) + 1$, *with multiplicity n, for $i = 2, 3, \ldots, m$.*
b. $\left((\gamma_i(G_1) + 1)(m+1) + r_2 \pm \sqrt{(m\gamma_i(G_1) - m - \gamma_i(G_1) + 2r_2 + 1)^2 + 4m}\right)/2$, *for $i = 1, 2, \ldots, n$*

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