

## **A Mathematical Model to Improve the Performance of Logistics Network**

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### **ABSTRACT**

*The role of logistics nowadays is expanding from just providing transportation and warehousing to offering total integrated logistics. To remain competitive in the global market environment, business enterprises need to improve their logistics operations performance. The improvement will be achieved when we can provide a comprehensive analysis and optimize its network performances. In this paper, a mixed integer linier model for optimizing logistics network performance is developed. It provides a single-product multi-period multi-facilities model, as well as the multi-product concept. The problem is modeled in form of a network flow problem with the main objective to minimize total logistics cost. The problem can be solved using commercial linear programming package like CPLEX or LINDO. Even in small case, the solver in Excel may also be used to solve such model.*

Keywords: *logistics network, integrated model, mathematical programming, network optimization*

### **Introduction**

Logistics deals with the planning and control of material flows as well as related information in organizations, and becomes a critical part of supply chain management. Its mission is to get the right materials to the right place at the right time (Ghiani, G. et.al, 2004). Logistics also deals with mobility concepts relating to tangible as well as intangible assets (Beamon, 1998).

Logistics activities connect and activate the objects in the supply chain in the form of a

logistics network. A logistics network consists of suppliers, manufacturing centers, warehouses, distribution centers, and retail outlets as well as channels for the flow of raw materials, work-in-process inventory, and finished products between the facilities (Simchi-Levi et al., 2000). On the other side, Tavasszy et.al (2003) stated that logistics deals also with the achievement of customer satisfaction at the minimum level of costs. It is a crucial problem in business nowadays due to the high proportion of logistics costs in the costs of goods sold (Bowersox, 1996).

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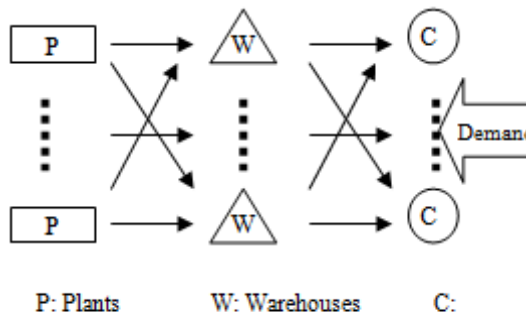


Figure 1. Multi-facilities network flow

The need to improve the network performance is also inline with the development of business nowadays. Business paradigm has changed. The new paradigm is to meet customer demands and minimize inventory while the old is to focus more on supply and keep large inventories. Therefore, business operations tend to move to be a network-based collaboration as well as global network, which means network of networks, to remain competitive in the global market. At this stage, the role of logistics is expanding from just providing transportation and warehousing to offering total integrated logistics, while optimizing a given performance measure and satisfying a given set of constraints (Ghiani, G. et.al, 2004).

The integrated logistics network is the integration of several business functions (procurement, manufacturing, and distribution) that could be drawn as an abstract of nodes and arcs. It covers either micro (within facility) or macro levels (between facilities). Those functions link an enterprise with its customers and suppliers (Bowersox, 1996). In many cases, the problems of each logistics functions are treated as isolated functions. In integrated analysis, the logistics functions are treated simultaneously rather than isolated, and the network performance optimality will be achieved when each component is optimum as well as the global optimum.

Recent progress in the industries displays the benefits produced in integrating the networks. It reduces cost, creates profits,

increases market share, strengthens competitive position, and enhances the value of the company (Lee, 2000). On the other hand, as logistics become more complex with increasing market volatility, the need to analyze and improve its performance becomes a crucial matter in logistics network.

In order to improve the logistics performance, there are several issues that should be encountered (e.g. Bramel, 1997; Bowersox, 1996; Lambert, 1998). Bowersox (1996) addressed 5 major problems as logistics competencies. There are: network design; information flow; transportation; inventory; and warehousing, material handling, and packaging. Many researchers have proposed the quantitative approach to improve and optimize the logistics system performance, such as Geoffrion, A.(1974), Blumenfeld, D.E. et al (1987), Brown, G.G. et al (1987), and Lakhal, S. et.al (2000).

The new approach to the analysis of logistics network and supply chains has been identified, which has proven to be of significant relevance to companies that have adopted it. This approach is based on the integration of different logistics functions into a single optimization model.

Integrated logistics network problem deals with a number of manufacturing plants; zero, one, or more distribution echelons with distribution centers; the customers; the suppliers of components and raw materials; recycling centers for used products and returned packaging containers; and the transportation channels that link all of the above components (Goetschalckx, et.al, 2002) as well as material flow and information (Blanchard, 1998). Therefore, the analysis to improve the network performance should elaborate procurement, production, transportation, and distribution decisions in one simultaneous analysis. Whereas, the basis of performance improvement in integrated logistics network is total cost analysis (Lambert, 1993). That is minimizing the total

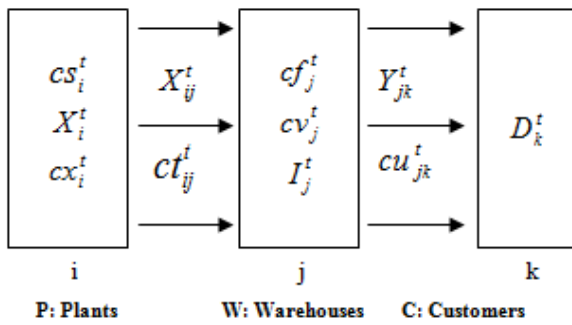


Figure 2. Network model representative

cost of transportation, warehousing, order processing and information, lot quantity, and inventory carrying cost.

In this paper, I consider an integrated logistics problem in minimizing the total logistics cost and provide model for logistics network optimization in form of a mathematical programming model. The major purpose of this paper is to investigate the application of integrated approach in solving the production and distribution-planning problem.

### Literature Review

Erenguc, et.al. (1999) provided a taxonomical framework for analyzing integrated production-distribution network in supply chains and proposed an future research on integrated approach to managing inventory decision.

Cohen and Lee (1988) addressed a methodology that measure tradeoff of cost, service, and flexibility in production-distribution system. The methodology considers relationship between production and distribution control policies that affect inventory control, plant production mix, and production scheduling.

Flipo and Finke (2001) interested in a multi-facility, multi-product and multi-period industrial problem. They stated that to solve a cost optimization problem in those problem

both production and distribution cost are significant and inter-related. Therefore they should be considered simultaneously. They developed their model in form of a network flow problem with relatively few additional 0-1 variables to describe the linking constraints between periods. They stated that the real size problem could be solved in reasonable time using commercial linier programming package.

Syam (2002) addressed an integrated model of logistics network that minimize total physical distribution costs by simultaneously determining optimal plants and warehouse locations, flows in the resulting network, shipment compositions and shipment frequencies in the network using heuristics methodology. The author proposed two methodologies that are simulated annealing and Lagrangian relaxation. Simulated annealing first is used to determine the optimal sets of open plants and warehouses. In turn, Lagrangian relaxation is used to solve the flow and consolidation problem in the resulting network.

Cordeau, et.al. (2002) formulated the deterministic logistics network in a single country and a single period that integrated location and capacity choices for plants and warehouses. In order to solve the logistics network design problem, two approaches are used, a simplex-based branch and bound and a Benders decomposition approach. Some valid constraints are also proposed to strengthen the LP relaxation of the problem. Introducing valid constraints in the master problem could dramatically reduce the number of cuts.

Lei lei (2003) concerned with the problem that is to determine the operation schedules to coordinate the production, inventory, and transportation routing operations so that the customer demand, transporter travel time and capacity constraints, plant production and inventory constraints are all satisfied, while the resulting operation cost (i.e., the sum of production, inventory and transportation cost) over a given planning horizon is minimized.

To solve that problem, two-phase solution approaches are used. At the first phase solves a mixed integer-programming model subject to the constraints in the original model except the transporter routings are restricted to direct shipment between facilities and customer demand centers. In the second phase applies a heuristic procedure to solve associated consolidation problem.

Daskin (2003) tried to minimize the total cost of transportation, inventory, and location by selecting the distribution center (DC) locations, assigning retailers to DCs, and determining order frequency and safety stock level at each DC.

Dong and Chen (2005) addressed several logistics aspects that should be addressed when we want to improve its integrated performance that are the network structure (serial, parallel, assembly and arbores-cent distribution), product structure (levels of Bill of Materials), transportation modes, and degree of uncertainty that they face.

Ambrosino and Scutella (2005) addressed an integrated distribution network model that involves facility location, transportation, and inventory decision to minimize the associated cost by defining the number and the location of the facilities in the network. There are two scenarios that have been investigated, both of them did not include inventories, and two types customer are addressed: clients and big clients.

Park (2005) presented the solution for integrated production and distribution planning and investigated the effectiveness of the integration through a computational study, in a multi-plant, multi-retailer, multi-item, and multi-period logistic environment where the objective is to maximize the total net profit.

## Model Structure and Formulation

In order to improve logistics network performance, several optimization methods can

be used, such as mathematical programming, genetic algorithm, simulated annealing, etc. This paper will use mathematical programming for the reason: it provides insight into problem, its characteristics and the linkages between the various interacting factors. Furthermore, there has been considerable progress in recent years with solving large-scale integer programming problems.

## Single-product multi-periods multi-facilities system

In this model (Fig.1), it is assumed that there are a number of plants that produce single product with a specific capacity over period time. The set-up cost is a fixed cost on a lot-for lot basis, not dependent on the realized volume. All products are assumed directly delivered to warehouse or retail outlet.

Products are delivered using a fleet homogeneous vehicle. The movement of vehicle is assumed incurs a variable transportation cost only.

The demand for an item in a period at warehouse is expressed as forecasted a real demand. It is assumed that the demands are given and backordering is not allowed. Each warehouse must keep a limited amount of inventory, with higher holding cost.

The objective is to optimize the production and distribution plan so as to minimize its total logistics cost. In this case, the optimization model will be:

*Minimize total cost* = production cost + transportation cost + inventory cost

*Subject to constraints:*

1. Production capacity,
2. Warehouse capacity,
3. Material flow requirements,
4. Balance constraints of inventory level

We shall represent the problem in the form of network (Fig.2). We define the network for the flow of products from their production points to customers through the warehouse as storage. This model then refers to three components: the production sites, indexed by  $i$ , the warehouse, indexed by  $j$ , and the customers, indexed by  $k$ .

A mixed integer model is then developed to solve the problem. The model comprises two cost model: (a) from Plant to Warehouse and (b) from Warehouse to Customers.

Notation that used in the model :

$X_i^t$  = amount of product produced at plant  $i$  in period  $t$

$X_{ij}^t$  = amount of product transported from plant  $i$  to warehouse  $j$  in period  $t$

$Y_{jk}^t$  = amount of product transported from warehouse  $j$  to customer  $k$  in period  $t$

$M_i$  = capacity of plant  $i$

$W_j$  = capacity of warehouse  $j$

$D_k^t$  = demand of customer  $k$  in period  $t$

$I_j^t$  =inventory level at warehouse  $j$  at the end of period  $t$

$cs_i^t$  = setup cost at plant  $i$  in period  $t$

$cx_i^t$  =unit cost of production at plant  $i$  in period  $t$

$ct_{ij}^t$  = unit cost of transportation to delivered product from plant  $i$  to warehouse  $j$  in period  $t$

$cu_{jk}^t$  = unit cost of transportation to delivered product from warehouse  $j$  to customer  $k$  in period  $t$

$cf_j^t$  =fixed cost at warehouse  $j$  in period  $t$

$cv_j^t$  =variable cost at warehouse  $j$  in period  $t$

### Decision Variable

$X_{ij}^t$  = amount of product transported from plant  $i$  to warehouse  $j$  in period  $t$

$Y_{jk}^t$  = amount of product transported from warehouse  $j$  to customer  $k$  in period  $t$

### Objective Function

The objective function (1a-1b) concerns the minimization of costs. It express the total cost includes the cost of production, transportation, and inventory over the time periods. (1a) shows the total cost incurs in stage 1 (from production site until products are delivered to warehouses), and (1b) is the total cost from warehouses to customer side.

$$\text{Min } \sum_i cs_i^t + cx_i^t X_i^t + \sum_i \sum_j X_{ij}^t ct_{ij}^t \quad (1a)$$

$$+ \sum_j cf_j^t + \sum_j I_j^t cv_j^t + \sum_j \sum_k Y_{jk}^t cu_{jk}^t \quad (1b)$$

### Model Constraints

$$\sum_j X_{ij}^t \leq M_i \quad \forall i, t \quad (2)$$

$$\sum_i X_{ij}^t + I_j^{t-1} - \sum_k Y_{jk}^t \leq W_j^t \quad \forall j, t \quad (3)$$

$$\sum_j X_{ij}^t \geq \sum_k Y_{jk}^t \quad \forall i, t \quad (4)$$

$$D_k^t = \sum_j Y_{jk}^t \quad \forall j, t \quad (5)$$

$$\sum_i X_{ij}^t + I_j^{t-1} = \sum_k D_k^t + I_j^t \quad \forall j, t \quad (6)$$

$$X_{ij}^t, Y_{jk}^t, I_j^t \geq 0 \quad (7)$$

$$M_i, W_i^t \geq 0 \quad (8)$$

From the model, constraints (2) is related to the capacity of production facility. It is also indicate the maximum number of product that can be produced during period t in plant i. Product flow from plants to warehouses must respected the throughput capacity of warehouse j, as indicated in (3). Constraint (4) related to product flow requirement in warehouse j. Total product delivered out from warehouse cannot exceed total product delivered to warehouse. Constraint (5) requires that warehouses must satisfy all demand. A Balance constraint of inventory level in warehouse j is provided by (6). (7) and (8) are non-negative value.

### Multi Periods Issue

Analysis of multi-period issue is mostly used to anticipate periods of high demand.

Therefore, it is necessary to store reasonable amount of product in advance, which in our model, this stock will be in warehouses. In this case, we need arcs between nodes representing a same stock during adjacent periods. The product flows on these arcs are products that stay in stock from one period to the next. As for that, there should be holding costs associated with these flows. Constraints (3) and (6) are used to control inventory flow in warehouses as well as its capacity.

### Multi-products multi-periods multi-facilities system

The model exposed in the previous section represent the model for single product. In the real industry application, most company produce not only singles product but many products. Therefore, to cover this problem, our mixed integer linier model will be copied to allow us to minimize total cost in multi-product application. To do this, the product type will be indexed by m and put in subscript that indicates the product model they are related to.

The objective function will become following:

$$\text{Min} \quad \sum_i \sum_m cs_{im}^t + cx_{im}^t X_{im}^t + \sum_i \sum_j \sum_m X_{ijm}^t ct_{ijm}^t \quad (1a^*)$$

+

$$\sum_j \sum_m cf_{jm}^t + \sum_j \sum_m I_{jm}^t cv_{jm}^t + \sum_j \sum_k \sum_m Y_{jkm}^t cu_{jkm}^t \quad (1b^*)$$

## Conclusion

In this paper, models to deal with single-product multi-facilities multi-periods that used to improve logistics network performance have been presented, as well as multi-products. By this model, we can optimize the network performance by minimizing its total cost, and plan the inventory at warehouses.

However, this model cannot be used to get the exact solution for larger problem that is composed with a very large number of lines and customers. Therefore, it may be need to develop heuristics to deal with such larger problems.

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