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On size multipartite Ramsey numbers for stars versus paths and cycles

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Abstract

Let $K_{l\times t}$ be a complete, balanced, multipartite graph consisting of l partite sets and t vertices in each partite set. For given two graphs G_1 and G_2, and integer $j \geq 2$, the size multipartite Ramsey number $m_j(G_1, G_2)$ is the smallest integer t such that every factorization of the graph $K_{j\times t} := F_1 \oplus F_2$ satisfies the following condition: either F_1 contains G_1 or F_2 contains G_2. In 2007, Syafrizal et al. determined the size multipartite Ramsey numbers of paths P_n versus stars, for $n = 2, 3$ only. Furthermore, Surahmat et al. (2014) gave the size tripartite Ramsey numbers of paths P_n versus stars, for $n = 3, 4, 5, 6$. In this paper, we investigate the size tripartite Ramsey numbers of paths P_n versus stars, with all $n \geq 2$. Our results complete the previous results given by Syafrizal et al. and Surahmat et al. We also determine the size bipartite Ramsey numbers $m_2(K_{1,m}, C_n)$ of stars versus cycles, for $n \geq 3, m \geq 2$.

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1. Introduction

Burger and Vuuren[1] studied one of generalizations of the classical Ramsey number problem. They introduced the size multipartite Ramsey number as follow. Let j, l, n, s and t be natural numbers with $n, s \geq 2$. The size multipartite Ramsey number $m_j(K_{n\times l}, K_{s\times t})$ is the smallest natural number ζ such that an arbitrary coloring of the edges of $K_{j\times\zeta}$, using the two colors red and blue, necessarily forces a red $K_{n\times l}$ or a blue $K_{s\times t}$ as a subgraph. They also determined the exact values of $m_1(K_{2\times 2}, K_{2\times 2})$ and $m_j(K_{2\times 2}, K_{3\times 1})$, for $j \geq 1$.

In [10], Syafrizal et al. generalized this concept by removing the completeness requirement as follows. For given two graphs G_1 and G_2, and integer $j \geq 2$, the *size multipartite Ramsey number* $m_j(G_1, G_2) = t$ is the smallest integer such that every factorization of graph $K_{j\times t} := F_1 \oplus F_2$ satisfies the following condition: either F_1 contains G_1 as a subgraph or F_2 contains G_2 as a subgraph. They also determined the size multipartite Ramsey numbers of paths versus other graphs, especially cycles and stars [10, 11, 12]. In this paper, we determine the size multipartite Ramsey numbers, $m_j(K_{1,m}, H)$, for $j = 2, 3$, where H is a path or a cycle on n vertices, and $K_{1,m}$ is a star of order $m + 1$.

Let G be a simple and finite graph. The *null* graph is the graph with n vertices and zero edges. A *matching* of a graph G is defined as a set of edges without a common vertex. The *maximum* degree of G is denoted by $\Delta(G)$, where $\Delta(G) = max\{d_G(v)|v \in V(G)\}$. The *minimum* degree of G is denoted by $\delta(G)$, where $\delta(G) = min\{d_G(v)|v \in V(G)\}$. A graph G of order n is called *Hamiltonian* if it contains a cycle of length n and it called *bipancyclic* if it contains cycles of all even lengths from 4 to n. A connected graph G is said to be *k-connected*, if it has more than k vertices and remains connected whenever fewer than k vertices are removed. A set U of vertices in a graph G is *independent* if no two vertices in U are adjacent. The maximum number of vertices in an independent set of vertices of G is called *independent number* of G and is denoted by $\alpha(G)$. For two vertices $x, y \in G$, if x is adjacent to y, then we denote by $x \sim y$. Otherwise, we denote by $x \nsim y$.

In this paper, we also use the following theorems to prove our results.

Theorem 1.1. *[4] If G is a graph of order n and the minimum degree of G, $\delta(G) \geq \frac{n}{2}$, then G is a Hamiltonian.*

Theorem 1.2. *[3] Let G be an s-connected graph with no independent set of $s + 2$ vertices. Then, G has a Hamiltonian path.*

Theorem 1.3. *[8] Let G be a balanced bipartite graph on $2n$ vertices. If the minimum degree of G, $\delta(G) \geq \frac{n+1}{2}$, then G is bipancyclic.*

2. Stars versus Paths

Hattingh and Henning gave the results for the size bipartite Ramsey numbers of stars versus paths, as follows.

Theorem 2.1. *[5] For positive integers $m, n \geq 2$,*

$$m_2(K_{1,m}, P_n) = \begin{cases} \frac{n}{2} + m - 1, & \text{for } m \leq \frac{n}{2} + 1, n \text{ is even,} \\ \frac{n-1}{2} + m, & \text{for } m \leq \frac{n-1}{2} + 1, n \text{ is odd, } m - 1 \equiv 0 \text{ mod}(\frac{n-1}{2}), \\ \frac{n-1}{2} + m - 1, & \text{for } m \leq \frac{n-1}{2} + 1, n \text{ is odd, } m - 1 \not\equiv 0 \text{ mod}(\frac{n-1}{2}), \\ 2m - 1, & \text{for } \frac{1}{2}\lfloor \frac{n}{2} \rfloor + 1 \leq m < \lfloor \frac{n}{2} \rfloor + 1, \\ \lfloor \frac{n+1}{2} \rfloor, & \text{for } m < \frac{1}{2}\lfloor \frac{n}{2} \rfloor + 1. \end{cases}$$

For positive integers $m, n \geq 1$, Christou et al. [2] determined the size bipartite Ramsey numbers of stars $K_{1,m}$ versus nP_2.

The size multipartite Ramsey numbers of paths P_n versus stars was determined only for $n = 2, 3$ by Syafrizal et al. [11] in 2007. Furthermore, Surahmat et al. [9] gave the size tripartite Ramsey numbers of paths P_n versus stars, for $n = 3, 4, 5, 6$. Lusiani et al. [7] gave the size tripartite Ramsey numbers of paths P_3 versus a disjoint union of m copies of a star. In this section, we investigate the size tripartite Ramsey numbers of paths P_n versus stars, with all $n \geq 2$. Our results complete the previous results given by Syafrizal et al. and Surahmat et al.

Theorem 2.2. *For positive integers $n \geq 2$, $m_3(K_{1,2}, P_n) = \lceil \frac{n}{3} \rceil$.*

Proof. For $n = 2, 3$, it is clear that $m_3(K_{1,2}, P_n) \geq 1$. To show that $m_3(K_{1,2}, P_n) \geq \lceil \frac{n}{3} \rceil$, for $n \geq 4$, let us consider a factorization the graph $K_{3\times(\lceil \frac{n}{3} \rceil - 1)} = F_1 \oplus F_2$. We choose F_1 as a matching, then $F_1 \not\supseteq K_{1,2}$. Since $|V(K_{3\times(\lceil \frac{n}{3} \rceil - 1)})| = |V(F_2)| = 3(\lceil \frac{n}{3} \rceil - 1) < n$, we obtain $F_2 \not\supseteq P_n$.

Now, we show that $m_3(K_{1,2}, P_n) \leq \lceil \frac{n}{3} \rceil$. For $n = 2, 3$, we know that in any red-blue coloring avoiding a red $K_{1,2}$, there will be a blue P_2 or a blue P_3. Therefore, $m_3(K_{1,2}, P_n) \leq 1$, for $n = 2, 3$. For $n \geq 4$, we consider a factorization $K_{3\times\lceil \frac{n}{3} \rceil} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,2}$, so $\Delta(G_1) \leq 1$. Then $\delta(G_2) \geq |V(G_2)| - \lceil \frac{n}{3} \rceil - \Delta(G_1) = 2\lceil \frac{n}{3} \rceil - 1 \geq \frac{3}{2}\lceil \frac{n}{3} \rceil = \frac{|V(G_2)|}{2}$. By Theorem 1.1, we have that G_2 is Hamiltonian which implies $G_2 \supseteq P_n$, for $n \geq 4$.

□

Theorem 2.3. *For positive integer $n \geq 2$,*

$$m_3(K_{1,3}, P_n) = \begin{cases} \lceil \frac{n}{3} \rceil + 1, & \text{if } 2 \leq n \leq 6, \\ \lceil \frac{n}{3} \rceil, & \text{if } n \geq 7. \end{cases}$$

Proof.

To show that $m_3(K_{1,3}, P_n) \geq t$, let $t = \begin{cases} 2, & \text{if } 2 \leq n \leq 3, \\ 3, & \text{if } 4 \leq n \leq 6, \\ \lceil \frac{n}{3} \rceil, & \text{if } n \geq 7. \end{cases}$

We consider a factorization the graph $K_{3\times(t-1)} = F_1 \oplus F_2$, where F_1 does not contain $K_{1,3}$. We consider the following three cases.

Case 1. For $2 \leq n \leq 3$.
We have $K_{3\times(t-1)} = K_3$. We can choose $F_1 = K_3$, which implies $F_1 \not\supseteq K_{1,3}$ and $F_2 \not\supseteq P_n$.

Case 2. For $4 \leq n \leq 6$.
We have $K_{3\times(t-1)} = K_{3\times 2}$. We can choose $F_1 = C_6$ and $F_2 = 2C_3$, see Figure 1. Then, $F_1 \not\supseteq K_{1,3}$ and the longest path in F_2 is a P_3.

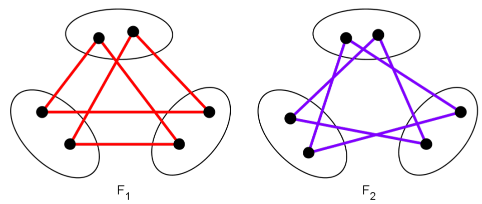

Figure 1. F_1 is a C_6 and F_2 is $2C_3$.

Case 3. For $n \geq 7$.
We have $K_{3\times(t-1)} = K_{3\times(\lceil\frac{n}{3}\rceil-1)}$. We can choose $F_1 = C_{3(\lceil\frac{n}{3}\rceil-1)}$, then $F_1 \not\supseteq K_{1,3}$. Since $|V(K_{3\times(t-1)})| = |V(F_2)| = 3(\lceil\frac{n}{3}\rceil - 1) < n$, we obtain $F_2 \not\supseteq P_n$.

Now, we show that $m_3(K_{1,3}, P_n) \leq t$, let $t = \begin{cases} 2, & \text{if } 2 \leq n \leq 3,\\ 3, & \text{if } 4 \leq n \leq 9,\\ \lceil\frac{n}{3}\rceil, & \text{if } n \geq 10.\end{cases}$
We consider a factorization $K_{3\times t} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,3}$, so $\Delta(G_1) \leq 2$. We consider the following three cases.

Case 1. For $2 \leq n \leq 3$.
We have $K_{3\times t} = K_{3\times 2}$. Since $\Delta(G_1) \leq 2$, then $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 6 - 2 - 2 = 2$, which implies that $G_2 \supseteq P_3$.

Case 2. For $4 \leq n \leq 9$.
We have $K_{3\times t} = K_{3\times 3}$. Since $\Delta(G_1) \leq 2$, then $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 9 - 3 - 2 = 4$. We will use Theorem 1.2 to show that G_2 is a Hamiltonian path. So, we will show that G_2 is a 2-connected graph with no independent set of 4 vertices. Let A, B, C be the three partities of G_2. Let $x \neq y$, where x, y be any vertices in G_2 and $S = N(x) \cap N(y)$. There are two possibilities:

1. Let x and y be in the same partite set. Since $\delta(G_2) \geq 4$, then $S \neq \emptyset$ and $|S| \geq 2$. So, two vertices of S together with x and y form a C_4.
2. Let x and y be in the different partite sets, say $x \in A$ and $y \in B$.
   (a) $x \sim y$. If $S = \emptyset$, then there exist $c_1, c_2 \in C, c_1 \neq c_2$ such that $x \sim c_1$ and $y \sim c_2$. Now, since $\delta(G_2) \geq 4$, $K = N(c_1) \cap N(c_2) \neq \emptyset$, say $b_2 \in K$. Then $\{x, y, c_1, c_2, b_2\}$ form a C_5. Also, If $S \neq \emptyset$, then x and y will be contained in a C_3.

(b) $x \nsim y$. Since $\delta(G_2) \geq 4$, then $|S| \geq 1$ and $S \subseteq C$. If $|S| \geq 2$, then x, y and two vertices of S will create a C_4. If $|S| = 1$, then $B - \{y\} \subseteq N(x)$ and $|N(y) \cap C| = 2$. Let $N(y) \cap C = \{c_1, c_2\}$. Since $\delta(G_2) \geq 4$, then $|N(c_1) \cap (B - \{y\})| \geq 1$. Therefore, select $b_1 \in B - \{y\}$ such that $c_1 \sim b_1$. Then, $xc_2yc_1b_1x$ is a C_5.

Since any two different vertices in G_2 belongs to a cycle, G_2 is a 2-connected graph. Now, we show that $\alpha(G_2) = 3$. Since G_2 is a factor of $K_{3\times 3}$, $\alpha(G_2) \geq 3$. $\alpha(G_2) \leq 3$, as any independent set of G_2 can have at most one element from each of the three partite sets. So, we have $\alpha(G_2) = 3$. Then, by Theorem 1.2, G_2 is a Hamiltonian path, which implies $G_2 \supseteq P_n$, for $4 \leq n \leq 9$.

Case 3. For $n \geq 10$.
We have $K_{3\times t} = K_{3\times \lceil \frac{n}{3} \rceil}$. Since $\Delta(G_1) \leq 2$, then $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 2\lceil \frac{n}{3} \rceil - 2 \geq \frac{3}{2}\lceil \frac{n}{3} \rceil = \frac{|V(G_2)|}{2}$. Thus, by Theorem 1.1, G_2 is Hamiltonian which implies $G_2 \supseteq P_n$, for $n \geq 10$. □

Theorem 2.4. *For positive integers* $4 \leq m \leq \frac{1}{2}\lceil \frac{n}{3} \rceil + 1$ *and* $n \geq 16, m_3(K_{1,m}, P_n) = \lceil \frac{n}{3} \rceil$.

Proof. To show that $m_3(K_{1,m}, P_n) \geq \lceil \frac{n}{3} \rceil$, let us consider a factorization graph $K_{3\times(\lceil \frac{n}{3} \rceil - 1)} = F_1 \oplus F_2$, where F_1 does not contain $K_{1,m}$. We can choose $F_1 = C_{3\lceil \frac{n}{3} \rceil - 3}$, then $F_1 \nsupseteq K_{1,m}$. Since $|V(K_{3\times(\lceil \frac{n}{3} \rceil - 1)})| = |V(F_2)| = 3\lceil \frac{n}{3} \rceil - 3 < n$, we obtain $F_2 \nsupseteq P_n$.

Now, we show that $m_3(K_{1,m}, P_n) \leq \lceil \frac{n}{3} \rceil$. We consider a factorization $K_{3\times \lceil \frac{n}{3} \rceil} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,m}$, so $\Delta(G_1) \leq m - 1$. Then, $\delta(G_2) \geq |V(G_2)| - \lceil \frac{n}{3} \rceil - \Delta(G_1) = 2\lceil \frac{n}{3} \rceil - (m-1)$. Since $\delta(G_2) \geq 2\lceil \frac{n}{3} \rceil - (m-1)$ and $2(m-1) \leq \lceil \frac{n}{3} \rceil$, then $\delta(G_2) \geq 2\lceil \frac{n}{3} \rceil - \frac{1}{2}\lceil \frac{n}{3} \rceil = \frac{3}{2}\lceil \frac{n}{3} \rceil = \frac{|V(G_2)|}{2}$. Then, by Theorem 1.1, G_2 is Hamiltonian which implies $G_2 \supseteq P_n$. □

3. Stars versus Cycles

The size multipartite Ramsey numbers for paths versus cycles of three or four vertices have been showed by Syafrizal et al. [12]. Recently, Lusiani et al. [6] showed the size multipartite Ramsey numbers for stars versus cycles. Now, we investigate the size bipartite Ramsey numbers for stars versus cycles. The research is inspired by the work of Hattingh and Henning on the size bipartite Ramsey numbers for stars versus paths. It seems that $m_2(K_{1,m}, C_n)$ is related to $m_2(K_{1,m}, P_n)$. However, since a complete bipartite graph does not contain odd cycles, then it is clear that $m_2(K_{1,m}, C_n) = \infty$. Now, we only consider $m_2(K_{1,m}, C_n)$, where n is even. To show this relation, in Theorem 3.1, we obtain the exact value of $m_2(K_{1,m}, C_n)$ for certain values of n.

Theorem 3.1. *Let* $m \geq 2$ *and* $n \geq 2m$, *where* n *is even. Then,*

$$m_2(K_{1,m}, C_n) = \begin{cases} 2m - 1, & \text{for } 2m \leq n \leq 4m - 4, \\ \lceil \frac{n}{2} \rceil, & \text{for } 4m - 2 \leq n. \end{cases}$$

Proof. Let $t = \begin{cases} 2m - 1, & \text{for } 2m \leq n \leq 4m - 4, \\ \lceil \frac{n}{2} \rceil, & \text{for } 4m - 2 \leq n. \end{cases}$

To show that $m_2(K_{1,m}, C_n) \geq t$, let us consider a factorization the graph $K_{2\times(t-1)} = F_1 \oplus F_2$, such that F_1 does not contain $K_{1,m}$. Then, $\Delta(F_1) \leq (m-1)$. We consider the following two cases.

Case 1. For $2m \leq n \leq 4m-4$.
We have $K_{2\times(t-1)} = K_{2\times(2m-2)}$. We can choose $F_1 = 2K_{2\times(m-1)}$. The complement of F_1 relative to $K_{2\times(2m-2)}$ is $2K_{2\times(m-1)}$. So, we get $F_2 = 2K_{2\times(m-1)}$, which implies $F_1 \not\supseteq K_{1,m}$ and $F_2 \not\supseteq C_n$ for $2m \leq n \leq 4m-4$.

Case 2. For $4m-2 \leq n$.
We have $K_{2\times(t-1)} = K_{2\times(\lceil\frac{n}{2}\rceil-1)}$. If we choose $F_2 = K_{2\times(\lceil\frac{n}{2}\rceil-1)}$, then F_1 is a null graph. So, $F_1 \not\supseteq K_{1,m}$. Since $|V(K_{2\times(t-1)})| = |V(F_2)| = 2(\lceil\frac{n}{2}\rceil - 1) < n$, we obtain $F_2 \not\supseteq C_n$.

Now, we show that $m_2(K_{1,m}, C_n) \leq t$. We consider a factorization $K_{2\times t} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,m}$, so $\Delta(G_1) \leq (m-1)$. We consider the following two cases.

Case 1. For $2m \leq n \leq 4m-4$.
We have $K_{2\times t} = K_{2\times(2m-1)}$. Since $\Delta(G_1) \leq (m-1)$, then $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 2m-1-(m-1) = m$. Then, by Theorem 1.4, G_2 is bipancyclic, which implies $G_2 \supseteq C_n$, for $2m \leq n \leq 4m-4$.

Case 2. For $4m-2 \leq n$.
We have $K_{2\times t} = K_{2\times(\lceil\frac{n}{2}\rceil)}$. Since $\Delta(G_1) \leq (m-1)$, then $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = \lceil\frac{n}{2}\rceil - (m-1) \geq \frac{1}{2}(\lceil\frac{n}{2}\rceil + 1)$, for $n \geq 4m-2$. Thus, by Theorem 1.3, G_2 is bipancyclic, which implies $G_2 \supseteq C_n$, for $n \geq 4m-2$. □

In the next two theorem, we consider $m_2(K_{1,m}, C_n)$ for certain values of m and n which are not included in Theorem 3.1. In particular, we prove that $m_2(K_{1,3}, C_4) = 5$ in Theorem 3.2 and $m_2(K_{1,4}, C_4) = 6$ in Theorem 3.3.

Theorem 3.2. $m_2(K_{1,3}, C_4) = 5$.

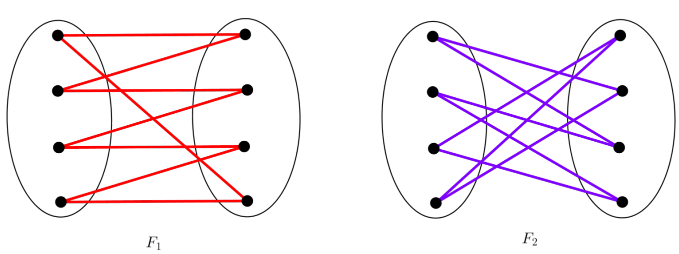

Figure 2. F_1 is a C_8 and F_2 does not contain a C_4.

Proof. To show that $m_2(K_{1,3}, C_4) \geq 5$, let us consider a factorization the graph $K_{2\times4} = F_1 \oplus F_2$, where F_1 does not contain $K_{1,3}$. If we choose $F_1 = C_8$, then F_2 does not contain a C_4, as shown in Figure 2. This implies that $F_1 \not\supseteq K_{1,3}$ and $F_2 \not\supseteq C_4$.

Now, we show that $m_2(K_{1,3}, C_4) \leq 5$. We consider a factorization $K_{2\times5} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,3}$, so $\Delta(G_1) \leq 2$. Then, $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 10 - 5 - 2 = 3$. Thus, by Theorem 1.3, G_2 is bipancyclic, which implies $G_2 \supseteq C_4$. □

Theorem 3.3. $m_2(K_{1,4}, C_4) = 6$.

Proof. To show that $m_2(K_{1,4}, C_4) \geq 6$, let us consider a factorization the graph $K_{2\times5} = F_1 \oplus F_2$, where F_1 does not contain $K_{1,4}$. Then, $\Delta(F_1) \leq 3$ and $\delta(F_2) \geq |V(F_2)| - t - \Delta(F_1) = 10 - 5 - 3 = 2$. We can choose $F_2 = C_{10}$. So, we get $F_2 \not\supseteq C_4$. Now, we show that $m_2(K_{1,4}, C_4) \leq 6$. We consider a factorization $K_{2\times6} = G_1 \oplus G_2$ such that G_1 does not contain $K_{1,4}$, so $\Delta(G_1) \leq 3$. Then, $\delta(G_2) \geq |V(G_2)| - t - \Delta(G_1) = 12 - 6 - 3 = 3$. Let A and B be the two partitie sets of $K_{2\times6}$. Let $a_1 \in A$ be adjacent to $b_i \in B$, $i \in \{1, 2, 3\}$ in G_2. Since $\delta(G_2) \geq 3$, then each b_i is adjacent to at least two vertices in $A - \{a_1\}$. By the Pigeonhole Principle, there exists at least one vertex in $A - \{a_1\}$ adjacent to two vertices in $\{b_1, b_2, b_3\}$. So, we get C_4 in G_2. □

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