# Susceptible-Infected-Recovered Epidemic Model on the Spread of Tuberculosis Disease in Central Java

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## ABSTRACT

**Background:** In order to understand tuberculosis (TB) trends, the epidemiologist need to focus on the long-term consequences of tuberculosis epidemics, which can be observed with the analysis of epidemiological models. Susceptible, infectious, recovered (SIR) model is an epidemiological models consist of compartments which represent sets of individuals grouped by TB status. This study aimed to examined SIR model on the spread of TB disease.
**Subjects and Method:** This was a cross sectional study. The study population was tuberculosis patients in Central Java, Indonesia, from 2018 to 2019. The study involved three sub-populations, included (1) suspected (high risk infected to TB), (2) infected (TB patients), (3) cured (TB survivors). TB cases were obtained from medical record. The data were analyzed by susceptible, infectious, recovered (SIR) model.
**Results:** The epidemic model found that an infected TB patient may disseminate TB cases to 2 suspected persons. SIR model can be used to calculate reproduction number of TB.
**Conclusion:** SIR model can be used to calculate reproduction number of TB.

**Keywords:** tuberculosis, SIR model, COVID-19

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## BACKGROUND

Tuberculosis (TB) or TB is an infectious disease of the lungs caused by the bacterium Mycobacterium tuberculosis. Altho-ugh commonly known as lung disease, other parts of the body, such as the kidneys, spine, and brain, can be damaged by tuberculosis. Tuberculosis was declared a viral epidemic for the first time on March 24, 1882 by a German scientist Robert Koch. Koch discovered that Mycobacterium tuberculosis is a pathogenic bacterium that transmits tuberculosis.

Since it was first declared as a disease caused by a virus, tuberculosis is still a world health problem today. Tuberculosis has become even more frightening during the COVID-19 pandemic. As we know, the corona virus also attacks the respiratory tract so that the risk of death will increase when the mycobacterium tuberculosis germs are already in the patient's body. Based on this, the Indonesian government is currently intensively vaccinating against COVID-19, but TB sufferers have several special rules before vaccinating. In tuberculosis patients who are under-


https://doi.org/10.26911/ICPHmanagement.FP.08.2021.20

going treatment, it is necessary to give anti-tuberculosis drugs first, at least 2 weeks before receiving the vaccine. This is done so that the risks that may occur after vaccination can be avoided to a minimum.

Central Java is one of the provinces in Indonesia which certainly cannot be separated from the same health problems. The Central Java government has also made various efforts to reduce the death rate from tuberculosis. However, based on data, there was a significant increase in the number of tuberculosis sufferers in Central Java between 2018 and 2019. This indicates the need to increase efforts to eradicate tuberculosis transmission in Central Java. Tubercu-losis can be transmitted through the air, so transmission can take place very quickly because of the many interactions between sufferers and susceptible people.

Several scientists have conducted research on the spread of the infectious disease tuberculosis by making mathematical models of its spread. Several studies have been conducted by Syafruddin Side (Side, 2015; Iskandar et al., 2017; Schipper et al., 2020; Das et al., 2021). The SIR Mathematical Model was first introduced by Kermack and McKendrick (1938).

Based on the things that have been described previously, we are interested in conducting research on the spread of TB disease in Central Java.

## SUBJECTS AND METHOD

### 1. Study Design

The factorial design method was carried out to determine the pattern of transmission of tuberculosis in Central Java for the 2018-2019 period.

### 2. Population and Sample

The population studied is the population of Central Java for the period 2018 and 2019 which includes 3 subpopulations, namely the suspected population, the infected population, and the recovered population.

### 3. Study Variables

The dependent variable is the number of suspected, infected, cured, and the basic reproduction rate. The independent variables are the transmission rate and the cure rate.

### 4. Operational Definition of Variables

**The suspicious population** is a group of people who have a risk of contracting tuberculosis.

**The infected population** is a group of tuberculosis sufferers.

**The recovered population** is the tuberculosis survivor group.

### 5. Data Analysis

The analysis used is to test the equilibrium point in the SIR model.


https://doi.org/10.26911/ICPHmanagement.FP.08.2021.20

## RESULTS

The illustration of the SIR model in Figure 1 can be explained as follows:

$$\frac{dS}{dt} = A - \beta SI - \mu S$$
$$\frac{dI}{dt} = \beta SI - \alpha I - \mu I$$
$$\frac{dR}{dt} = \alpha I - \mu R$$
$$A = \mu S = \mu I = \mu R = 0$$

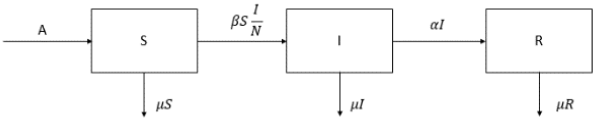


**Table 1. SIR Model Compartment Diagram**

**Table 1. The number of the population of Central Java infected and cured of tuberculosis**

| Year | Numbers Of Population | | |
|---|---|---|---|
| | Total Population | Infected | Recovered |
| 2018 | 34490835 | 49616 | 15325 |
| 2019 | 34718204 | 73171 | 17283 |

Table 1 showed the number of residents of Central Java, both infected and recovered in 2018 and 2019.

Based on these data, then we substituteinto the specific solution the sub-population of the number of people who are susceptible to tuberculosis, so that: $S(t) = S(0)e^{\left(\frac{\alpha}{\beta}\right)R(t)}$ .

Based on these data, then substituted into a special solution from the sub-population of the number of people who are susceptible to tuberculosis, namely S so that S (t) represents the number of people who are susceptible to tuberculosis and S (0) represents the initial population and R(t) states that the number of people infected with TB disease obtained the following results:

$$\frac{\beta}{\alpha} = 3.8 \times 10^{-7}$$

As we know, the cure and death rate from TB is 10 years so:

$$\alpha = \frac{1}{10} = 0.1.$$

The result, obtained a model of the spread of Tuberculosis:

$$\begin{cases} \frac{dS}{dt} = 34490835 - 3.8 \times 10^{-8} S(t)I(t) \\ \frac{dI}{dt} = 3.8 \times 10^{-8} S(t)I(t) - 0.1I(t) \end{cases} \quad (1)$$

Model analysis is needed to obtain research results on the spread of tuberculosis in Central Java using the SIR model.

### Mathematical Model Analysis

#### 1. Critical Point

To find the critical point, the system (1) is made at a constant position with respect to time, that is, a condition where $\frac{dS}{dt} = 0, \frac{dI}{dt} = 0,$ dan $\frac{dR}{dt} = 0.$


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## 2. Equilibrium point and stability

If we assume I in the second equation in equation (2), we can get the conclussion $7.6 \times 10^{-8}S - 0{,}1 = 0$, So $S = 2631580$.

Then, the equilibrium point of the System of Equation (2), (S,I)= (26315 80,344908350) is obtained which states that the number of individuals who are susceptible and infected with Tuberculosis are 2631580 people and 344908350 people, respectively.

For example, $\frac{dS}{dt} = f_1(S,I)$ and $\frac{dI}{dt} = f_2(S,I)$, Then a stability test will be carried out using the first derivative of each function on each variable in the jacobian matrix.

$$Jf(S,I) = \begin{bmatrix} -3.8 \times 10^{-8}I & -3.8 \times 10^{-8}S \\ 3.8 \times 10^{-8}I & 3.8 \times 10^{-8}S - 0.1 \end{bmatrix}$$

$$Jf(2631580,344908350) = \begin{bmatrix} -13.106 & -0.1 \\ 13.106 & 0 \end{bmatrix}$$

Based on $Jf(S,I)$, following the following characteristic equation:

$$\lambda^2 + 13.106 + 1.3106 = 0$$

Earned λ_1=-13.00523 and λ_2=-0.100775, we can see $\lambda_{1,2} < 0$, so obtainned local asymptotic equilibrium point (S, I)= (2631580,344908350) This showed that the number of susceptible individuals and the number of infected individuals is very small, so that with increasing time the population reaches a balance point (S,I) = (2631580,344908350) which means that Tuberculosis is still present in the community.

## 3. Basic Reproduction Number

$\beta A$ is the basic reproduction rate that is influenced by the birth or migration rate and the transmission rate of tuberculosis so that:

$$R_0 = \beta A = 3.8 \times 10^{-8} \times 34490835 = 1.31065$$

From the above formula it can be seen that one infected individual can, on average, infect 1 to 2 susceptible individuals in the TB disease population.

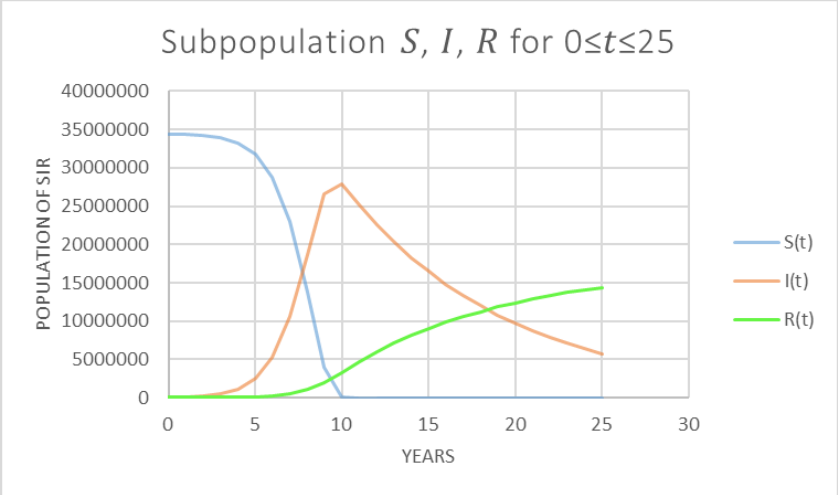


**Figure 2. The Spread of The Tuberculosis Virus in Central Java in the Next 25 Years Using the SIR Model**

Based on Figure 2, it can be seen that after 25 years it can be seen that the infected population will increase even more than the susceptible population, so that the disease becomes endemic.


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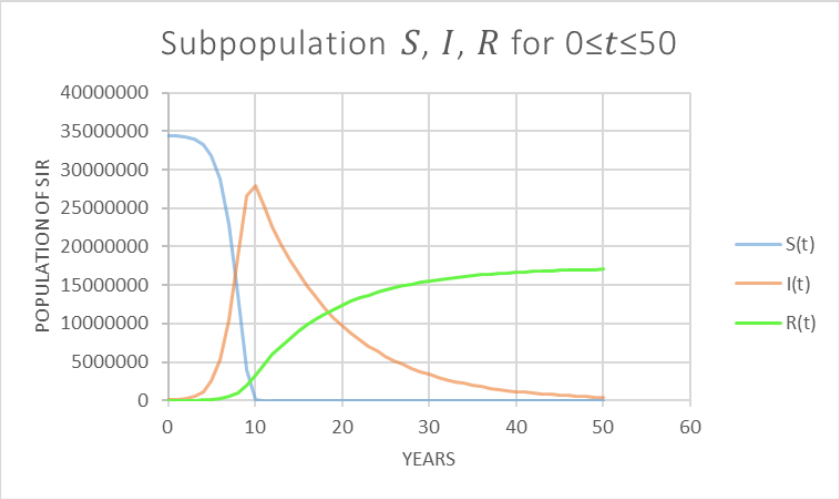


**Figure 2. The results of the AUC classification process for heart failure datasets using the naïve Bayes Algorithm**

Based on Figure 3, it can be seen that the number of people who are susceptible to tuberculosis will be smaller and closer to zero. Another interesting thing is the decrease in the number of people infected and the increase in the number of people who have recovered. This shows that there is still hope that tuberculosis will no longer be an endemic disease and even disappear in Central Java.

## DISCUSSION

In this study, the model used shows that the basic reproduction number denoted R_0 has a value of more than 1. It can also be seen that when the value of R_0>1, the model used shows that the local asymptotic endemic equilibrium point. So it can be concluded that tuberculosis will still be an endemic disease in Central Java Province.

This is in line with research that has been conducted in several cities in Central Java. In a study conducted by May Fadilla at a public health center (PusKesMas) in the city of Purbalingga, it was shown that it was still necessary to increase public awareness of the importance of a Clean and Healthy Lifestyle and improvement of the sanitation system in every home that has patients infected with tuberculosis. (Fadilla and Utomo 2018) This research can also be used as a rough description of several factors for the spread of tuberculosis in other cities in Central Java.

Another research is Baharo's research on the spread of tuberculosis in Kudus Regency which shows the same points as this study. In this study, the researchers found that using the SIR model, it was found that the endemic equilibrium point was stable and an indication that tuberculosis will still be an endemic disease in Kudus Regency (Bahari et al., 2022).

Based on the research that has been done, we found that Central Java has the potential to be free from tuberculosis in the future. Our research on the spread of tuberculosis in the Central Java area can still be developed.

There are several things that might be considered in further


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research, namely using more complex models such as the SEIR and SEIR models with vaccination which are also used by several researchers including Rahmat Syam (Syam et al., 2021; Side et al., 2017; Roudhotillah and Chandra, 2021).

In addition to the development of research models regarding the spread of tuberculosis, the development can be carried out with a larger scope of research, for example the island of Java or throughout Indonesia.

## AUTHOR CONTRIBUTION

Muhammad Faudzi Bahari as the main author is in charge of managing team work in general, dividing tasks, explainning research concepts, searching for data, performing calculations on case examples used, and compiling research into a journal to be published. Ade Ima Hafifa Hamiyati and M. Adib Jauhari Dwi Putra as the second and third authors helped explain to Putri Indriyani regarding the calculation mechanism using the SIR model face-to-face, in collaboration with LPPM UMKU regarding research publications. Putri Indriyani is tasked with finding data, performing calculations on the case examples used.

## CONFLICT OF INTEREST

There is no conflict of interest in this study.


## FUNDING AND SPONSORSHIP

This research was funded by The Institution of Research and Community Services Universitas Muhammadiyah Kudus.

## ACKNOWLEDGMENT

The authors grateful to Central Statistics Agency for providing the data needed to conduct this research. We also thank the Institute for Research and Community Service, Universitas Muhammadiyah Kudus, Central Java for facilitating this research.


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