

DIGITAL CORRELATION OF FIRST ORDER SPACE TIME IN A FLUCTUATING MEDIUM

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ABSTRACT

DIGITAL CORRELATION OF FIRST ORDER SPACE TIME IN A FLUCTUATING MEDIUM. The study of fluctuating medium has been of great interest through the use of the correlation techniques. A laser beam is known to form a coherent beam which can be made to propagate within the fluctuating medium. This will allow the study of the outgoing beam using digital correlation technique. Based on the power spectrum, the integral transformation of the correlation function, one can obtain for instance the radius and mass of the particles executing Brownian motion in the dispersed solution. To correlate the laser beam directly may not allow the detection of signals by electronic means. A method of digitizing the light signals by means of light beat heterodyne technique is therefore adopted. The temporal and spatial correlation functions can be measured.

Keywords: fluctuating medium, Brownian motion, laser

INTRODUCTION

Correlation techniques have played an important role in the study of fluctuating media. In particular laser light is a good probe due to its coherent properties. However the light frequencies range between 10^{13} and 10^{15} , make it impossible to make the correlation directly, because no such electronic devices could handle such high frequencies. With the rapid development in light beating techniques [3,4], this digital correlation method is available. Since the correlation of the scattered light carries significant information, about the nature of the fluctuating media, one can apply this method directly. An example is the measurement of the dynamic viscosity, radius and mass of molecules undergoing Brownian motions, etc.[5]. Correlation techniques can also be applied to study the properties of normal fluid, liquid near critical point, stable plasma, laminar flow etc. [6]. The studies may be too complicated if they are carried out by conventional methods.

THEORETICAL ANALYSIS

The correlation function of electric field $E_1(R_1, t), E_2(R_2, t)$ at different positions R_1, R_2 undergoing fluctuations can be written as [7]

$$\Gamma_{12}(R_1, R_2, \tau) = \langle E_1(R_1, t) E_2^*(R_2, t + \tau) \rangle \quad (1)$$

The brackets indicate the time average and τ is the time delay at R_2 taken to be later than R_1 . The above expression is called the ‘mutual correlation’ or mutual coherent function of the light signals from R_1 and R_2 . This is a more general form of the first order correlation function, for the special case when $R_1 = R_2$ the function reduces to self correlation function. Written as

$$\Gamma_{11}(\tau) = \langle E_1(R_1, t) E_1^*(R_1, t + \tau) \rangle \quad (2)$$

If τ is to be zero, then $\Gamma_{11}(0)$ is the intensity of light at R_1 . Most of the studies are based on information of $\Gamma_{11}(\tau)$ or $\Gamma_{11}(0)$. Little has been made on the studies based on $\Gamma_{12}(\tau)$, since it involves the problem of measuring such quantity, and then the problem of interpretation of the data obtained. The purpose of this study is to show how $\Gamma_{12}(\tau)$ can be measured digitally, so that the corresponding power spectrum that contained physical information can be obtained.

Using a beam splitter, a helium neon laser beam with frequency ω_0 can be split into two beams. The first beam is modulated by an ultrasonic modulator of frequency Ω_1 so that the frequency becomes $\omega_1 = \omega_0 + \Omega_1$. The beam is then expanded by a beam expander (BE1) into 12.5 mm in diameter, and is made to pass through a quarter wave plate (QP1) as a reference beam. The axis of the plate is positioned in such a way it makes an angle β^0 to the direction of the linearly polarized beam. The axis of the plate is chosen to be the direction x-axis as reference of analysis, while y-axis is perpendicular to this axis, anti clock wisely. In the case $\beta^0 = 45^\circ$, the electric field component of the outgoing beam will be circularly polarized otherwise it is elliptical. In general therefore the electric field of this reference beam taken to be zero (on the laser beam axis, $R_1 = 0$) can be written as

$$\vec{E} = \hat{x}E_x \exp[i(\omega_1 t + \delta)] + \hat{y}E_y \exp[i(\omega_1 t + \pi/2 + \delta)] \quad (3)$$

\hat{x} and \hat{y} being the normal vectors along the x and y axes respectively, while δ is a constant phase factor.

The second split beam is also modulated by an ultrasonic modulator of frequency Ω_2 so that the electric field of the beam after expander and passed through a quarter wave plate (QP2) can be written as

$$\vec{F} = \hat{x}F_x \exp[i(\omega_2 t + \eta)] + \hat{y}F_y \exp[i(\omega_2 t + \pi/2 + \eta)] \quad (4)$$

Before the beam passes a fluctuating medium. Here again η is a phase factor, $\omega_2 = \omega_0 + \Omega_2$. After the beam passes through a fluctuating medium the field experiences fluctuations, let it be indicated by \vec{G} , satisfying

$$\vec{G} = \tilde{A}(\xi, t) \vec{F} \quad (5)$$

where $\tilde{A}(\xi, t)$ is a complex function satisfying $\tilde{A}(\xi, t) = A \exp(i\phi)$ and where $A = |\tilde{A}(\xi, t)|$ $\phi = \arg(A)$. The variable ξ measures the distance of a particular measurement position from the axis of the laser beam. We could correlate the reference beam, and the fluctuating beam either \hat{x} or \hat{y} components and can be shown to satisfy the following

$$\begin{aligned} I_x &= |E_x'|^2 + |AF_x'|^2 + 2E_x' F_x' A(0, t) \cos[(\omega_1 - \omega_2)t + \phi(0, t) + \rho] \\ J_x &= |E_x''|^2 + |AF_x''|^2 + 2E_x'' F_x'' A(\xi, t) \cos[(\omega_1 - \omega_2)t + \phi(\xi, t) + \sigma] \\ J_y &= |E_y''|^2 + |AF_y''|^2 + 2E_y'' F_y'' A(\xi, t) \sin[(\omega_1 - \omega_2)t + \phi(\xi, t) + \sigma] \end{aligned}$$

ρ and σ are some phase constants, which if there is no polarization changes, their difference is always zero for $r=0$ [7]. The components along \hat{x} and \hat{y} are made by adjusting the polarizer in the experimental set up. If the correlation is for the same component (e.g. \hat{x} and \hat{x}) it is called the auto correlation, otherwise it is called cross correlation. (\hat{x} and \hat{y}).

We have the auto correlation function

$$\begin{aligned} C_x(\xi, \tau) &= \langle I_x(0, t) J_x(\xi, t + \tau) \rangle = \\ & K_x \langle A(0, t) A(\xi, t + \tau) \cos[(\omega_1 - \omega_2)\tau + \phi(\xi, t + \zeta) + \rho - \sigma] \rangle \end{aligned} \quad (7)$$

and the cross correlation function

$$\begin{aligned} C_y(\xi, \tau) &= \langle I_x(0, t) J_y(\xi, t + \tau) \rangle = \\ & K_y \langle A(0, t) A(\xi, t + \tau) \sin[(\omega_1 - \omega_2)\tau + \phi(\xi, t + \zeta) + \rho - \sigma] \rangle \end{aligned} \quad (8)$$

$\rho - \sigma$ should be made zero or π by adjusting the axes of the polarizer or/and the quarter wave plates. K_x, K_y are constants namely $K_x = 2E_x' E_x'' F_x' F_x''$ and $K_y = 2E_x' E_y'' F_x' F_y''$. In practice these constants can be determined (in arbitrary unit) by measuring C_x and C_y in the absence of

fluctuating medium. In this case $C'_x = C_x / K_x$ and $C'_y = C_y / K_y$ are the required normalized auto and cross correlation functions.

The following rearrangement can be made

$$C'_x + iC'_y = \langle \tilde{A}(0, t) \tilde{A}^*(\xi, t + \tau) \rangle \exp[i(\omega_1 - \omega_2)\tau] \quad (9)$$

Where the last term on the right hand side has been taken out from the time average, since it is not function of t . The term in the bracket is the normalized first order space-time mutual correlation functions (SPMCF) given by

$$\tilde{\gamma}(\xi, \tau) = \langle \tilde{A}(0, t) \tilde{A}^*(\xi, t + \tau) \rangle = \gamma(\xi, \tau) \exp[i\theta(\xi, \tau)] \quad (10)$$

with $\gamma(\xi, \tau) = \sqrt{C_x'^2 + C_y'^2}$

and $\theta(\xi, \tau) = \tan^{-1}(C'_y / C'_x) - (\omega_1 - \omega_2)\tau$

Physical information contained in the power spectrum that can be derived by Fourier transforming the SPMCF

$$\tilde{\gamma}(\xi, \omega) = \int_{-\infty}^{\infty} \tilde{\gamma}(\xi, \tau) \exp[-i(\omega\tau)] d\tau \quad (11)$$

This $\tilde{\gamma}(\xi, \tau)$ represents $\Gamma_{12}(\tau)$, where the sub-indices have been represented by ξ .

EXPERIMENTAL SET UP

The experimental set up is based on then optical heterodyne technique where the beat signals are separated into two components, the \hat{x} and \hat{y} components, called the phase quadrature technique [8]. A Mach-Zender heterodyne interferometer is set up completed with the electronic measuring system. In this experiment the fluctuating medium is artificially made by blowing thermal air across the beam path. The initial beam coming from Helium-Neon laser is chosen such that the direction of electric field polarization is along a reference axis. The signals detected by two detectors D1 and D2 (see figure -1) are filtered to suppress unwanted frequencies. The filters allow to pass the frequency carrier ranging from 10 to 70 kHz. In this experiment, is chosen to be 54.935 kHz. In selecting the sampling period, the correlation can be observed by modulating the signals in the range of 100 to 200 Hz. The correlation functions observed are shown in fig 2, while in figure 3 we have the first order space time correlation functions. It is not clear yet what the physical interpretation of these functions.

RESULTS AN DISCUSSIONS

Measurements have been made for $\xi = 1, 2, 3, 4, mm$. This limitation is due to the size of the detector aperture having the diameter 1mm, and the beam expansion which has 12.5 mm in diameter. The Helium-Neon laser has the maximum output of 2mW. The beam intensity decreases with respect to ξ . should follow a Gaussian profile $I(\xi) = I_0 \exp(-\mu\xi^2)$, where I_0, μ are constants. There seems to be an anomaly at $\xi = 1mm$ by showing a stronger time and spatial correlation function decay compared to other different location. The reason is not understood yet. The thermal fluctuating medium is made by blowing thermal air across the beam path. The temperature of the medium is around $65^{\circ}C$. There seems to be a significant different in the function obtained, if the temperature is reduced to room temperature. The decay pattern of $\gamma(\xi, \tau)$ is thus dictated by two variables at least namely the temperature and the wind velocity of the fluctuating air turbulence. Figure 3 shows the experimental measurements of $\gamma(\xi, \tau)$ as function of ξ and τ . In the case $\tau = 0$ the function $\gamma(\xi, 0)$ is called mutual correlation function (MCF). This laboratory simulation experiment can be extended to the measurement of atmospheric air turbulence, in which the decay pattern of MCF is due to the aerosol and molecular turbulences. The function obtained does not follow the pattern $M(\xi) \propto \exp(-\beta\xi^{5/3})$ as shown in reference [9], because they have different condition too.

CONCLUSIONS

Using digital correlation techniques, the space-time correlation function can be measured successfully, its modulus and phase. The technique can be applied for other studies such as in the measurement of micro-emulsions, an important studies in micro-biology, pharmaceutical, and others. In particular, the measurement of this simulated atmospheric turbulence can be extended to the real air atmospheric turbulence in an effort to understand the nature of atmospheric behavior.

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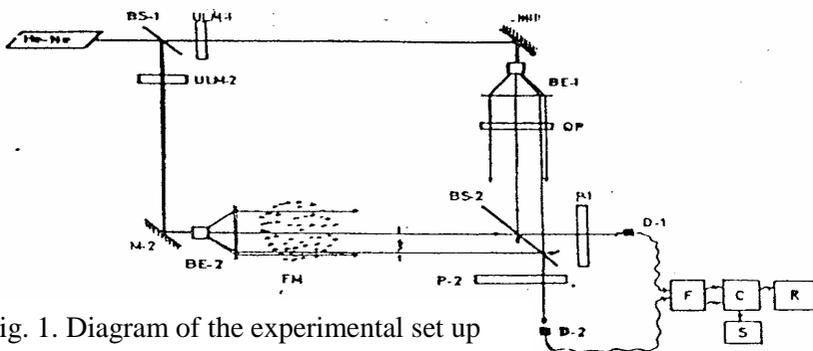


Fig. 1. Diagram of the experimental set up

- BS – beam splitter
- M – mirror
- QP – quarter wave plate
- FM – fluctuating medium
- D – detector, F – electronic filter
- C – correlator, R – recorder, S – oscilloscope
- ULM – ultrasonic modulator, BE – beam expander
- P – polarizer

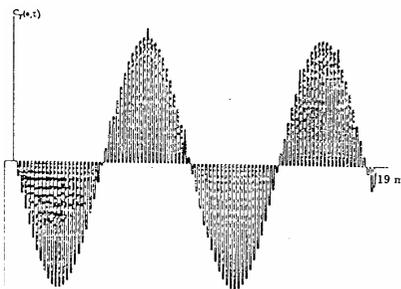


Fig. 2a. Auto correlation with no turbulence Vertical axis is $C_x^i(0, \tau)$ and horizontal axis is τ maximum 19 msec.

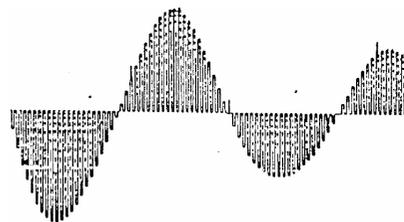


Fig. 2b. Auto correlation with turbulence.

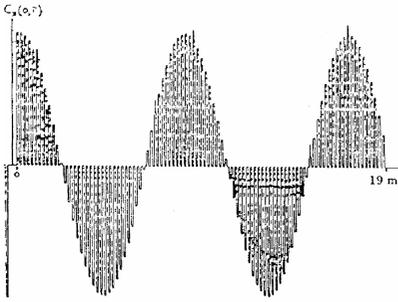


Fig. 2c. Cross correlation with turbulence. Vertical axis is $C_y(0, \tau)$ and horizontal axis is τ maximum 19 msec.

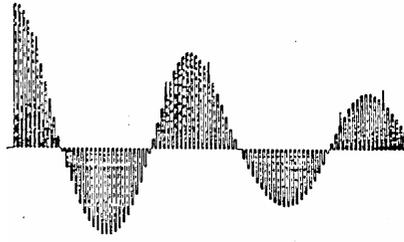


Fig. 2d. Cross correlation with turbulence.

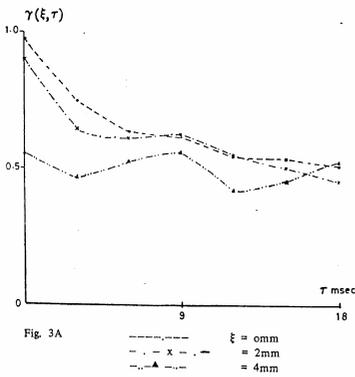


Fig. 3a. First Order Space time Correlation.

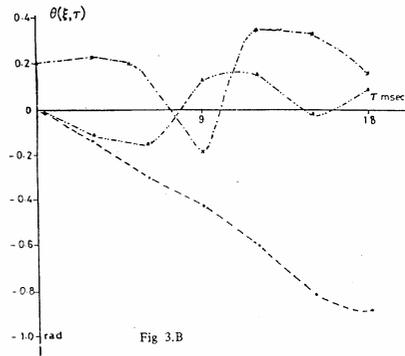


Fig. 3b. Phases of First Order Space-time Correlation.