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DEVELOPMENT OF THE STRENGTH STATISTICAL CHARACTERISTICS OF MATERIALS, WHICH TAKES INTO ACCOUNT THE FEATURES OF THEIR BRITTLE FRACTURE

The object of the research is the algorithm for determining the finding of the most probable, mean value, dispersion and coefficient of failure loading variation of a stochastically defective plate under conditions of comprehensive tensile-compression. The material of the plate is considered as a continuous medium in which evenly distributed defects such as rectilinear cracks that do not interact with each other. It is isotropic and has the same crack resistance. Let's believe that the plate consists of primary elements, each of which can be weakened by one defect.

To predict the strength and failure conditions of plates made of such material, it is natural to use, on the one hand, the results of the theory of limit equilibrium of individual determined defects and their development, and on the other hand, probabilistic-statistical methods that take into account the randomness of defects. This comprehensive approach makes it possible to calculate the statistical characteristics of strength and fracture based on data on the structure of the material defect and its resistance to the emergence and development of cracks.

The main content of this paper is the algorithm for calculating and research the strength statistical characteristics of stochastically defective plate structural elements taking into account some deterministic features of their brittle fracture. Based on the deterministic failure criterion, which takes into account the initial direction of crack propagation, the ratio is obtained to find the most probable, mean value, dispersion and coefficient of variation of failure loading. The dependences of the specified strength statistical characteristics on the type of applied loading, the number of defects (body size) and structural inhomogeneity of the material, as well as the effect of taking into account the initial direction of crack propagation are investigated.

The obtained results allow to more adequately assessing the reliability of structural materials under conditions of complex stress state, taking into account the stochastic of their structure. This is due to the fact that the use of the approach to determine the limit applied stresses, which takes into account the initial direction of the crack propagation, improves the algorithm for finding strength statistical characteristics.

Keywords: *plate, isotropic material, rectilinear crack, distribution density, failure loading, strength statistical characteristics.*

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1. Introduction

Most theoretical studies of fracture mechanics are based on the theory of mechanics of continuous media, which refers to homogeneous materials and does not take into account the inhomogeneous microstructure of a solid body. In real material, however, there are many defects of different sizes, shapes and orientations. Taking into account this chaotic internal structure requires the application of probabilistic-statistical approach, which allows studying the phenomenon of real materials failure and effectively modeling it. The integrated application of deterministic solutions of brittle fracture mechanics and methods of probability theory and mathematical statistics allows making an adequate assessment of their reliability. The problem of strength reliability of structural materials in the probabilistic aspect, in particular with the use of experimental data, has been studied in the works of a number of authors.

In the paper [1], a two-dimensional finite element simulation-based approach was developed to assess the pore-pore interactions and their impact on fracture statistics of isotropic microstructures. The statistical properties of fracture strength of brittle materials described in terms of the Weibull distribution are researched [2]. In the article [3], the probabilistic fracture mechanical computing codes are compared. The probability distribution of fishnet strength is calculated as a sum of a rapidly convergent series of the failure probabilities after the rupture of one, two, three, etc., links [4]. A modified Weibull failure probability model that considers the impact of compressive stress on cladding failure probability is deduced, and then it is used to calculate the failure probability of different cladding designs [5]. The failure modes, the probabilistic model of multiple surface cracking are studied in [6]. In the work [7], a data-driven approach based on a Gaussian process for regression is developed to determine the probability of axle failure caused by crack growth in railway axles. In the paper [8], a reliability analysis of fatigue crack growth for a pearlitic steel subject to the growth of multiple cracks is presented.

Therefore, a model of the material designed to describe the strength and fracture of bodies, which takes into account its defects in the probabilistic aspect, is relevant.

Thus, *the object of research* is the algorithm for determining the finding of the most probable, mean value, dispersion and coefficient of failure loading variation of a stochastically defective plate under conditions of comprehensive tensile-compression. *The subject of research* is selected brittle model materials under complex stress conditions using the deterministic criterion of fracture, which takes into account the initial direction of crack propagation. Analysis of the strength statistical characteristics of this model dependence on the type of applied loading, the number defects (body size) and the material structural inhomogeneity is carried out.

The aim of research is constructing an algorithm for finding strength statistical characteristics of stochastically defective plates, taking into account the peculiarities of the deterministic fracture criterion of plate weakened by a crack-type defect under conditions of complex stress state.

2. Research methodology

Consider an algorithm for determining the strength statistical characteristics in the case of a plane stress state of lamellar bodies weakened by stochastically distributed rectilinear defects-cracks (let's assume that their number is equal to a certain number N), which penetrate the normal thickness. The model material is isotropic and has the same crack resistance. The plate is under the action of a uniform loading P and $Q<\eta P$ (biaxial tension, compression or tension-compression of two mutually perpendicular directions) (Fig. 1). Loading P and Q can be considered as the main stresses in the plane stress state. The number of defects N is proportional to the plate area S: $N<N_0S$, where N_0 is the number of defects per unit area. Let's assume that the possible size of defects in the material is limited.

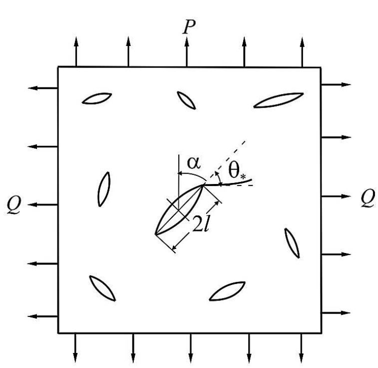

Fig. 1. Stochastically defective plate model

Cracks are evenly distributed, do not interact with each other and are characterized by two statistically independent parameters. The first random crack parameter is the angle of inclination α relative to the direction of force P action $(-\pi/2\le\alpha\le\pi/2)$, and the second is the length $2l$ $(0\le 2l\le d$, d is the finite structural characteristic). Since the plate material is isotropic, the random variable α can be set by the density of uniform probability distribution: $f(\alpha)=1/\pi$ [9]. The probability distribution density of a random variable l is chosen in the form of a generalized β-distribution [9]:

$$f(l)=\frac{r+1}{d}\left(1-\frac{l}{d}\right)^r,$$

where $r\ge 0$ is the structural parameter of the material (with its increasing are more probable small cracks).

Consider a plate as a set of N primary defective elements (with one crack). The failure loading for it coincides with the failure loading for its weakest element (Weibull weakest link hypothesis).

For a plate with a rectilinear crack under conditions of tensile-compression, let's choose a deterministic criterion of failure, which takes into account the initial direction of its propagation [10]:

$$|P_*|=\frac{K}{\sqrt{l}}\Phi_i(\eta,\alpha,\rho,\theta_*),\ K=\frac{K_{Ic}}{\sqrt{\pi}}\ (i=1,2), \tag{1}$$

where P_*, $Q_*=\eta P_*$ is the failure loading; K_{Ic} is the stress intensity factor (constant, which characterizes the resistance of the material to crack propagation); ρ is the crack edge friction coefficient $(0\le\rho\le 1)$, the angle $\theta=\theta_*$ (Fig. 1) determines the initial direction of crack propagation. It is found by the following formula:

$$\theta_*=2\operatorname{arctg}\frac{1-\sqrt{1+8b^2}}{4b}, \tag{2}$$

where $b=\dfrac{(1-\eta)\sin 2|\alpha|}{2(\sin^2\alpha+\eta\cos^2\alpha)}$.

Depending on the type of crack, the function $\Phi_i(\eta,\alpha,\rho,\theta_*)$ has the following analytical representation:

a) for open cracks $(\sigma_n\ge 0)$:

$$\Phi_1(\eta,\alpha,0,\theta_*)=\sec^2\frac{\theta_*}{2}\begin{pmatrix}\cos\frac{\theta_*}{2}(\sin^2\alpha+\eta\cos^2\alpha)-\\ -\frac{3}{2}(1-\eta)\sin\frac{\theta_*}{2}\sin 2|\alpha|\end{pmatrix}^{-1}, \tag{3}$$

where σ_n are the normal stress to the crack line $(\sigma_n=$ $=P\sin^2\alpha+Q\cos^2\alpha)$;

b) for closed cracks $(\sigma_n<0)$:

$$\Phi_2(\eta,\alpha,\rho,0)=\frac{\sqrt{3}}{4}\begin{pmatrix}(1-\eta)\sin 2|\alpha|+\\ +2\rho\, sign\, P(\sin^2\alpha+\eta\cos^2\alpha)\end{pmatrix}^{-1}. \tag{4}$$

3. Research results and discussion

3.1. Distribution function of failure loading. In article [9], based on the analytical representation of the deterministic failure criterion (1)–(4), let's obtain the expressions of the distribution function of failure loading for a plate element with one crack, taking into account the initial direction of its propagation. Let's write these expressions in the form convenient for finding strength statistical characteristics for such cases of the applied loading:

– for biaxial tension $0\le\eta\le 1$ $(P>0)$:

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_1}^{\pi/2}\left(1-\frac{K^2}{P^2d}\Phi_1^2(\eta,\alpha,0,\theta_*)\right)^{r+1}d\alpha,$$

$$\frac{K}{\sqrt{d}}\le P\le\frac{K}{\eta\sqrt{d}}\ (\eta\ne 1), \tag{5}$$

$$F_1(P,\eta)=\frac{2}{\pi}\int_{0}^{\pi/2}\left(1-\frac{K^2}{P^2d}\Phi_1^2(\eta,\alpha,0,\theta_*)\right)^{r+1}d\alpha,$$

$$\frac{K}{\eta\sqrt{d}}\le P<\infty\ (\eta\ne 0); \tag{6}$$

– for tension-compression $-1\le\eta<0$ $(P>0,\ Q<0)$:

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_3}^{\pi/3}\left(1-\frac{K^2}{P^2d}\Phi_2^2(\eta,\alpha,\rho,0)\right)^{r+1}d\alpha,$$

$$\frac{\sqrt{3}K}{8\sqrt{|\eta|d}}\le P<\frac{\varphi_1(\eta)K}{\sqrt{d}}\ (\eta\ne -1), \tag{7}$$

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_3}^{\alpha_0}\left(1-\frac{K^2}{P^2d}\Phi_2^2(\eta,\alpha,\rho,0)\right)^{r+1}d\alpha+$$

$$+\frac{2}{\pi}\int_{\alpha_0}^{\pi/3}\left(1-\frac{K^2}{P^2d}\Phi_1^2(\eta,\alpha,0,\theta_*)\right)^{r+1}d\alpha,\ \frac{\varphi_1(\eta)K}{\sqrt{d}}\le P<\infty; \tag{8}$$

– for predominant compression-tension $(-\infty<\eta<-1)$:

1) when $-\left(\rho+\sqrt{1+\rho^2}\right)^2\le\eta<-1$:

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_3}^{\pi/3}\left(1-\frac{K^2}{P^2d}\Phi_2^2(\eta,\alpha,\rho,0)\right)^{r+1}d\alpha,$$

$$\frac{\sqrt{3}K}{8\sqrt{|\eta|d}}\le P\le\frac{\varphi_2(\eta)K}{\sqrt{d}}, \tag{9}$$

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_3}^{\alpha_0}\left(1-\frac{K^2}{P^2d}\Phi_2^2(\eta,\alpha,\rho,0)\right)^{r+1}d\alpha+$$

$$+\frac{2}{\pi}\int_{\alpha_0}^{\alpha_1}\left(1-\frac{K^2}{P^2d}\Phi_1^2(\eta,\alpha,0,\theta_*)\right)^{r+1}d\alpha,\ \frac{\varphi_2(\eta)K}{\sqrt{d}}\le P<\infty; \tag{10}$$

2) when $-\infty<\eta\le-\left(\rho+\sqrt{1+\rho^2}\right)^2$:

$$F_1(P,\eta)=\frac{2}{\pi}\int_{\alpha_3}^{\alpha_4}\left(1-\frac{K^2}{P^2d}\Phi_2^2(\eta,\alpha,\rho,0)\right)^{r+1}d\alpha,$$

$$\frac{\sqrt{3}K}{4\sqrt{d}\left((1-\eta)\sqrt{1+\rho^2}+\rho(1+\eta)\right)}\le P\le\frac{\sqrt{3}K}{8\sqrt{|\eta|d}}. \tag{11}$$

Here let's introduce the notation $\alpha_0=\mathrm{arctg}\sqrt{-\eta},\alpha_1\in(0,\pi/2)$ is the solution of equation:

$$\frac{K^2\Phi_1^2(\eta,\alpha,0,\theta_*)}{P^2}=d,$$

$$\alpha_2=\mathrm{arctg}\frac{\eta-1+\sqrt{(1-\eta)^2-4\eta\rho^2}}{2\rho},$$

$$\alpha_3=$$

$$=\mathrm{arctg}\frac{(1-\eta)P\sqrt{d}-\sqrt{(1-\eta)^2P^2d-4\left(\sqrt{3}K/8-\rho P\sqrt{d}\right)\left(\sqrt{3}K/8-\eta\rho P\sqrt{d}\right)}}{2(\sqrt{3}K/8-\rho P\sqrt{d})},$$

$$\alpha_4=$$

$$=\mathrm{arctg}\frac{(1-\eta)P\sqrt{d}+\sqrt{(1-\eta)^2P^2d-4\left(\sqrt{3}K/8-\rho P\sqrt{d}\right)\left(\sqrt{3}K/8-\eta\rho P\sqrt{d}\right)}}{2(\sqrt{3}K/8-\rho P\sqrt{d})},$$

$$\varphi_1(\eta)=\Phi_1\left(\frac{\pi}{3},\eta\right),\ \varphi_2(\eta)=\Phi_1(\alpha_0,\eta).$$

In the case $\frac{\sqrt{3}K}{8\sqrt{|\eta|d}}\le P\le\frac{\varphi_2(\eta)K}{\sqrt{d}}$ the distribution function $F_1(P,\ \eta)$ has the form (9).

For $\frac{\varphi_2(\eta)K}{\sqrt{d}}\le P<\infty$ the distribution function $F_1(p,\eta)$ is determined by formula (10).

If $1<\eta<\infty$ $(P>0)$ or $-\infty<\eta<0$ $(Q>0)$, then by substituting P for Q and η for $\eta_1=1/\eta$ in the corresponding expressions for $F_1(P,\ \eta)$, let's obtain the distribution function $F_1(Q,\ \eta_1)$.

3.2. Distribution density probabilities of failure loading. Distribution density probabilities of failure loading for a plate with a stochastic distribution of N defects is determined by the following ratio [11]:

$$f_N(P,\eta)=N(1-F_1(P,\eta))^{N-1}\frac{dF_1(P,\eta)}{dP}. \tag{12}$$

The most probable value of failure loading (mode), which corresponds to the loading level, in which the distribution density probabilities $f_N(P,\ \eta)$ reaches a maximum, is determined from the equation [11]:

$$\frac{d^2\left(1-\left(1-F_1(P,\eta)\right)^N\right)}{dP^2}=0. \tag{13}$$

Equation (13) can also be written as:

$$(1-N)(F_1'(P,\eta))^2+\left(1-F_1(P,\eta)\right)F_1''(P,\eta)=0. \tag{14}$$

Substituting in formula (12) the analytical representations of the distribution function (5)–(11), let's obtain formulas that determine the distribution density probabilities of failure loading for a plate with randomly distributed defects-cracks for different cases of stress state.

Consider partial cases: uniaxial tension $(\eta<0,\ P>0,\ Q<0)$, biaxial symmetric tension $(\eta<1,\ P<Q>0)$ and biaxial symmetric tension-compression $(\eta=-1,\ P>0,\ Q=-P)$.

For uniaxial tension, the distribution density probabilities of failure loading are defined as follows:

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$$f_N(P,0)=\frac{4NK^2(r+1)}{\pi P^3 d}(1-F_1(P,0))^{N-1}\times$$
$$\times\int_{\alpha_1}^{\pi/2}\left(1-\frac{K^2}{P^2 d}\Phi_1^2(0,\alpha,0,\theta_*)\right)^r \Phi_1^2(0,\alpha,0,\theta_*)\,\mathrm{d}\alpha,\ \frac{K}{\sqrt{d}}\le P<\infty. \quad (15)$$

For biaxial symmetric loading there is:

$$f_N(P,1)=\frac{2NK^2(r+1)}{P^3 d}(1-F_1(P,1))^{N-1}\left(1-\frac{K^2}{P^2 d}\right)^r,$$
$$\frac{K}{\sqrt{d}}\le P<\infty. \quad (16)$$

For biaxial symmetric tension-compression let's obtain:

$$f_N(P,-1)=\frac{4NK^2(r+1)}{\pi P^3 d}(1-F_1(P,-1))^{N-1}\times$$
$$\times\left(\begin{array}{l}\int_{\alpha_3}^{\alpha_0}\left(1-\frac{K^2}{P^2 d}\Phi_2^2(-1,\alpha,\rho,0)\right)^r \Phi_2^2(-1,\alpha,\rho,0)\,\mathrm{d}\alpha+\\ +\int_{\alpha_0}^{\pi/3}\left(1-\frac{K^2}{P^2 d}\Phi_1^2(-1,\alpha,0,\theta_*)\right)^r \Phi_1^2(-1,\alpha,0,\theta_*)\,\mathrm{d}\alpha\end{array}\right),$$
$$\frac{\varphi_1(\eta)K}{\sqrt{d}}\le P<\infty. \quad (17)$$

According to expressions (15)–(17) in Fig. 2–4 the distribution density probabilities of failure loading $f_N(P,\eta)$ graphs for a plate with stochastic N crack distribution under different types of stress state are constructed.

In Fig. 2, 3, solid lines correspond to the case of taking into account the initial direction of cracks propagation, and dashed lines – without taking it into account $(\theta_*=0)$.

In Fig. 4, the distribution density probabilities of failure loading curves are constructed taking into account the initial direction of crack propagation for materials of various inhomogeneity. Solid lines correspond to the case of uniaxial tension, dashed lines to the case of biaxial symmetric tension, dash dotted lines to the case of biaxial symmetric tension-compression.

The distributions of the failure loading random variable will be unimodal. The threshold value of strength is not equal to zero and depends on the type of loading.

3.3. Strength statistical characteristics. Let's find and investigate some strength statistical characteristics of plates with stochastic distribution of defects. The mean value of the failure loading is found by the formula [10]:

$$\langle p\rangle = p_{\min}(\eta)+\int_{p_{\min}(\eta)}^{p_{\max}(\eta)}\left(1-F_1(p,\eta)\right)^N \mathrm{d}p. \quad (18)$$

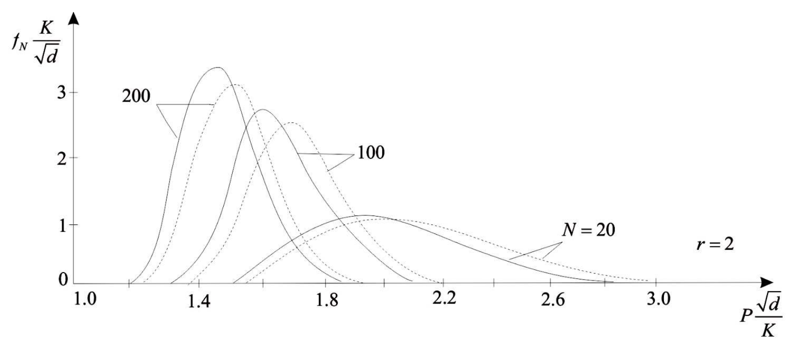

Fig. 2. The distribution density probabilities of failure loading in the case of uniaxial tension

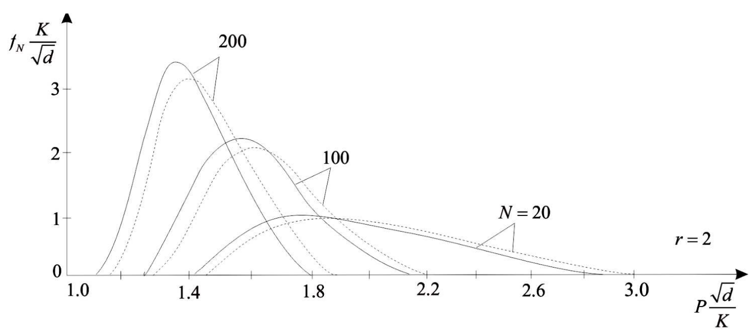

Fig. 3. The distribution density probabilities of failure loading in the case of biaxial symmetric tension-compression

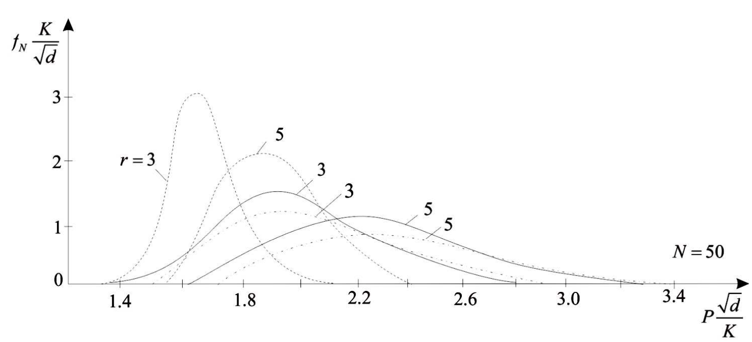

Fig. 4. The distribution density probabilities of failure loading for materials of various inhomogeneity

The dispersion and coefficient of variation of failure loading are determined by the following relations [10]:

$$D(p)=p_{\min}^2(\eta)+2\int_{p_{\min}(\eta)}^{p_{\max}(\eta)}\left(1-F_1(p,\eta)\right)^N p\mathrm{d}p-\langle p\rangle^2, \quad (19)$$

$$W(p)=\frac{\sqrt{D(p)}}{\langle p\rangle}. \quad (20)$$

Substituting in formulas (18)–(20) analytical representations of the distribution function (5)–(11), let's obtain relations that determine specified strength statistical characteristics for plates with randomly distributed defects-cracks in different cases of stress state.

Let's consider the above partial cases. Let's make a change of variables:

$$x=\frac{K^2}{P^2 d}.$$

Let's obtain the ratio to determine the mean value and dispersion of the failure loading.

For uniaxial tension:

$$\langle P\rangle\frac{\sqrt{d}}{K}=$$

$$=1+\frac{1}{2}\int_0^1\left(1-\frac{2}{\pi}\int_{\alpha_1}^{\pi/2}\left(1-x\Phi_1^2(0,\alpha,0,\theta_*)\right)^{r+1}\mathrm{d}\alpha\right)^N\frac{\mathrm{d}x}{\sqrt{x^3}}, \quad (21)$$

$$D(P)\frac{d}{K^2}=$$

$$=1+\int_0^1\left(1-\frac{2}{\pi}\int_{\alpha_1}^{\pi/2}\left(1-x\Phi_1^2(0,\alpha,0,\theta_*)\right)^{r+1}\mathrm{d}\alpha\right)^N\frac{\mathrm{d}x}{x^2}-\langle P\rangle^2\frac{d}{K^2}. \quad (22)$$

For biaxial symmetrical tension there is:

$$\langle P\rangle\frac{\sqrt{d}}{K}=1+\frac{1}{2}\int_0^1\left(1-(1-x)^{r+1}\right)^N\frac{\mathrm{d}x}{\sqrt{x^3}}, \quad (23)$$

$$D(P)\frac{d}{K^2}=1+\int_0^1\left(1-(1-x)^{r+1}\right)^N\frac{\mathrm{d}x}{x^2}-\langle P\rangle^2\frac{d}{K^2}. \quad (24)$$

For biaxial symmetric tension-compression let's obtain:

$$\langle P\rangle\frac{\sqrt{d}}{K}=\frac{\sqrt{3}}{8}+$$

$$+\frac{1}{2}\int_0^{\phi_1^{-2}(\eta)}\left(\begin{array}{l}1-\frac{2}{\pi}\int_{\alpha_3}^{\alpha_0}\left(1-x\Phi_2^2(-1,\alpha,\rho,0)\right)^{r+1}\mathrm{d}\alpha-\\-\frac{2}{\pi}\int_{\alpha_0}^{\pi/3}\left(1-x\Phi_1^2(-1,\alpha,0,\theta_*)\right)^{r+1}\mathrm{d}\alpha\end{array}\right)^N\frac{\mathrm{d}x}{\sqrt{x^3}}, \quad (25)$$

$$D(P)\frac{d}{K^2}=\frac{3}{64}+$$

$$+\int_0^{\phi_1^{-2}(\eta)}\left(\begin{array}{l}1-\frac{2}{\pi}\int_{\alpha_3}^{\alpha_0}\left(1-x\Phi_2^2(-1,\alpha,\rho,0)\right)^{r+1}\mathrm{d}\alpha-\\-\frac{2}{\pi}\int_{\alpha_0}^{\pi/3}\left(1-x\Phi_1^2(-1,\alpha,0,\theta_*)\right)^{r+1}\mathrm{d}\alpha\end{array}\right)^N\frac{\mathrm{d}x}{x^2}-\langle P\rangle^2\frac{d}{K^2}. \quad (26)$$

According to expressions (20)–(26) in Fig. 5–7 graphs of statistical characteristics of the failure loading under different types of stress state for materials with different number of defects and different inhomogeneity. Solid lines correspond to the case of taking into account the initial direction of crack propagation, and dashed lines – without taking it into account $(\theta_*=0)$.

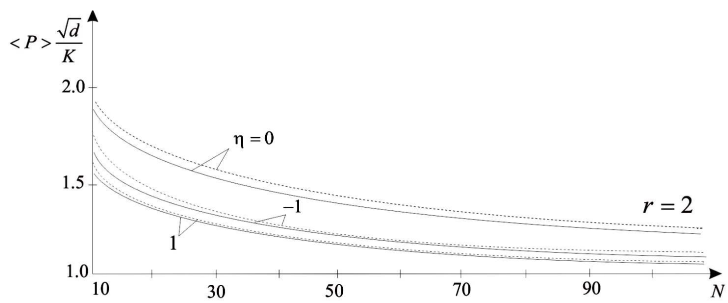

Fig. 5. The failure loading mean value

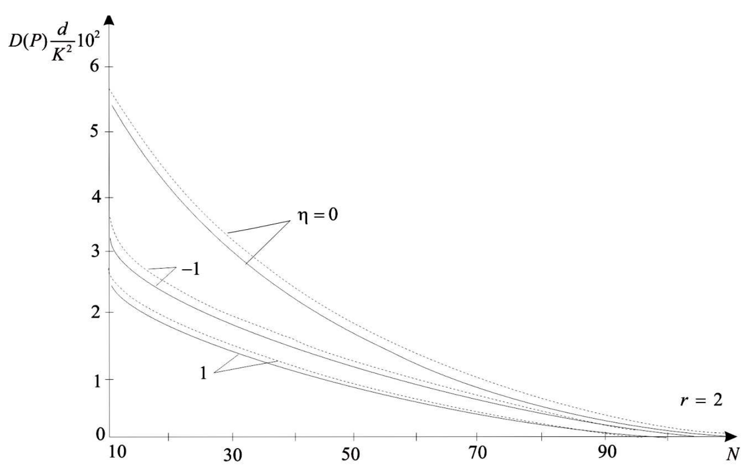

Fig. 6. Dispersion of failure loading

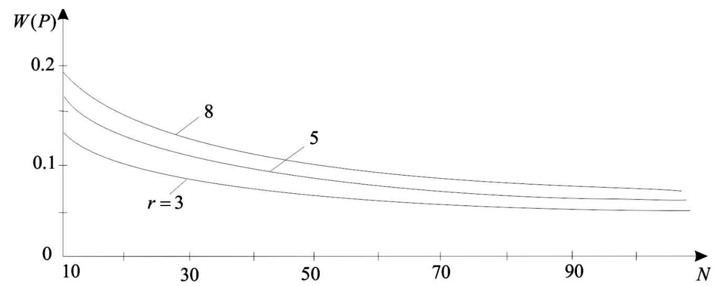

Fig. 7. Coefficient of failure loading variation

In Fig. 2–4, let's consider the influence of the applied loading ratio (parameter η), number of cracks N in the plate (plate area with the same number of cracks), angle θ_*, which determines the initial direction of crack propagation and the law of crack length distribution (parameter r) on the most probable value of failure loading (mode) $Mo\left(P\frac{\sqrt{d}}{K}\right)$.

From Fig. 2, 3, it is possible to see that with an increasing in the number of cracks in the plate, both for uniaxial tensile and biaxial symmetric tensile-compression, there is a decreasing in the most probable value of failure loading.

Taking into account the initial direction of crack propagation (angle θ_*) in both cases of loading, leads to a decreasing in the value $Mo\left(P\frac{\sqrt{d}}{K}\right)$.

As can be seen from Fig. 4, increasing the value of the parameter r (increasing the homogeneity of the material) leads to an increasing in the most probable value of failure loading. Also, with the change of the parameter r, the shape of the distribution density curve changes. The lowest value $Mo\left(P\frac{\sqrt{d}}{K}\right)$ is observed for the biaxial symmetrical tension $(\eta=1)$, the highest for the biaxial symmetric tension-compression $(\eta=-1)$. Similar conclusions were made in [11].

Let's note that the maximum ordinate of the distribution curve is directly proportional to the parameter N and inversely proportional to the parameter r. This feature does not depend on the type of applied loading. Therefore, in the case of increasing the parameter N, the maximum value of the distribution density also increases and decreases with increasing parameter r.

In Fig. 5, the influence of the loading ratio, the number of cracks and taking into account the initial direction of crack propagation on the mean value of the failure loading is considered. Taking into account the initial direction of crack propagation leads to a decreasing the value $\langle P\rangle\frac{\sqrt{d}}{K}$. As the parameter N increases, the mean value of the failure loading decreases regardless of the type of stress state. Its greatest value will be in the case of uniaxial tensing. Let's note that there is a certain range of body sizes for which the strength with an asymptotic approach to its threshold value is almost independent of the number of defects.

Fig. 6 shows the dependence of the dispersion of the failure loading on the loading ratio, the number of cracks and taking into account the initial direction of crack propagation. The dispersion of the failure loading is a decreasing function of the argument N. At a certain interval of N change it is possible to see a significant decreasing in the value $D(P)\frac{d}{K^2}$. The nature of this decreasing does not depend on the type of loading and parameter θ_*.

As the parameter N changes, the dispersion of failure loading changes by an amount that is almost independent of the type of stress state. As for the case of the mean value $\langle P\rangle\frac{\sqrt{d}}{K}$, there is a certain range of body sizes for which the dispersion of failure loading is almost independent of the number of defects [12]. The dispersion of failure loading decreases taking into account the parameter θ_*.

In Fig. 7, the influence of material homogeneity and number of cracks on the coefficient of variation of failure loading $W(P)$ under biaxial symmetric tension is investigated. It is established that in this case the value $W(P)$ is an invariant with respect to the change of θ_* and depends only on the homogeneity of the material and the size of the plate. In the case of uniaxial tension and biaxial symmetric compression tension, the effect of the parameter is insignificant. The value $W(P)$ increases with increasing of parameter r and decreases with increasing of parameter N. There is a certain range of body sizes, for which let's observe a significant change in value $W(P)$ and asymptotic approach to a certain threshold value. Similar patterns can be traced for other types of stress.

A limitation of this study is the flat model of the defective body. Therefore, its generalization to the spatial case is relevant. This will make it possible to make a probabilistic description of the known experimental statistical regularities and strength characteristics of stochastically defective materials.

In this paper, the algorithm for determining of the strength statistical characteristics in the case of a flat stress state of lamellar bodies weakened by stochastically distributed defects of one grade was considered. A possible development of this study is to construct a generalized algorithm for the case when the body is weakened by defects of different varieties that do not interact with each other.

4. Conclusions

The study shows the influence of the ratio of applied loading, the number of defects-cracks in the plate (plate area with the same number of cracks), the angle that determines the initial direction of crack propagation and the law of crack length distribution on the most probable value of failure loading. It is established that with the increase in the number of cracks in the plate, both in uniaxial tension and in biaxial symmetric compression-tension, there is a decrease in the most probable value of the failure loading. Taking into account the initial direction of crack propagation leads to a decrease in the value of the most probable value of the failure loading, and increasing the homogeneity of the material leads to its increase.

It is found that the influence of the initial direction of crack propagation leads to a decrease in the mean value of the failure loading. As the number of cracks increases, the mean value of the failure loading decreases regardless of the type of stress state. Its greatest value will be in the case of uniaxial tension. A certain range of body sizes is established, for which the strength with asymptotic approach to its threshold value is almost independent of the number of defects.

It is found that the dispersion of the failure loading is a decreasing function of the argument (number of cracks), in particular, at a certain interval of its change there is a significant decrease in the value of the dispersion. The nature of this reduction does not depend on the type of loading and the angle that determines the initial direction of the crack propagation. With the change of the argument, the dispersion of the failure loading changes to a value that almost does not depend on the type of stress state, in particular, there is a certain range of body sizes for which the dispersion of the failure loading is almost independent of the number of defects taking into account the initial direction of crack propagation, the dispersion of the failure loading decreases.

It is established that the coefficient of variation of the failure loading under biaxial symmetric tension is an invariant with respect to the change of angle, which determines the initial direction of crack propagation and depends only on the homogeneity of the material and plate dimensions. In the case of uniaxial tension and biaxial symmetric compression-tension, the effect of this parameter is insignificant. The value of the coefficient of variation of the failure loading increases with increasing homogeneity of the material and decreases with increasing number of defects.

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