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## MATHEMATICAL MODEL OF MECHANICAL SUBSYSTEM OF TRACTION ELECTRIC DRIVE OF AN ELECTRIC LOCOMOTIVE

*The mathematical model of the mechanical subsystem of the traction electric drive of the four - axle section of the locomotive is developed in the article. The peculiarity of the model is taking into account the interconnected vertical oscillations of the locomotive and torsional oscillations in traction transmission and the influence of the anti-discharge device on the processes in the traction electric drive. This allows taking into account the relationship of oscillations of the crew of the locomotive, which have a significant impact on the processes in the electrical part of the traction drive. The model describes the oscillations of the locomotive body, two carts, four wheel pairs, traction motors and gearboxes. When developing the model, the assumption of linear dependence of stiffness and damping coefficients on displacements and velocities is accepted. Coupling of a wheel with a rail is described by means of the approximated dependence. The oscillations of the rails and the bases of the rail track are introduced into the model. As perturbation for vertical oscillations the roughness of a rail track is accepted. The model takes into account the influence of traction force and vertical oscillations of the crew on the unloading of the axles of the locomotive. This will allow us to study the processes of realization of the locomotive's maximum traction forces in the traction mode and in the mode of electrodynamic braking.*

*The use of the developed mathematical model of the mechanical subsystem will allow taking more fully into account the mutual influence of electrical processes and mechanical oscillations on each other, which will increase the accuracy of modeling. The model can be used for research of freight electric locomotives such as VL10, VL11, VL82, VL80, 2EL4, 2EL5 in order to further improve their traction electric drives, in particular, to determine the rational modes of application of the anti-unloading device in traction and braking modes. Further application of the mathematical model is possible to assess the performance of the traction drive and the locomotive in general in the study of modern traction drives, in particular, asynchronous, the use of which is possible on the above locomotives.*

**Keywords:** electric locomotive, traction electric drive, bogie, oscillation

**Introduction.** The National Economic Strategy for the period up to 2030 [1] provides for the renewal and modernization of the rolling stock of JSC Ukrzaliznytsia. The latter will be most effective if, as a result of modernization, the key technical characteristics of the rolling stock are brought closer to the level of modern models. Taking into account the work on the electrification of the railways of Ukraine, it seems advisable to improve the main-line electric freight locomotives of the VL10, VL11, VL80, VL82 types, as well as the 2EL5K and 2ES5K electric locomotives in order to increase their traction and energy performance.

Prediction of the characteristics of modernized electric locomotives can be achieved through mathematical modeling, which allows one to study processes in various systems and predict the properties of an electric locomotive as a whole. To carry out such work, appropriate mathematical models are required that reliably describe the processes in the systems under study.

The modern approach to modeling processes in such a complex electromechatronic system as a traction electric drive of a locomotive is to develop a complex mathematical model [2, 3] using specialized software that allows modeling with a high degree of detail of the studied locomotive systems [4-6]. However, for problems where electrical processes in a traction electric drive and its components are studied as a priority, it is sufficient to use simplified, less detailed mathematical models of the mechanical subsystem. For example, in [7, 8], the mechanical subsystem is described in the form of equations of torsional vibrations in the traction drive of a locomotive. At the same time, it was shown in [9] that torsional vibrations are interconnected with vertical vibrations of the locomotive undercarriage assembly, therefore, models that take this circumstance into account will be more reliable. Such models are used in [10, 11]. In this case, the assumption is made that horizontal and vertical oscillations are independent of each other [9, 12]. Models of this type - taking into account the vertical vibrations of the locomotive and torsional vibrations of the traction drive - seem to be more suitable for tasks in which the main research is directed to the processes in the electrical subsystem, but there is a significant mutual influence of the electrical and mechanical subsystems of the traction electric drive.

A mathematical model of the mechanical subsystem of an electric traction drive of an electric locomotive can be developed using the recommendations that are used to study the dynamics of a locomotive and assess its driving performance [13-15]. This model is supplemented by a description of the traction drive torsional vibrations, containing the equations of the traction motor and gearbox vibrations. External forces will be traction and braking forces (with

electrodynamic braking), resistance forces to motion, as well as axle loading forces during the operation of counter-unloading devices, which are equipped with the considered electric locomotives. It should be noted that there are practically no studies concerning the effect of anti-unloading devices on dynamic processes in the mechanical subsystem and the traction electric drive as a whole. It also seems necessary to take into account the redistribution of axle loads that occurs when traction (braking) forces are realized.

Thus, the **aim** of the article is to develop a mathematical model of the mechanical subsystem of the traction electric drive of the four-axle section of an electric locomotive, taking into account the vertical vibrations of the locomotive and torsional vibrations in the traction drive of the four-axle section, as well as the operation of the anti-unloading device.

**Research results.** A mathematical model is being developed for a four-axle section of a mainline freight electric locomotive. A general description of the design of electric locomotives can be found in the relevant literature ([16-18], etc.), here we only note the design features that affect the development of a mathematical model. The electric locomotives are equipped with a first-class traction drive, in which the traction motor and the traction gearbox have axial support. The traction motor is supported on the axle of the wheelset through plain bearings, on the frame through a rubber-metal support. The gear train is double-sided (spaced chevron), there are no elastic elements in the gear wheel design. The wheel-motor unit is connected to the bogie frame through leashes with rubber-metal bushings. The transfer of traction forces from the bogie to the body is carried out through a ball connection, which does not allow the body and bogies to move along the longitudinal axis of the electric locomotive. A distinctive feature of electric locomotives is the use of anti-unloading devices to increase the use of the adhesion weight of the locomotive: anti-unloading devices provide additional loading of the extreme axes of the electric locomotive section when traction forces are applied (the first wheelset is loaded) and electrodynamic braking (the last wheelset is loaded along the way).

When developing the model, the following assumptions were made [12, 13]:

- 1) the body, bogies and unsprung elements of the locomotive undercarriage assembly are considered absolutely rigid bodies;
- 2) the movement of wheelsets on the rails is uninterrupted;
- 3) the upper structure of the track is presented in the form of a concentrated mass with elastic and dissipative parameters;
- 4) the crew is symmetrical, there are no technological deviations and clearances;
- 5) fluctuations are small;
- 6) the disturbing effect on the crew is assumed to be random;
- 7) is considered freight traffic at speeds of no more than 100 km/h in straight horizontal sections of the track.

We also neglect the elastic-dissipative properties of the wheelset axle [9, 19], which makes it possible to exclude a separate mathematical description of the movement of the left and right wheels. We assume that the spring suspension and vibration dampers of the body stage are installed in a vertical transverse plane passing through the center of the ball connection of the body and the bogie, and are replaced by devices with equivalent parameters. We also assume that the spring suspension and vibration dampers of the axle-box stage are installed in vertical and horizontal planes passing through the axis of rotation of the wheelset and are replaced by devices with equivalent parameters. The counter-unloading device is modeled by an elastic-dissipative element with constant stiffness and damping coefficients.

Figure 1 shows the computational model of the section of the investigated electric locomotive.

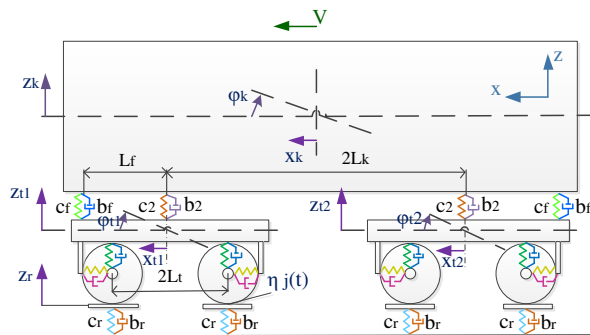


Figure 1 - Design model of a four-axle section of an electric locomotive

( $2L_k$  – body base,  $2L_t$  – bogie base,  $L_f$  – distance from the pivot assembly to the end beam of the bogie)

The design model of the wheel-motor unit was adopted in accordance with the recommendations given in [9, 14, 19, 20]. The elasticity of the gearing of the traction reducer has been added to the model. The design model diagram is shown in Fig. 2.

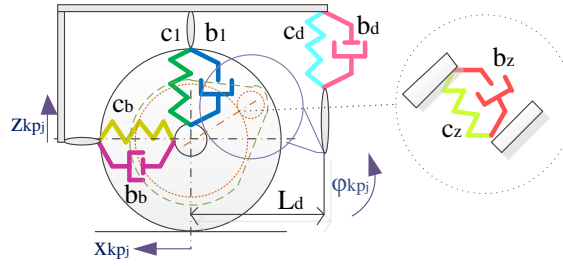


Figure 2 - Design model of the wheel-motor unit

Differential equations of oscillations are compiled using the Lagrange equations of the second kind

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} + \frac{\partial \Pi}{\partial q_i} = Q_i, \quad (1)$$

where  $T$  – kinetic energy;

$\Pi$  – potential energy;

$\Phi$  – dissipative function;

$q_i$  – generalized coordinate;

$\dot{q}_i$  – generalized speed;

$Q_i$  – external force acting along the coordinate.

The kinetic energy is calculated by the expression

$$T = \frac{1}{2} (M_k \dot{x}_k^2 + M_k \dot{z}_k^2 + J_k \dot{\phi}_k^2) + \frac{1}{2} \sum_{i=1}^2 (M_i \dot{x}_{ti}^2 + M_i \dot{z}_{ti}^2 + J_i \dot{\phi}_{ti}^2) + \frac{1}{2} \sum_{j=1}^4 (M_{kp} \dot{x}_{kpj}^2 + M_{kp} \dot{z}_{kpj}^2 + J_{kp} \dot{\phi}_{kpj}^2) + \frac{1}{2} \sum_{j=1}^4 (M_d \dot{x}_{dj}^2 + M_d \dot{z}_{dj}^2 + J_d \dot{\phi}_{dj}^2) + \frac{1}{2} \sum_{j=1}^4 J_a \dot{\phi}_{aj}^2 + \frac{1}{2} \sum_{j=1}^4 M_r \dot{z}_{rj}^2, \quad (2)$$

where  $M_k$  – body weight,

$J_k$  – body moment of inertia,

$M_i$  – bogie frame weight,

$J_i$  – moment of inertia of the bogie frame,

$M_{kp}$  – weight of the wheelset (with axle boxes and toothed wheels of traction gearbox),

$J_{kp}$  – moment of inertia of a wheel pair (with gear wheels of traction gearbox),

$M_d$  – the mass of the traction motor (taking into account the mass of the casings and gears of the traction gearbox),

$J_d$  – moment of inertia of the traction motor (taking into account the casings, gears of the traction reducers and the reduced moment of inertia of the armature) relative to the axis of the wheelset,

$J_a$  – moment of inertia of the armature (taking into account the gears of the traction reducers) relative to the axis of rotation of the electric motor,

$M_r$  – reduced mass of the rail-sleeper grate and track base (per wheelset),

$\dot{x}_k, \dot{z}_k, \dot{\phi}_k$  – speed of the center of mass of the body in the longitudinal and vertical directions and the speed of rotation of the body when galloping, respectively,

$\dot{x}_{ti}, \dot{z}_{ti}, \dot{\phi}_{ti}$  – the speed of the center of mass of the bogie frame in the longitudinal and vertical directions and the speed of rotation of the frame when galloping the bogie, respectively ( $i = 1, 2$ ),

$\dot{x}_{kpj}, \dot{z}_{kpj}, \dot{\phi}_{kpj}$  – speed of the center of mass of the wheelset in the longitudinal and vertical directions and the angular speed of the wheelset, respectively ( $j = 1, 4$ ),

$\dot{x}_{dj}, \dot{z}_{dj}, \dot{\phi}_{dj}$  – speed of the center of mass of the electric motor in the longitudinal and vertical directions and the speed of rotation of the electric motor relative to the axis of the wheelset, respectively,

$\dot{\phi}_{aj}$  – motor armature rotation speed,

$\dot{z}_{rj}$  – vertical speed of the rail-sleeper grid.

The horizontal speed of the bogies is expressed in terms of the speed of the body

$$\dot{x}_{ti} = \dot{x}_k \pm \dot{\phi}_k H_k, \quad (3)$$

where  $H_k$  – the distance from the center of the ball link on the bogie to the center of mass of the body. The second term takes into account the twitching of the cart when the body is galloping.

Horizontal speed of the center of mass of the traction motor (neglecting the clearances in the motor-axial bearings)

$$\dot{x}_{dj} = \dot{x}_{kpj} . \quad (4)$$

The vertical speed of the center of mass of the traction motor [21]

$$\dot{z}_{dj} = \frac{\dot{z}_{ti}L_1 + \dot{z}_{kpj}L_2}{L_d} , \quad (5)$$

where  $L_1, L_2, L_d = L_1 + L_2$  – horizontal distance from the center of mass of the electric motor to the point of its attachment to the bogie frame, the axle of the wheelset, respectively, and the total length of the engine mount. In the calculations, we assume that the fastening of the electric motor to the bogie frame is located in a plane passing through the center of the pivot assembly.

Speed of rotation of the traction motor frame around the axis of the wheelset [21]

$$\dot{\phi}_{dj} = \frac{\dot{z}_{kpj} - \dot{z}_{ti}}{L_d} . \quad (6)$$

Since the rolling of the wheelset on the rail is continuous, the vertical speed of the track element under the wheelset is expressed as follows

$$\dot{z}_{rj} = \dot{z}_{kpj} - \dot{\eta}_j , \quad (7)$$

where  $\dot{\eta}_j = \dot{\eta}(x_{kpj})$  – time derivative of the unevenness of the track ( $x_{kpj}$  – longitudinal displacement of the wheelset).

The potential energy of the system is calculated by the expression

$$\begin{aligned} \Pi = & \frac{1}{2} \sum_{i=1}^2 c_2 (\Delta z_{2i})^2 + \frac{1}{2} \sum_{i=1}^2 k_{fi} c_f (\Delta z_{fi})^2 + \frac{1}{2} \sum_{j=1}^4 c_1 (\Delta z_{1j})^2 + \frac{1}{2} \sum_{j=1}^4 c_b (\Delta x_{kpj})^2 + \\ & + \frac{1}{2} \sum_{j=1}^4 c_{b\phi} (\Delta \phi_{b\phi j})^2 + \frac{1}{2} \sum_{j=1}^4 c_d (\Delta z_{dj})^2 + \frac{1}{2} \sum_{j=1}^4 c_z (\Delta \phi_{zj})^2 + \frac{1}{2} \sum_{j=1}^4 c_r (\Delta z_{rj})^2 \end{aligned} , \quad (8)$$

where  $c_2$  – coefficient of rigidity of the spring suspension of the body stage (equivalent),

$c_f$  – coefficient of rigidity of the anti-unloading device,

$c_1$  – coefficient of rigidity of the spring suspension of the axle-box stage (equivalent),

$c_b$  – coefficient of rigidity of the leaders (equivalent),

$c_{b\phi}$  – coefficient of torsional rigidity of the leaders (equivalent),

$c_d$  – coefficient of rigidity of electric motor suspension,

$c_z$  – coefficient of rigidity of the gearing of the traction gearbox (equivalent),

$c_r$  – coefficient of rigidity of the rail-sleeper lattice and the track base,

$\Delta z_{2i}$  – movement of the spring suspension of the body stage,

$\Delta z_{fi}$  – movement of the support of the anti-unloading device,

$\Delta z_{1j}$  – vertical movement of the spring suspension in the axle-box stage,

$\Delta x_{kpj}$  – horizontal movement of the rubber elements of the leashes,

$\Delta \phi_{b\phi j}$  – the angle of twisting of the rubber elements of the leashes,

$\Delta z_{dj}$  – deformation of the rubber support of the traction motor,

$\Delta \phi_{zj}$  – deformation in the gearing of the traction gearbox,

$\Delta z_{rj}$  – deformation of the rail-sleeper grid,

$k_{fi}$  – boolean variable. If the anti-unloading device is on, then  $k_{fi} = 1$  otherwise  $k_{fi} = 0$ . In this case, the simultaneous switching on of the anti-unloading device of the front and rear bogies is not allowed.

The movement of the spring suspension of the body stage is calculated by the expression (neglect the galloping of the bogie)

$$\Delta z_{2i} = (z_k \pm \varphi_k L_k) - z_{ti} , \quad (9)$$

where  $\varphi_k$  is the pitching angle of the locomotive body,

$z_{ti}$  – vertical movement of the bogie frame,

$L_k$  – half the distance between the centers of the ball joints on the body.

The "+" sign is taken for the first trolley in the direction of travel, the "-" – for the second trolley in the direction of movement.

Vertical movement of the counter-unloading device hinge

$$\Delta z_{fi} = (z_k \pm \varphi_k (L_k + L_f)) - (z_{ti} \pm \varphi_{ti} L_f), \quad (10)$$

where  $\varphi_{ti}$  – bogie galloping angle,

$L_f$  – the distance from the center of the ball joint to the lining for supporting the anti-unloading device roller on the end beam of the bogie.

Signs "+" are taken for the first bogie in the direction of travel, signs "-" – for the second bogie in the direction of travel.

Vertical movement of the spring suspension in the axle-box step

$$\Delta z_{1j} = (z_{ti} \pm \varphi_{ti} L_t) - z_{kpj}, \quad (11)$$

where  $z_{kpj}$  is the vertical movement of the wheelset. The "+" sign is taken for the first wheelset in the direction of travel for each bogie, the "-" sign – for the second wheelset in the direction of travel of each bogie.

Horizontal deformation of the leader rubber elements

$$\Delta x_{kpj} = x_{kpj} - (x_{ti} \pm \varphi_{ti} H_t) = x_{kpj} - (x_k \pm \varphi_k H_k \pm \varphi_{ti} H_t), \quad (12)$$

where  $H_t$  – distance from the axle of the wheelset to the center of mass of the bogie.

The "+" sign in front of the term  $\varphi_k H_k$  is for the first and second wheelsets, the "-" sign in front of the term  $\varphi_k H_k$  is for the third and fourth wheelsets. The "+" sign in front of the term  $\varphi_{ti} H_t$  is taken for the first and third wheelset, the "-" sign in front of the term  $\varphi_{ti} H_t$  is taken for the second and fourth wheelset.

The angle of twisting of the rubber elements of the leaders is calculated by the expression

$$\Delta \varphi_{bpj} = \frac{z_{ti} - z_{kpj}}{L_b} \quad (13)$$

where  $L_b$  – the distance between the axes of the rubber elements in the leader.

Vertical deformation of the traction motor support (neglect the galloping of the bogie)

$$\Delta z_{dj} = z_{ti} - z_{kpj} \quad (14)$$

Deformation in gearing

$$\Delta \varphi_{cj} = R_2 \varphi_{kpj} - R_1 \varphi_{dj}, \quad (15)$$

where  $\varphi_{kpj}, \varphi_{dj}$  – the angle of rotation of the  $j$ -th wheelset and the armature of the  $j$ -th electric motor, respectively  
 $R_1, R_2$  – radii of the gear and toothed wheel of the traction gearbox.

Deformation of the rail-sleeper lattice

$$\Delta z_{rj} = z_{kpj} - \eta_j \quad (16)$$

Here  $\eta_j = \eta(x_{kpj})$  is the unevenness of the path.

The dissipative function is calculated by the expression

$$\begin{aligned} \Phi = & \frac{1}{2} \sum_{i=1}^2 b_2 (\dot{\Delta z}_{2i})^2 + \frac{1}{2} \sum_{i=1}^2 k_{fi} b_f (\dot{\Delta z}_{fi})^2 + \frac{1}{2} \sum_{j=1}^4 b_1 (\dot{\Delta z}_{1j})^2 + \frac{1}{2} \sum_{j=1}^4 b_b (\dot{\Delta x}_{kpj})^2 + \\ & + \frac{1}{2} \sum_{j=1}^4 b_{bp} (\dot{\Delta \varphi}_{bpj})^2 + \frac{1}{2} \sum_{j=1}^4 b_d (\dot{\Delta z}_{dj})^2 + \frac{1}{2} \sum_{j=1}^4 b_z (\dot{\Delta \varphi}_{cj})^2 + \frac{1}{2} \sum_{j=1}^4 b_r (\dot{\Delta z}_{rj})^2 \end{aligned} \quad (17)$$

where  $b_2$  – damping coefficient of the spring suspension of the body stage (equivalent);

$b_f$  – damping coefficient of the anti-unloading device (equivalent);

$b_1$  – damping coefficient of the axle-box spring suspension (equivalent);

$b_b$  – compression damping coefficient of the leader;

$b_{bp}$  – torsional damping coefficient of the leader;

$b_d$  – damping coefficient of the traction motor suspension;

$b_z$  – damping coefficient of the gearing of the traction reducer (equivalent);

$b_r$  – damping coefficient of the rail-sleeper lattice.

In expression (17), it is assumed that the speed is calculated as  $\dot{\Delta} = \frac{d}{dt} \Delta$ . Then using expressions (9) – (16) we

obtain

$$\dot{\Delta z}_{2i} = (\dot{z}_k \pm \dot{\varphi}_k L_k) - \dot{z}_{ti} \quad (18)$$

$$\dot{\Delta z}_{fi} = (\dot{z}_k \pm \dot{\varphi}_k (L_k + L_f)) - (\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_f) \quad (19)$$

$$\dot{\Delta z}_{1j} = (\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_t) - \dot{z}_{kpj} . \quad (20)$$

$$\dot{\Delta x}_{kpj} = \dot{x}_{kpj} - (\dot{x}_k \pm \dot{\varphi}_k H_k \pm \dot{\varphi}_{ti} H_t) . \quad (21)$$

$$\dot{\Delta \varphi}_{bpj} = \frac{\dot{z}_{ti} - \dot{z}_{kpj}}{L_b} . \quad (22)$$

$$\dot{\Delta z}_{dj} = (\dot{z}_{ti} \pm \dot{\varphi}_{ti} L_t) - \dot{z}_{kpj} . \quad (23)$$

$$\dot{\Delta \varphi}_{zj} = R_2 \dot{\varphi}_{kpj} - R_1 \dot{\varphi}_{aj} . \quad (24)$$

$$\dot{\Delta z}_{rj} = \dot{z}_{kpj} - \dot{\eta}_j . \quad (25)$$

To calculate the generalized forces, we introduce the generalized coordinates:

- longitudinal horizontal and vertical movements and pitching angle of the body  $x_k, z_k, \varphi_k$ ,
- vertical movements and angles of galloping of frame of bogies  $z_{ti}, \varphi_{ti}$ ,
- horizontal and vertical displacements and angles of rotation of wheelsets  $x_{kpj}, z_{kpj}, \varphi_{kpj}$ ,
- angle of rotation of the traction motor armature  $\varphi_{aj}$ .

Thus, the system will have 23 degrees of freedom.

We introduce generalized forces as follows (Fig. 3). Consider the traction operation mode.

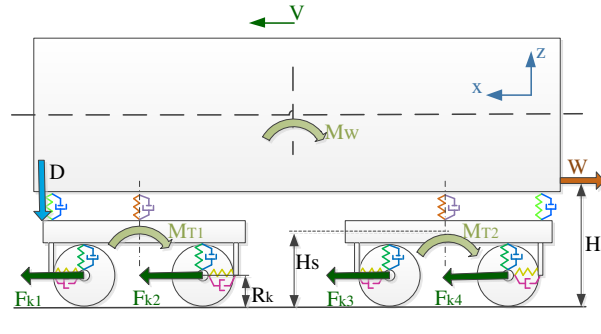


Figure 3 - Scheme of application of generalized forces and moments  
( $H_a$  – coupling height,  $H_s$  – height of the pivot assembly,  $R_{kp}$  – wheel radius)

The traction forces act along the coordinate  $x_k$

$$F_A = \sum_{j=1}^4 F_{kpj} = \frac{1}{R_{kp}} \sum_{j=1}^4 M_{kpj} \quad (26)$$

where  $M_{kpj}$  – coupling moment of the j-th wheelset and the force of the main resistance to movement  $W$ , which we neglect

On the coordinate  $\varphi_k$ , the moment  $M_W$  from the traction force acts on the resistance coupler

$$M_W = F_A (H_a - H_s) , \quad (27)$$

where  $H_a$  and  $H_s$  are the automatic couplings, the height and distance from the level of the rail head to the center of the bogie ball connection. We accept these values as unchanged when the locomotive vibrates.

By the coordinates  $z_{t1}$  and  $z_{t2}$  the forces act  $D_1$  and  $D_2$ , accordingly, from the anti-unloading device:  $D_i = k_{fi} (D_{0i} - c_{fd} \Delta f_i)$  – the additional loading force from the anti-unloading device, here  $D_{0i}$  – the initial force,  $\Delta f_i$  – the piston stroke of the cylinder of the anti-unloading device of the i-th bogie,  $c_{fd}$  – coefficient of rigidity.

On the coordinate  $\varphi_{t1}$ , the moment from the force of the anti-unloading device and the traction forces of the first bogie act

$$M_{t1} = -D_1 L_f + (F_{kp1} + F_{kp2}) (H_s - R_{kp}) \quad (28)$$

The moment from the traction force of the second bogie acts along the coordinate  $\varphi_{t2}$

$$M_{t2} = D_2 L_f + (F_{kp3} + F_{kp4}) (H_s - R_{kp}) \quad (29)$$

The moments  $M_{aj}$  of the electric motors act on the coordinates  $\varphi_{aj}$ .

The adhesion moments  $M_{kpj}$  act on the coordinates  $\varphi_{kpj}$ .

The adhesion moment of the j-th wheelset is calculated taking into account the recommendations [9, 19, 23-26]

$$M_{kpj} = R_{kp} \Psi_0 K_{kpj} (N_0 + \Delta N_{kpj} \pm \Delta N_{Mkpj}) \quad (30)$$

where  $K_{kpj} = K(u_{kpj})$  – dependence of the coefficient of adhesion on the sliding speed for the  $j$ -th wheelset,

$\Psi_0$  – potential adhesion coefficient (determined by [22]),

$N_0$  – static component of the normal reaction of the rails to the wheelset, equal to 235 kN,

$\Delta N_{kpj}$  – the dynamic component of the normal reaction of the rails to the wheelset,

$\Delta N_{Mkpj}$  – change in the load on the wheelset when the traction (braking) force is realized.

The sliding speed of the  $j$ -th wheelset is calculated by the expression

$$u_{kpj} = R_{kp} \dot{\phi}_{kpj} - \dot{x}_{kpj} \quad (31)$$

Coefficient  $K(u_{kpj})$  is approximated by the following formulas depending on the relative sliding speed  $\frac{u}{\hat{V}_E}$  ( $\hat{V}_E$  – speed expressed in m/s ( $\hat{V}_E = \dot{x}$ );  $V_E$  – speed expressed in km/h ( $V_E = 3,6\hat{V}_E$ )):

if  $0 < u \leq 0,14\% \hat{V}_E$

$$K = 1491,25 \frac{u}{V_E}, \quad (32)$$

if  $0,14\% \hat{V}_E < u \leq 1,14\% \hat{V}_E$

$$K = \frac{381,6u - V_E}{360u - 0,04V_E}, \quad (33)$$

if  $1,14\% \hat{V}_E < u \leq 2,5\% \hat{V}_E$

$$K = 1,06 - 16,36 \frac{u}{V_E}, \quad (34)$$

if  $u \geq 2,5\% \hat{V}_E$

$$K = 0,57e^{-0,68(u-u_{kr})} + 0,36e^{-0,0036(u-u_{kr})} + 0,02e^{-1,5(u-u_{kr})}, \quad (35)$$

where  $u_{kr} \cong 2,5\% \hat{V}_E$ .

The dynamic component of the normal reaction of the rails to the wheelset is calculated by the expression [9]

$$\Delta N_{kpj} = \frac{M_r (b_l \dot{\Delta z}_{lj} + c_l \Delta z_{lj}) - M_{kp} (b_r \dot{\Delta z}_{rj} + c_r \Delta z_{rj})}{M_{kp} + M_r}. \quad (36)$$

Change in the load on the rails when the wheelset implements the traction force [14]

$$\Delta N_{Mkpj} = \frac{F_{kpj} R_{kp}}{L_t}. \quad (37)$$

In this case, unloading takes place if the traction motor is located behind the wheelset in the direction of travel, and additional loading takes place if the traction motor is located in front of the wheelset.

After performing calculations according to expressions (1) - (25), we obtain 23 differential equations describing the longitudinal, vertical and angular vibrations of the locomotive and torsional vibrations in the traction drive:

– longitudinal body vibrations

$$(M_k + 2M_t) \ddot{x}_k + (4b_b) \dot{x}_k - b_b \dot{x}_{kp1} - b_b \dot{x}_{kp2} - b_b \dot{x}_{kp3} - b_b \dot{x}_{kp4} + (4c_b) x_k - c_b x_{kp1} - c_b x_{kp2} - c_b x_{kp3} - c_b x_{kp4} = F_A - W \quad (38)$$

– vertical body vibrations

$$M_k \ddot{z}_k + (2b_2 + (k_{f1} + k_{f2}) b_f) \dot{z}_k + (-b_2 - k_{f1} b_f) \dot{z}_{t1} - (k_{f1} b_f L_f) \dot{\phi}_{t1} + (-b_2 - k_{f2} b_f) \dot{z}_{t2} + (k_{f2} b_f L_f) \dot{\phi}_{t2} + (2c_2 + (k_{f1} + k_{f2}) c_f) z_k + (-c_2 - k_{f1} c_f) z_{t1} - (k_{f1} c_f L_f) \phi_{t1} + (-c_2 - k_{f2} c_f) z_{t2} + (k_{f2} c_f L_f) \phi_{t2} = 0 \quad (39)$$

– body angular vibrations

$$\begin{aligned}
& \left( J_k + 2M_t H_k^2 \right) \ddot{\phi}_k + \left( 2b_2 L_k^2 + b_f (k_{f1} + k_{f2}) (L_k + L_f) - 4b_b H_k^2 \right) \dot{\phi}_k + \left( -b_2 L_k - b_f k_{f1} (L_k + L_f) \right) \dot{z}_{t1} + \\
& + \left( -b_f L_f (L_k + L_f) k_{f1} \right) \dot{\phi}_{t1} + \left( b_2 L_k + b_f (L_k + L_f) k_{f2} \right) \dot{z}_{t2} + \left( -b_f L_f (L_k + L_f) k_{f2} \right) \dot{\phi}_{t2} + \\
& + b_b H_k \dot{x}_{kp1} + b_b H_k \dot{x}_{kp2} - b_b H_k \dot{x}_{kp3} - b_b H_k \dot{x}_{kp4} + \left( c_f (L_k + L_f) (k_{f1} + k_{f2}) \right) z_k + \\
& + \left( 2c_2 L_k^2 + 4c_b H_k^2 + c_f (L_k + L_f)^2 (k_{f1} + k_{f2}) \right) \phi_k + \left( -c_2 L_k - c_f (L_k + L_f) k_{f1} \right) z_{t1} + \\
& + \left( -c_f L_f (L_k + L_f) k_{f1} \right) \phi_{t1} + \left( c_2 L_k - c_f (L_k + L_f) k_{f2} \right) z_{t2} + \left( -c_f L_f (L_k + L_f) k_{f2} \right) \phi_{t2} + \\
& + c_b H_k (x_{kp1} + x_{kp2} - x_{kp3} - x_{kp4}) = F_A (H_a - H_s)
\end{aligned} \tag{40}$$

vertical vibrations of bogies

$$\begin{aligned}
& \left( M_t + 2M_d \frac{L_1^2}{L_d^2} + 2 \frac{J_d}{L_d^2} \right) \ddot{z}_{t1} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{kp1} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{kp2} + \left( -b_2 - b_f k_{f1} \right) \dot{z}_k + \\
& + \left( -b_2 L_k - b_f (L_k + L_f) k_{f1} \right) \dot{\phi}_k + \left( -b_2 + b_f k_{f1} + 2b_1 + 2b_{b\phi} \frac{1}{L_b^2} + 2b_d \right) \dot{z}_{t1} + \left( b_f L_f k_{f1} \right) \dot{\phi}_{t1} + \\
& + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{kp1} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{kp2} + \left( -c_2 - c_f k_{f1} \right) z_k + \left( -c_2 L_k - c_f (L_k + L_f) k_{f1} \right) \phi_k + \\
& + \left( c_2 + 2c_1 + 2c_{b\phi} \frac{1}{L_b^2} + 2c_d + k_{f1} c_f \right) z_{t1} + \left( c_f L_f k_{f1} \right) \phi_{t1} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{kp1} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{kp2} = -D_1
\end{aligned} \tag{41}$$

$$\begin{aligned}
& \left( M_t + 2M_d \frac{L_1^2}{L_d^2} + 2 \frac{J_d}{L_d^2} \right) \ddot{z}_{t2} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{kp3} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{kp4} + \left( -b_2 - b_f k_{f2} \right) \dot{z}_k + \\
& + \left( b_2 L_k + b_f (L_k + L_f) k_{f2} \right) \dot{\phi}_k + \left( b_2 + b_f k_{f2} + 2b_1 + 2b_{b\phi} \frac{1}{L_b^2} + 2b_d \right) \dot{z}_{t2} + \left( -b_f k_{f2} L_f \right) \dot{\phi}_{t2} + \\
& + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{kp3} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{kp4} + \left( -c_2 - c_f k_{f2} \right) z_k + \left( c_2 L_k - c_f (L_k + L_f) k_{f2} \right) \phi_k + \\
& + \left( c_2 + 2c_1 + 2c_{b\phi} \frac{1}{L_b^2} + 2c_d + c_f k_{f2} \right) z_{t2} + \left( c_f L_f k_{f2} \right) \phi_{t2} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{kp3} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{kp4} = -D_2
\end{aligned} \tag{42}$$

– angular vibrations of bogies

$$\begin{aligned}
& J_t \ddot{\phi}_{t1} + \left( -b_f L_f k_{f1} \right) \dot{z}_k + \left( -b_f (L_k + L_f) L_f k_{f1} \right) \dot{\phi}_k + \left( b_f L_f k_{f1} \right) \dot{z}_{t1} + \left( b_f L_f^2 k_{f1} + 2(b_1 + b_d) L_t^2 + 2b_d L_t^2 \right) \dot{\phi}_{t1} + \\
& + (b_b H_t) \dot{x}_{kp1} + \left( -(b_1 + b_d) L_t \right) \dot{z}_{kp1} + \left( -b_b H_t \right) \dot{x}_{kp2} + \left( (b_1 + b_d) L_t \right) \dot{z}_{kp2} + \left( -c_f L_f k_{f1} \right) z_k + \left( -c_f (L_k + L_f) L_f k_{f1} \right) \phi_k + \\
& + \left( c_f L_f k_{f1} \right) z_{t1} + \left( 2c_1 L_t^2 - 2c_b H_t^2 + c_f L_f^2 k_{f1} \right) \phi_{t1} + \left( c_b H_t \right) x_{kp1} + \left( -c_b H_t \right) x_{kp2} + \left( -c_1 L_t \right) z_{kp1} + \left( c_1 L_t \right) z_{kp2} = M_{t1}
\end{aligned} \tag{43}$$

$$\begin{aligned}
& J_t \ddot{\phi}_{t2} + \left( -k_{f2} b_f L_f \right) \dot{z}_k + \left( b_f (L_k + L_f) L_f k_{f2} \right) \dot{\phi}_k + \left( k_{f2} b_f L_f \right) \dot{z}_{t2} + \left( -b_f L_f^2 k_{f2} + 2(b_1 + b_d) L_t^2 + 2b_d L_t^2 \right) \dot{\phi}_{t2} + \\
& + (b_b H_t) \dot{x}_{kp3} + \left( -(b_1 + b_d) L_t \right) \dot{z}_{kp3} + \left( -b_b H_t \right) \dot{x}_{kp4} + \left( (b_1 + b_d) L_t \right) \dot{z}_{kp4} + \left( -c_f L_f k_{f2} \right) z_k + \left( -c_f (L_k + L_f) L_f k_{f2} \right) \phi_k + \\
& + \left( c_f L_f k_{f2} \right) z_{t2} + \left( 2c_1 L_t^2 - 2c_b H_t^2 + c_f L_f^2 k_{f2} \right) \phi_{t2} + \left( c_b H_t \right) x_{kp3} + \left( -c_b H_t \right) x_{kp4} + \left( -c_1 L_t \right) z_{kp3} + \left( c_1 L_t \right) z_{kp4} = M_{t2}
\end{aligned} \tag{44}$$

– longitudinal vibrations of wheelsets

$$(M_{kp} + M_d) \ddot{x}_{kp1} + (-b_b) \dot{x}_k - (b_b H_k) \dot{\phi}_k - (b_b H_t) \dot{\phi}_{t1} + (b_b) \dot{x}_{kp1} + (-c_b) x_k + (-c_b H_k) \phi_k + (-c_b H_t) \phi_{t1} + (c_b) x_{kp1} = F_{kp1} \tag{45}$$

$$(M_{kp} + M_d) \ddot{x}_{kp2} + (-b_b) \dot{x}_k - (b_b H_k) \dot{\phi}_k + (b_b H_t) \dot{\phi}_{t1} + (b_b) \dot{x}_{kp2} + (-c_b) x_k - (c_b H_k) \phi_k + (c_b H_t) \phi_{t1} + (c_b) x_{kp2} = F_{kp2} \tag{46}$$

$$(M_{kp} + M_d) \ddot{x}_{kp3} + (-b_b) \dot{x}_k + (b_b H_k) \dot{\phi}_k - (b_b H_t) \dot{\phi}_{t2} + (b_b) \dot{x}_{kp3} + (-c_b) x_k + (c_b H_k) \phi_k + (-c_b H_t) \phi_{t2} + (c_b) x_{kp3} = F_{kp3} \tag{47}$$



$$(M_{kp} + M_d) \ddot{x}_{kp4} + (-b_b) \dot{x}_k + (b_b H_k) \dot{\phi}_k + (b_b H_t) \dot{\phi}_{t2} + (b_b) \dot{x}_{kp4} + (-c_b) x_k + (c_b H_k) \phi_k + (c_b H_t) \phi_{t2} + (c_b) x_{kp4} = F_{kp4} \quad (48)$$

– vertical vibrations of wheelsets

$$\begin{aligned} & \left( M_{kp} + M_d \frac{L_2^2}{L_d^2} + M_r \right) \ddot{z}_{kp1} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{t1} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{t1} + (-b_1 L_t) \dot{\phi}_{t1} + \\ & + \left( b_1 + b_{b\phi} \frac{1}{L_b^2} + b_d + b_r \right) \dot{z}_{kp1} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{t1} + (-c_1 L_t) \phi_{t1} + \left( c_1 + c_{b\phi} \frac{1}{L_b^2} + c_d + c_r \right) z_{kp1} = \\ & = (M_r) \ddot{\eta}_1 + (b_r) \dot{\eta}_1 + (c_r) \eta_1 \end{aligned} \quad (49)$$

$$\begin{aligned} & \left( M_{kp} + M_d \frac{L_2^2}{L_d^2} + \frac{J_d}{L_d^2} + M_r \right) \ddot{z}_{kp2} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{t1} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{t1} + (b_1 L_t) \dot{\phi}_{t1} + \\ & + \left( b_1 + b_{b\phi} \frac{1}{L_b^2} + b_d + b_r \right) \dot{z}_{kp2} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{t1} + (c_1 L_t) \phi_{t1} + \left( c_1 + c_{b\phi} \frac{1}{L_b^2} + c_d + c_r \right) z_{kp2} = \\ & = (M_r) \ddot{\eta}_2 + (b_r) \dot{\eta}_2 + (c_r) \eta_2 \end{aligned} \quad (50)$$

$$\begin{aligned} & \left( M_{kp} + M_d \frac{L_2^2}{L_d^2} + \frac{J_d}{L_d^2} + M_r \right) \ddot{z}_{kp3} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{t2} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{t2} + (-b_1 L_t) \dot{\phi}_{t2} + \\ & + \left( b_1 + b_{b\phi} \frac{1}{L_b^2} + b_d + b_r \right) \dot{z}_{kp3} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{t2} + (-c_1 L_t) \phi_{t2} + \left( c_1 + c_{b\phi} \frac{1}{L_b^2} + c_d + c_r \right) z_{kp3} = \\ & = (M_r) \ddot{\eta}_3 + (b_r) \dot{\eta}_3 + (c_r) \eta_3 \end{aligned} \quad (51)$$

$$\begin{aligned} & \left( M_{kp} + M_d \frac{L_2^2}{L_d^2} + \frac{J_d}{L_d^2} + M_r \right) \ddot{z}_{kp4} + \left( M_d \frac{L_1 L_2}{L_d^2} - \frac{J_d}{L_d^2} \right) \ddot{z}_{t2} + \left( -b_1 - b_{b\phi} \frac{1}{L_b^2} - b_d \right) \dot{z}_{t2} + (b_1 L_t) \dot{\phi}_{t2} + \\ & + \left( b_1 + b_{b\phi} \frac{1}{L_b^2} + b_d + b_r \right) \dot{z}_{kp4} + \left( -c_1 - c_{b\phi} \frac{1}{L_b^2} - c_d \right) z_{t2} + (c_1 L_t) \phi_{t2} + \left( c_1 + c_{b\phi} \frac{1}{L_b^2} + c_d + c_r \right) z_{kp4} = \\ & = (M_r) \ddot{\eta}_4 + (b_r) \dot{\eta}_4 + (c_r) \eta_4 \end{aligned} \quad (52)$$

– angular vibrations of wheelsets

$$(J_{kp}) \ddot{\phi}_{kp1} + (b_z R_2^2) \dot{\phi}_{kp1} + (-b_z R_1 R_2) \dot{\phi}_{a1} + (c_z R_2^2) \phi_{kp1} + (-c_z R_1 R_2) \phi_{a1} = M_{kp1} \quad (53)$$

$$(J_{kp}) \ddot{\phi}_{kp2} + (b_z R_2^2) \dot{\phi}_{kp2} + (-b_z R_1 R_2) \dot{\phi}_{a2} + (c_z R_2^2) \phi_{kp2} + (-c_z R_1 R_2) \phi_{a2} = M_{kp2} \quad (54)$$

$$(J_{kp}) \ddot{\phi}_{kp3} + (b_z R_2^2) \dot{\phi}_{kp3} + (-b_z R_1 R_2) \dot{\phi}_{a3} + (c_z R_2^2) \phi_{kp3} + (-c_z R_1 R_2) \phi_{a3} = M_{kp3} \quad (55)$$

$$(J_{kp}) \ddot{\phi}_{kp4} + (b_z R_2^2) \dot{\phi}_{kp4} + (-b_z R_1 R_2) \dot{\phi}_{a4} + (c_z R_2^2) \phi_{kp4} + (-c_z R_1 R_2) \phi_{a4} = M_{kp4} \quad (56)$$

– angular vibrations of traction motor armatures

$$(J_a) \ddot{\phi}_{a1} + (-b_z R_1 R_2) \dot{\phi}_{kp1} + (b_z R_1^2) \dot{\phi}_{a1} + (-c_z R_1 R_2) \phi_{kp1} + (c_z R_1^2) \phi_{a1} = M_{a1} \quad (57)$$

$$(J_a) \ddot{\phi}_{a2} + (-b_z R_1 R_2) \dot{\phi}_{kp2} + (b_z R_1^2) \dot{\phi}_{a2} + (-c_z R_1 R_2) \phi_{kp2} + (c_z R_1^2) \phi_{a2} = M_{a2} \quad (58)$$

$$(J_a) \ddot{\phi}_{a3} + (-b_z R_1 R_2) \dot{\phi}_{kp3} + (b_z R_1^2) \dot{\phi}_{a3} + (-c_z R_1 R_2) \phi_{kp3} + (c_z R_1^2) \phi_{a3} = M_{a3} \quad (59)$$

$$(J_a) \ddot{\phi}_{a4} + (-b_z R_1 R_2) \dot{\phi}_{kp4} + (b_z R_1^2) \dot{\phi}_{a4} + (-c_z R_1 R_2) \phi_{kp4} + (c_z R_1^2) \phi_{a4} = M_{a4} \quad (60)$$

Thus, equations (38) - (60) form a simplified mathematical model of the mechanical part of a four-axle section of an electric locomotive. The resulting model is intended to be included in a complex mathematical model of a traction electric drive in order to take into account the mutual influence of electrical processes and mechanical vibrations on each other.

**Conclusions.** A mathematical model of the mechanical subsystem of the traction electric drive of the four-axle section of the electric locomotive has been developed. A feature of the model is to take into account the interrelated vertical vibrations of the locomotive and torsional vibrations in the traction drive and the influence of the anti-unloading device on the processes in the traction drive. The use of the developed mathematical model of the mechanical subsystem will make it possible to more fully take into account the mutual influence of electrical processes and mechanical vibrations on each other, which will increase the accuracy of modeling. Further improvement of the developed model is possible by introducing nonlinear elastic-dissipative connections between the elements of the mechanical subsystem.

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У статті розроблено математичну модель механічної підсистеми тягового електроприводу чотирьохвісної секції локомотиву. Особливістю моделі є врахування взаємопов'язаних вертикальних коливань локомотива і крутильних коливань у тяговій передачі та впливу роботи протирозвантажувального пристрою на процеси в тяговому електроприводі. Це дозволяє врахувати взаємозв'язок коливань екіпажної частини локомотиву, які мають значущий вплив на процеси в електричній частині тягового електроприводу. Модель описує коливання кузова локомотиву, двох візків, чотирьох колісних пар, тягових двигунів та редукторів. При розробці моделі прийнято припущення про лінійну залежність коефіцієнтів жорсткості та демпфування від переміщень та швидкостей. Зчеплення колеса з рейкою описано за допомогою апроксимованої залежності. У модель введено коливання рейок та основи рейкової колії. У якості збурення для вертикальних коливань приймається нерівність рейкової колії. У моделі враховано вплив сили тяги і вертикальних коливань екіпажу на розвантаження осей локомотива. Це дозволить досліджувати процеси при реалізації локомотивом граничних тягових зусиль у тяговому режимі та у режимі електродинамічного гальмування.

Використання розробленої математичної моделі механічної підсистеми дозволить більш повно врахувати взаємний вплив електричних процесів і механічних коливань один на одного, що підвищить точність моделювання. Модель може бути застосовано для досліджень вантажних електровозів типу ВЛ10, ВЛ11, ВЛ82, ВЛ80, 2ЕЛ4, 2ЕЛ5 з метою подальшого удосконалення їх тягових електроприводів, зокрема, визначенню раціональних режимів застосування протирозвантажувального пристрою у тягових та гальмівних режимах. Подальше застосування математичної моделі можливе для оцінки показників тягового електроприводу і локомотиву в цілому при дослідженні сучасних тягових електроприводів, зокрема, асинхронного, застосування яких можливе на вищезазначених локомотивах

**Ключові слова:** електровоз, тяговий електропривод, візок, коливання

В статье разработана математическая модель механической подсистемы тягового электропривода четырехосной секции локомотива. Особенностью модели является учет взаимосвязанных вертикальных колебаний локомотива и крутильных колебаний в тяговой передаче и влияния работы противоразгрузочного устройства на процессы в тяговом электроприводе. Это позволяет учесть взаимосвязь колебаний экипажной части локомотива, которые оказывают значимое влияние на процессы в электрической части тягового электропривода. Модель описывает колебания кузова локомотива, двух тележек, четырех колесных пар, тяговых двигателей и редукторов. При разработке модели принято предположение о линейной зависимости коэффициентов жесткости и демпфирования от перемещений и скоростей. Сцепления колеса с рельсом описано с помощью аппроксимированной зависимости. В модель введено колебания рельсов и основы рельсового пути. В модели учтено влияние силы тяги и вертикальных колебаний экипажа на разгрузку осей локомотива. Это позволит исследовать процессы при реализации локомотивом предельных тяговых усилий в тяговом режиме и в режиме электродинамического торможения.

Использование разработанной математической модели механической подсистемы позволит более полно учесть взаимное влияние электрических процессов и механических колебаний друг на друга, что повысит точность моделирования. Модель может быть применено для исследований грузовых электровозов типа ВЛ10, ВЛ11, ВЛ82, ВЛ80, 2ЕЛ4, 2ЕЛ5 с целью дальнейшего совершенствования их тяговых электроприводов, в частности, определению рациональных режимов применения противоразгрузочного устройства в тяговых и тормозных режимах. Дальнейшее применение математической модели возможно для оценки показателей тягового электропривода и локомотива в целом при исследовании современных тяговых электроприводов, в частности, асинхронного, применение которых возможно на вышеупомянутых локомотивах.

**Ключевые слова:** электровоз, тяговой электропривод, тележка, колебания

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