

## Robust Control of a Brushless Servo Motor Using Sliding Mode

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### Abstract

*The application of sliding mode techniques the position control of a brushless servo motor is discussed. Such control laws are well suited for electric power inverter. However, high frequency commutations are avoided due to the mechanical systems. Various recent schemes are studied and operated to derive control solutions which are technically feasible. In spite of straightforward applications the resulting systems show robust performances to parametric variations and disturbances. Robustness studied with respect to rotor flux uncertainties and to stator resistance which varies with the temperature of the motor.*

*Keywords: non linier control, brushless servo motor, sliding mode, robust control*

### 1. Introduction

This journal is written through system of technical controls non linear with *sliding mode*. Sliding control theory mode constitutes one of classic control technical for non linear system. With method controls sliding mode, trajectory is situation is restrained that follows trajectory basis (reference) one that is established. This method is divided two phases which is trajectory is pulled making for an area has already is defined previous and surface named sliding (*sliding surface*) and trajectory moves by glide(*sliding*) at previous upper surface. Severally sliding control than method this mode is:

- Can be utilized for system syntheses non tall order linear.
- Can be utilized for MIMO'S system (*Multi Input, Multi Output*).
- Resulting control system robust to parameter and perturbation variation (trouble).
- Can be utilized for system what does have uncertainty so can solve inaccurate problem face to face.

Result of sliding' method this mode will be applied on one system which is Brushless dc's motor. *Brushless motor* are three phase synchronous motor that utilize electronic commutation to processes *switching* current among phase or utilizes silicone rectifier as component of static commutation synchronous starting motor permanent magnet has idiosyncrasy as follows:

Rotor construction brushless just consisting of magnet just so not requires care.

1. Very tall approachable top speed.
2. Moment of inertia of bottommost rotor for angular speed motor or for torsi motor.
3. Motor construction without brush utterly being closed to water and airproof.

On scientific journal writing this, will do operation result analysis to motor speed, direct current(  $i_d$ ), voltage quadratic ( $V_q$ ), voltage direct ( $V_d$ ) and *robustness* to flux face value change rotor(  $\Phi_f$ ) and stator prisoner(  $R_s$ )

### 2. Brushless Motor DC

State Space of state from d-q transforms with model assuming taken from by rotor default model. Division gap flux airs that resulting by field circumference asymmetrically lies around field pole diameter. this pole is wicked named route or direct pole. pole of armature wave lies 90 degrees of field pole is named quadratic pole. Written numeric's data as follows

$V_d$  = direct Voltage (Volt)  
 $V_q$  = quadratic Voltage (Volt)  
 $\Phi_d$  = Direct Flux  
 $\Phi_q$  = Quadratique Flux  
 $L_d$  = Stator Inductance (0.0014 H)  
 $L_q$  = Rotor Inductance (0.0028 H)  
 $R_s$  = Stator Resistance (0.6Ω)  
 $J$  = moment of inertia (expenses and motor) ( $11 \times 10^{-4}$  kg m<sup>2</sup>)  
 $P$  = poles (4)  
 $F_v$  = viscous damping coefficient ( $1.4 \times 10^{-3}$  kg m<sup>2</sup>s<sup>-1</sup>)  
 $\Omega$  = Rotor speed (rad /sec)  
 $\Phi_f$  = Rotor flux (constant) (0.1194 Wb)  
 $C_L$  = Torsi expenses (constant) (Nm)

Model equation's consisting of electrical equation and mechanical equation. Relationship among voltage and flux is

$$V_d = R_s \cdot i_d - p\Omega\phi_d \quad (1)$$

$$V_q = R_s \cdot i_q - p\Omega\phi_d \quad (2)$$

With :

$$\phi_d = L_d \cdot i_d + \phi_f, \phi_q = L_q \cdot i_q \quad (3)$$

And electrical torsi:

$$C_m = p[(L_d - L_q) i_d + \phi_f] i_q \quad (4)$$

Mechanic Equations:

$$\theta = \phi_d \quad (5)$$

$$J \Omega = C_m + C_L - f_v \Omega \quad (6)$$

Servo motor dynamic equations:

$$\dot{X} = \begin{bmatrix} \dot{\theta} \\ \dot{\Omega} \\ \dot{i}_d \\ \dot{i}_q \end{bmatrix} = \begin{bmatrix} \phi \\ \frac{P}{J} [(L_d - L_q) i_d + \phi_f] i_q - \frac{C_L}{J} - \frac{f_v}{J} \Omega \\ -\frac{R_s}{L_d} i_d + p \frac{L_q}{L_d} \Omega i_q \\ -p \frac{\phi_f}{L_q} \Omega - p \frac{L_d}{L_q} \Omega i_d - \frac{R_s}{L_q} i_q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \quad (7)$$

### 3. Sliding's Mode control for Brushless servo motor

The system has two inputs which is  $V_d$  and  $V_q$ . torsi count (4) point out that inductances' difference ( $L_d - L_q$ ) play one role on system if current  $i_d$  zero unlike. Strategy that enables to control brushless servo's position or speed motor via sliding's method mode is make  $i_d$  as zero. Besides this strategy current rule pock phase.  $i_d$  and  $i_q$  restrained separately. Current  $i_d$  and  $i_q$  determined lies loop's outboard so sliding is mode happens on switching's surface  $s_i = 0$ . On generally  $i_d$  determined at the price zero, since  $i_d$  not result motor torsi, therefore motor don't depend  $i_d$ . [5]

Two surface sliding we can determine while two intent dots get concurrently been reached. That thing is said apropos while two self supporting entry is gotten which is  $V_d$  and  $V_q$ . That looking at  $x$  determine part of system,  $V$  determine one of part of  $V_d$  or  $V_q$  and  $S(x)$  are surface sliding. Sliding control mode can be reached if available law controls with condition  $s\dot{S} < 0$ .

$$V = V_{eq} + V_n \quad (8)$$

While  $V_{eq}$  are solution of  $\dot{S}(x, v) = 0$ .  $V_{eq}$  will can control while  $S(x) = 0$  and so-called equivalent control in standard terms. Design of  $V_{eq}$  on equation (8) rather doctrinaire: more tend to exact linearization from output  $S(x)$  with one pole place (*pole placement*) to provenance.  $V_n$  designed for meeting situation condition *trajectory* to pull to surface  $S(x) = 0$ . Controls problem

in common be divide into two subs about problems which is solved wholly independently. It becomes maybe while  $i_d$  made as zero and will make a abode little in many situation. Sliding mode successful control  $i_d$ .

#### 4. Direct Current Method ( $i_d$ )

Sliding Surface:

$$S_i = i_{d,ref} - i_d = -i_d \quad (9)$$

$$\dot{S}_i = -\dot{i}_d \quad (10)$$

Substitute (7) and (10)

$$\dot{S}_i = \frac{R_s}{L_d} i_d - p\Omega \frac{L_q}{L_d} i_q - \frac{V_d}{L_d} \quad (11)$$

To calculate  $V_{eq}$ , we solve equation  $S_i = 0$

$$V_{d,eq} = R_s i_d - p\Omega L_q i_q \quad (12)$$

With  $V_d = V_{d,eq} + V_{d,n}$  sliding mode condition  $S_i \dot{S}_i < 0$

$$\frac{-V_{dn}}{L_d} S_i < 0 \quad (13)$$

Then

$$V_d = V_{d,eq} + K_d \text{sign}(S_i) \quad (14)$$

Where  $K_d$  is real positive constant.  $K_d$  taken according to point  $V_{d,eq}$  and for point what do fit for motor tension entry.

#### 5. Speed Control Method

$\Omega_{ref}$  constituting constant reference of speed rotation.  $S_v = \Omega_{ref} - \Omega$  wide of the mark

for sliding's surface definition while input is not explicit ala lay in  $\dot{S}$ . Define back  $S_v = K_v e_v + \dot{e}_v$  with  $e_v = \Omega_{ref} - \Omega$  and with  $K_v$  positive regular reality results

$$\dot{S}_v = -K_v \Omega + \frac{d}{dt} \left( \frac{-f_v \dot{\Omega}}{J} + \frac{p[(L_d - L_q)i_d + \phi_f]}{J} i_q - \frac{C_L}{J} \right) \quad (15)$$

In consequence

$$V_{q,eq} = \frac{JL_q}{p[(L_d - L_q)i_d + \phi_f]} \left( \frac{f}{J} - K_v \right) \dot{\Omega} + p\Omega \phi_f + p\Omega L_d i_d + R_s i_q \quad (16)$$

$V_q = V_{q,eq} + V_{q,n}$  condition of  $S_v \dot{S}_v < 0$  declare

$$-\frac{V_{q,nP} [(L_d - L_q)i_d + \phi_f]}{JL_q} < 0 \quad (17)$$

and

$$V_q = V_{q,eq} + K_q \text{sign}(S_v) \quad (18)$$

$K_q$  is real constant positive.

## 6. Sliding's Mode application

Surface on Sliding mode is prescribed by appreciative  $\epsilon$ . Meanwhile  $e$  pointing out factor's deviation speed (velocity error). Speeds bias factor can be set down of equation:

$$e = \Omega_{ref} - \Omega \quad (19)$$

Working out on equation 4.1 can be figured on fasa's area ( $\dot{e}, e$ ) while methods to conduct equivalent is applied on *boundary layer* ( $-\epsilon < S < \epsilon$ ). Boundary layer sliding's explanation this mode is pointed out. On sliding surface, speed deviation is gotten of equation:

$$\dot{e} = K_v \times e \quad (20)$$

Speed factor deviation can form asymptote that approach zero if  $K_v > 0$  and system will wend *dot* of fasa's area as on Figure no2. Price  $K_v$  causing speed as convergent. Coefficient  $K_q$  determined as  $K_v$ . On simulation by use of Matlab is determined score for  $K_v = 1000$ ,  $K_q = 24$ ,  $K_d = 500$  and  $\epsilon = 0$ . Robustness analysis of controller is still was taken into account since not yet available point change to rotor flux ( $\Phi_r$ ) and Stator prisoner ( $R_s$ ).

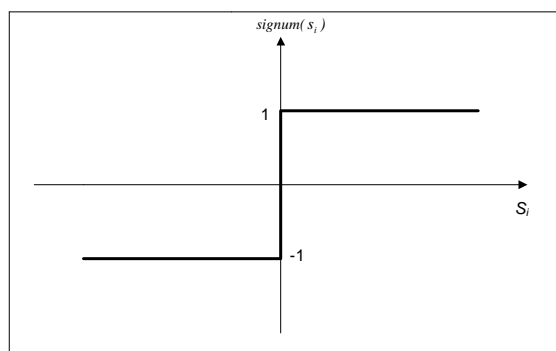


Figure 1. signum ( $s_i$ ) function

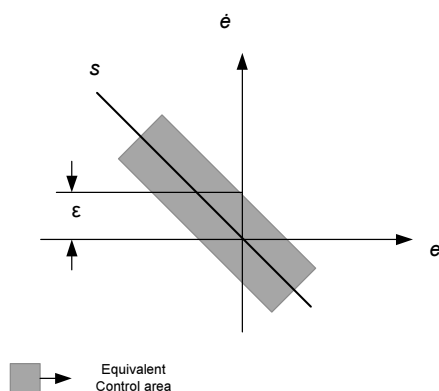


Figure 2. Sliding Modes: *Boundary Layer*

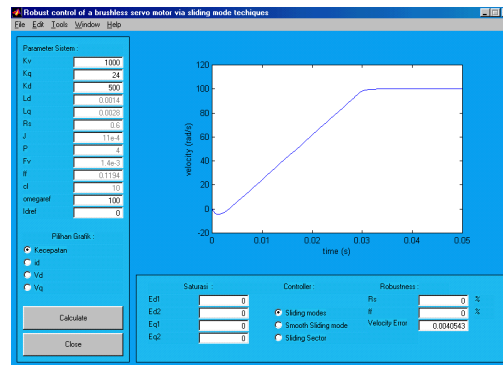
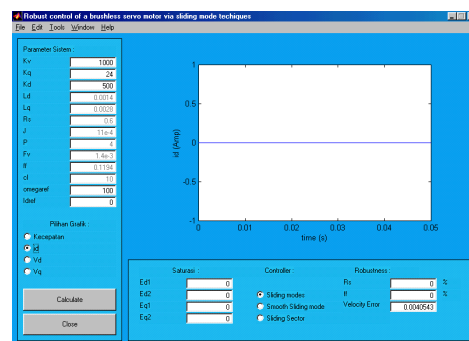
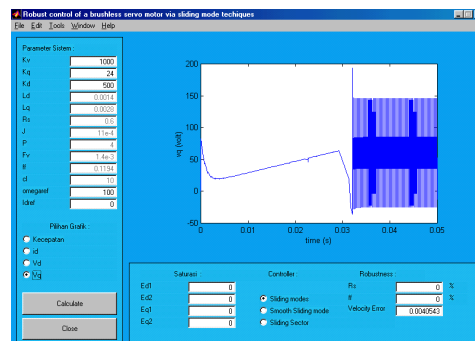
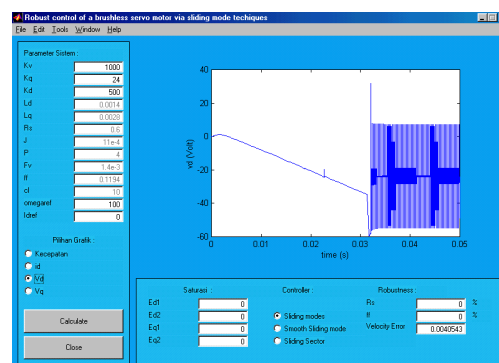


Figure 3. Sliding Modes: Speed response to time

Figure 4. Sliding Modes:  $i_d$  response to timeFigure 5. Sliding Modes:  $V_q$  t response to timeFigure 6. Sliding Modes:  $V_d$  response to time

Simulation result appears that marks sense function *signum* on controller will cause effect *chattering* on response  $V_d$  and  $V_q$ . To avoid phenomenon *chattering* this, function *signum* can be substituted by one ala one function continue approximate.

## 7 Smooth Sliding Modes

Signum's function as upon can evoke effect '*chattering*', which is a phenomenon where happens changing control with frequency that very tall while trajectory available surface around sliding and while sign's price often arbitrary. To avoid this *chattering* phenomenon, function *signum* can be substituted by one ala one function continue approximate. Method conducts this was gotten by replaces function *sign* ( $s_i$ ) with function *sign<sub>2</sub>* one that constitute sat's function *sat*( $s_i$ ). On fasa's area ( $\dot{e}, e$ ), boundary layer becomes like on Figure 8.

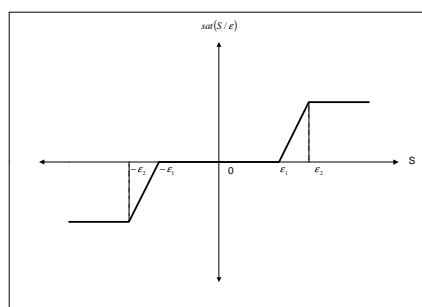


Figure 7. Smooth sliding mode with  $\text{sat}(s_i/\epsilon)$  function

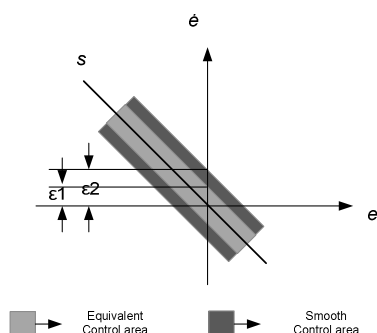


Figure 8. Smooth Sliding Modes: Boundary layer

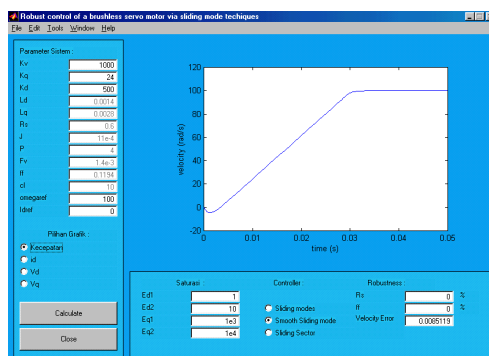
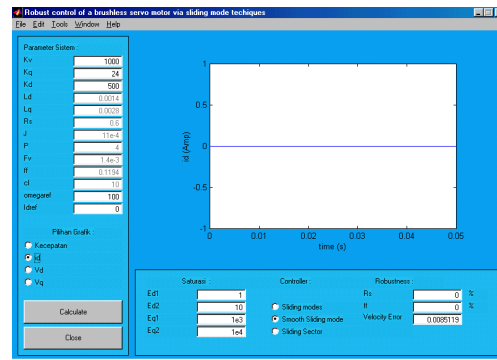
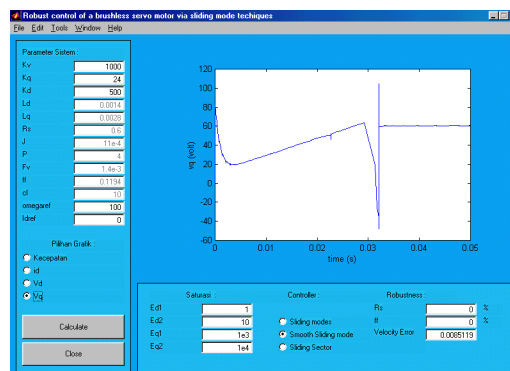
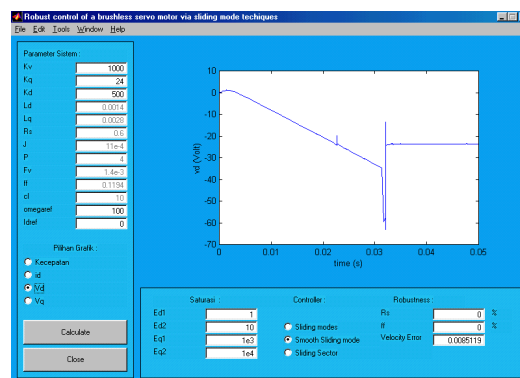


Figure 9. Smooth Sliding Modes: Speed response to time

Figure 10. Smooth Sliding Modes:  $i_d$  response to timeFigure 11. Smooth Sliding Modes:  $V_q$  response to timeFigure 12: Smooth Sliding Modes:  $V_d$  response to time

On sign's simulation 2, price for unchanged parameter from sign1 which is:  $\Omega_{ref} = 100$  rad seconds,  $i_{dref} = 0$  ampere,  $K_v = 1000$  and  $K_q = 24$ . Score of  $\epsilon_{d1}$ ,  $\epsilon_{d2}$ ,  $\epsilon_{q1}$  and  $\epsilon_{q2}$  determined to reduce phenomenon *chattering*. Appreciative elect for  $\epsilon_{d1} = 1$ ,  $\epsilon_{d2} = 10$ ,  $\epsilon_{q1} = 10^3$ ,  $\epsilon_{q2} = 10^4$  show that effect *chattering* on graph  $V_d$  and  $V_q$  can be reduced. Graph to respond speed unchanged to responds from first method (*sign1*).

## 8. Robustness Test

Study robustness is addressed to measure in as much as which tech sturdiness conducts that is used to parameter or perturbation change. After gets method to conduct for nominal situation, applied by that method to system already being given by variation.

Tested did by gives variation to flux appreciative change rotor ( $\Phi_f$ ) and stator prisoner ( $R_s$ ). Robustness of controller is analyzed by use of  $s_{ign_2}$ , since that controller response gets minimize effect *chattering*.

On robustness's examination change assesses rotor flux ( $\Phi_f$ ) and stator prisoner ( $R_s$ ) are as follows; changing point s rotor flux ( $\Phi_f$ ) as  $\pm 10\%$  from point nominal ( $\Phi_f = 0.1194$  Wb)

- Changing stator prisoner point ( $R_s$ ) as big as 50% of face value ( $R_s = 0.6 \Omega$ ). Face value ascension  $R_s$  because of ascension temperature motor Determined by parameter  $\Omega_{ref}$ ,  $i_{d_{ref}}$ ,  $K_v$ ,  $K_q$ ,  $K_d$ ,  $\varepsilon_{d1} = 1$ ,  $\varepsilon_{d2} = 10$ ,  $\varepsilon_{q1} = 10^3$  dan  $\varepsilon_{q2} = 10^4$ .

Shown that response of flux point change rotor is as follows:

- For rotor flux change ( $\Phi_f$ ) as big as + 10% will be shown that controller will hasten motor speed response. Constant controller *robust* while is face value  $R_s$  raised as big as 50%
- For rotor flux change ( $\Phi_f$ ) as big as 10% will be shown that controller will slow motor speed response. Constant controller *robust* while is face value  $R_s$  raised as big as 50%.

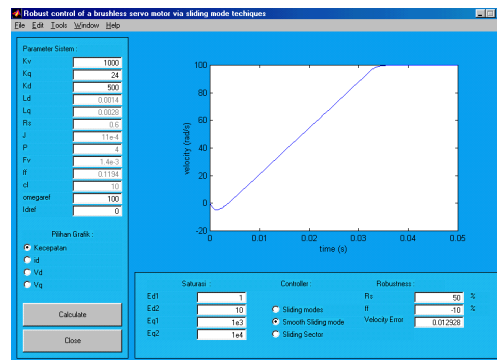


Figure 13. Speed robustness study with rotor flux change as big as +10%

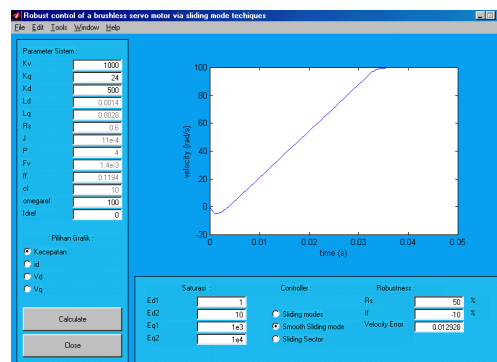


Figure 14. Speed robustness - with rotor flux change as big as 10%

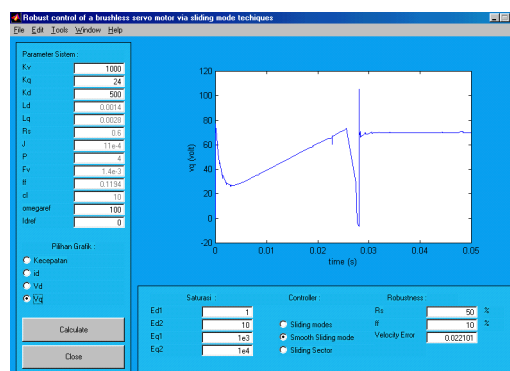
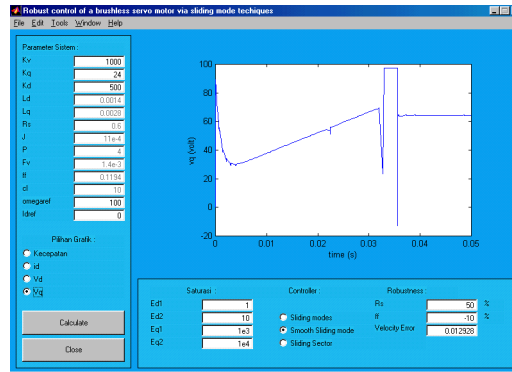
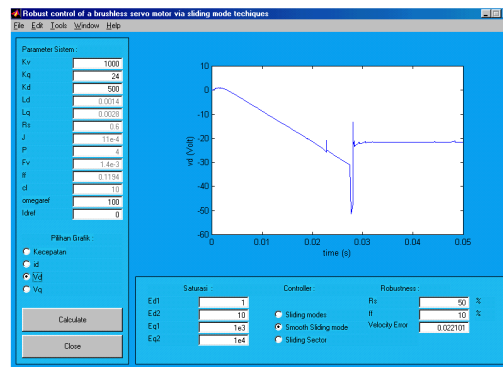
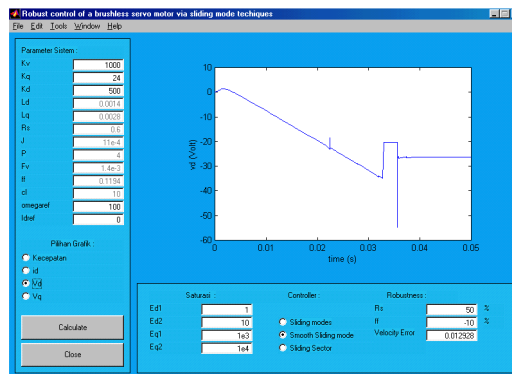


Figure 15.  $V_q$  response to time with changed rotor flux +10%



Figure 16.  $V_q$  response to time with changed rotor flux -10%Figure 17.  $V_d$  to time with changed rotor flux +10%Figure 18.  $V_d$  response to time with changed rotor flux -10%

## 9. Conclusion

Strategy to control position or speed motor servo without brush via tech sliding mode is make  $i_d$  equal to zero. Current for  $i_d$  determined at the price zero, since  $i_d$  doesn't result torsi motor, so motor not depends  $i_d$ .

1. Simulation result appears that prescribed speed accords at the price reference which be determined in the period of less than 0.04 second. There is effect even *chattering* on sliding mode can be reduced by replaces function  $\text{sign}(s_i)$  with function  $\text{sat}(s_i)$  or function  $\text{sat}(s_i/\epsilon)$ .
2. sliding controller mode can be utilized for system multivariable which is by restrains  $V_d$  and  $V_q$  separately
3. Resulting control systems have robust character to parameter and perturbation variation (trouble), which is change to rotor flux and stator prisoner. Flux face value ascension rotor will hasten speed response and flux face value decrease rotor will slow speed response..

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