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Study of Dzarrah: Analysis of Wave Functions Potential Non Central Coulombic Rosen Morse

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Abstract

It has been obtained a graph of the wave function of Coulombic's non-central potential Schrodinger equation Rosen Morse which is systematically an embodiment of hydrogen atoms and their isotopes. Schrodinger's equation is solved through variable spatial techniques with each variable resolved using a different method so that the radial and angular part wave functions are visualized by Matlab-based computer programming. Visualization of the radial wave function that is formed describes the probability value of the discovery of electrons which are small parts of an atom, while the visualization of the formed angular wave function shows the movement of electrons in an energy level. The mention of the atom itself is already in the Qur'an, Allah says in surah As-Saba verse 3. According to the interpretation of Quraish Shihab the word dzarrah in Arabic refers to a very small object, the size of an ant's child or fine dust. The phrase mitsqalu dzarrah in this verse means the weight of an atom. This implies the presence of a compound whose specific gravity is lighter than an atom.

Keywords: *non central Coulombic Rosen Morse, dzarrah.*

Introduction

Research has been carried out on various things from the beginning of the development of science to the present. The virtue of doing research and understanding of all things that occur are mentioned in the Quran, found in the verse Surah Al-Imran 190 which reads:

إِنَّ فِي خَلْقِ السَّمَاوَاتِ وَالْأَرْضِ وَاخْتِلَافِ اللَّيْلِ وَالنَّهَارِ
لَآيَاتٍ لِأُولِي الْأَلْبَابِ

Meaning: "*Indeed, in the creation of heaven and earth, and the alternation of night and day there are signs (greatness of Allah) for those who have reason*" (Team Translator, 2004).

Research arises from curiosity and ideas for various phenomena that occur both from simple to complex. The various objects of categorized research are very diverse especially in physics, both in very large sizes such as planets, stars, to sizes exceeding the solar system, to very small sizes such as particles, atoms, and other elements. Allah says in surah Yunus verse 61:

وَمَا تَكُونُ فِي شَأْنٍ وَمَا تَتْلُو مِنْهُ مِنْ قُرْآنٍ وَلَا تَعْمَلُونَ
مِنْ عَمَلٍ إِلَّا كُنَّا عَلَيْكُمْ شُهُودًا إِذْ تُفِيضُونَ فِيهِ ۚ وَمَا
يَعْزُبُ عَنْ رَبِّكَ مِنْ مِثْقَالِ ذَرَّةٍ فِي الْأَرْضِ وَلَا فِي
السَّمَاءِ وَلَا أَصْغَرَ مِنْ ذَٰلِكَ وَلَا أَكْبَرَ إِلَّا فِي كِتَابٍ مُبِينٍ

Meaning: "*You are not in a situation and do not read a verse from the Qur'an and you are not doing a job, but We are witnesses of you when you do it. Do not escape the knowledge of your Lord even though it is as big as zarrah (atom) on earth or in the sky. There is nothing smaller and no (also) greater than that, but (all recorded) in a real book (Lauh Mahfuzh)* "(Anonim, 2004).

According to Ibn Kathir's interpretation of the two verses above is about how Allah tells His prophet that Allah always knows all the circumstances and events of His people and all beings in every second of time. There is nothing hidden from His knowledge of things as large as an atom in heaven and on earth, and nothing is smaller or greater than that, except that everything is recorded in the real book. Ibn Kathir's interpretation of the two verses above can be understood and leads to a conclusion. Research is a nature that is carried out by humans as beings who have the mind in order to fulfill God's commands and also in order to find the good and welfare of human civilization both for individuals and for the whole of humanity. Everything in the universe is under God's supervision and is an ordinance, so that humans are easily facilitated in learning, researching, and gaining knowledge of what is in the universe.

The mention of heaven and earth can be analogous to how the object of research. Unlimited sky can be regarded as a macroscopic object consisting of very large components such as stars, planets, solar systems and various physical properties. The earth which is a stretched and very small stretch compared to the sky can be considered a microscopic object like a quantum object. The alternation of day and night illustrates the dynamics of the universe and the mention of dzarrah also leads to microscopic physical phenomena.

Based on the history of physics, after there were two important sectors in the form of Einstein's theory of relativity and the achievement of experimental methods at the level of atomic and subatomic structures, physics which was originally known as classical physics entered a new phase of modern physics, also known as quantum physics. Based on the hypothesis proposed by de Broglie regarding wave-particle duality, Schrodinger built the theory of wave mechanics which includes the movement of microscopic particles known as wave functions. Wave function is a mathematical equation that describes one or all of the conditions of a particle from an isolated quantum system, also shows the wave properties of particles. In general, the wave function of a system can be expressed in various physical quantities such as momentum, position, energy and so on. The wave function contains probability amplitude that displays the possibility of finding particles from a system.

Research on the completion of wave functions in the Schrodinger equation is a very important in modern physics. Various methods of solving the Schrodinger equation for the motion of charged particles at central and non-central potentials with a vector potential or an integral scalar potential have been developed. The various methods that have been developed include the Supersymmetry method, the method of horrorization, the shape invariant method, the Nikiforov-Uvarov (NU) method, and the Romanovski polynomial (Yanuarief et al. 2012).

This research focuses on the completion of the wave function of an electron moving inside Coulombic Rosen Morse non-central potential which is a combination of Coulomb potential and potential non-central Rosen Morse. The 3-dimensional Schrodinger equation is separated into 3 variables, namely the radial part, the polar part and the azimutal part. The radial part is solved by the NU method and in the polar part is solved by the Romanovski polynomial.

The Basic Theory of Nikiforov-Uvarov (NU)

The general equations of Nikiforov-Uvarov are stated as follows (Bekdemir, 2012),

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0, \quad (1)$$

with,

$$\psi(s) = \varphi(s)\chi(s), \quad (2)$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ a are order 2 polynomials, and $\bar{\tau}(s)$ are order 1 polynomials.

From equation (1), the equation for hypergeometry is obtained,

$$\sigma(s)\chi''(s) + \tau(s)\chi'(s) + \lambda\chi(s) = 0, \quad (3)$$

where $\varphi(s)$ is defined as,

$$\frac{\varphi\prime(s)}{\varphi(s)} = \frac{\pi(s)}{\sigma(s)}, \quad (4)$$

and section $\chi(s)$ is a function of hypergeometry with the completion of polynomials using the Rodrigues equation,

$$\chi_n(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}\left(\sigma^n(s)\rho(s)\right), \quad (5)$$

where B_n is the normalization constant with the weight factor $\rho(s)$, fulfilling the condition,

$$\frac{d}{ds}\left(\sigma(s)\rho(s)\right) = \tau(s)\rho(s), \quad (6)$$

The function $\pi(s)$ and parameter λ are needed for the NU method which is defined as,

$$\pi(s) = \frac{\sigma\prime - \bar{\tau}}{2} \pm \sqrt{\left(\frac{\sigma\prime - \bar{\tau}}{2}\right)^2 - \bar{\sigma} + k\sigma}, \quad (7)$$

$$\lambda = k + \pi'(s). \quad (8)$$

Basic Theory of the Romanovski Polynomials

The general equation for hypergeometry is stated as follows (Yanuarief et al. 2012),

$$\sigma(x)\frac{d^2 y_n(x)}{dx^2} + \tau(x)\frac{dy_n(x)}{dx} + \lambda_n y_n(x) = 0, \quad (9)$$

with, $\sigma(x) = ax^2 + bx + c; \quad \tau = dx + e,$

$$\frac{d(\sigma(x)w(x))}{dx} = \tau(x)w(x), \quad (10)$$

weighting factors obtained from solving Pearson differential equations, i.e.

$$w(x) = (1 + x^2)^{-p} e^{q tan^{-1}(x)}, \quad (11)$$

$$P(\theta) = g_n(x) = (1 + x^2)^{\frac{\beta}{2}} e^{\frac{-\alpha}{2} tan^{-1} x} R_n^{(-\beta,-\alpha)}(x), \quad (12)$$

Differential equations are fulfilled by Romanovski polynomials, i.e.

$$(1 + x^2) \frac{\partial^2 R_n^{(p,q)}(x)}{\partial x^2} + \{2x(-p + 1) + q\} \frac{\partial R_n^{(p,q)}(x)}{\partial s} - \{n(n-1) + 2n(1-p)\} R_n^{(p,q)}(x), \quad (13)$$

$$R_n^{(p,q)}(x) = R_n^{(-\beta,-\alpha)}(x) = \frac{1}{w^{(-\beta,-\alpha)}(x)} \frac{d^n}{dx^n} \{(1 + x^2)^n w^{(-\beta,-\alpha)}(x)\}. \quad (14)$$

Determine the Wave Function

Non-central potential which is a combination of Coulomb potential and potential non-central Rosen Morse is

$$V(r, \theta) = \frac{-e^2}{r} + \frac{\hbar^2}{2mr^2} \left(\frac{\nu(\nu+1)}{\sin^2 \theta} - 2\mu \cot \theta \right). \quad (15)$$

with $\nu > 0, \mu > 0$

he Schrodinger 3D equation for non-central potentials in equation (5) is written as follows,

$$-\frac{\hbar^2}{2m} \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 sin\theta} \frac{\partial}{\partial \theta} \left(sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 sin^2\theta} \frac{\partial^2}{\partial \varphi^2} \right) \psi(r, \theta, \varphi) + \left\{ \frac{-e^2}{r} + \frac{\hbar^2}{2mr^2} \left(\frac{\nu(\nu+1)}{sin^2\theta} - 2\mu cot\theta \right) \right\} \psi(r, \theta, \varphi) = E\psi(r, \theta, \varphi). \quad (16)$$

From equation (16) is obtained,

$$\frac{1}{R} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{2mr^2}{\hbar^2} \frac{e^2}{r} + \frac{2mr^2}{\hbar^2} E = \lambda, \quad (17a)$$

$$-\frac{1}{\Upsilon sin\theta} \frac{\partial}{\partial \theta} \left(sin\theta \frac{\partial}{\partial \theta} \right) - \frac{1}{\phi sin^2\theta} \frac{\partial^2}{\partial \varphi^2} + \left(\frac{\nu(\nu+1)}{sin^2\theta} - 2\mu cot\theta \right) = \lambda, \quad (17b)$$

with λ is a separation constant with the value $\lambda = l(l + 1)$ for the motion of electrons in the Coulomb potential.

While solving the wave equation in the azimuth section is obtained from equation (17b), namely:

$$\phi = A_m e^{im\varphi}, \quad (18)$$

with A_m is the normalization constant and $m = \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

Completion of the radial part of the Schrodinger equation

From the Schrodinger equation the radial part in equation (17a) can be rewritten as,

$$\frac{\partial^2 R}{\partial r^2} + \frac{2}{r} \frac{\partial R}{\partial r} + \left(\frac{\beta^2}{r} - \epsilon^2 - \frac{l(l+1)}{r^2} \right) R = 0, \quad (19)$$

with,

$$\frac{2m}{\hbar^2} E = -\epsilon^2, \frac{2me^2}{\hbar^2} = \beta^2, \quad (20)$$

The completion of the radial part wave function uses the NU equation (7), where,

$$\pi = \left(\frac{\sigma' - \tilde{\tau}}{2} \right) \pm \sqrt{(\frac{\sigma' - \tilde{\tau}}{2})^2 - \tilde{\sigma} + k\sigma} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + l(l+1) - \beta^2 r + \epsilon^2 r^2 + kr}, \quad (21)$$

$$\pi = -\frac{1}{2} \pm \epsilon \left\{ r + \frac{k - \beta^2}{2\epsilon^2} \right\}, \quad (22)$$

and, $(k - \beta^2)^2 - 4(\epsilon^2) \left(l + \frac{1}{2} \right)^2 = 0$,
so that it is obtained,

$$k = \beta^2 \pm 2\epsilon \left(l + \frac{1}{2} \right). \quad (23)$$

To fulfill the completion of a bound state condition, then,

$$\pi = -\epsilon r - \frac{2\epsilon\left(l + \frac{1}{2}\right)}{2\epsilon} - \frac{1}{2} \text{ for, } k = \beta^2 + 2\epsilon \left(l + \frac{1}{2} \right). \quad (24)$$

So that the energy equation is obtained, namely,

$$E_n = -\frac{m_e e^4}{2\hbar^2 (n_r + l + 1)^2}, \quad (25)$$

with n_r is a radial quantum number, $n = n_r + l + 1$. $n = 1,2,3, \ldots$.. n is the main quantum number, l is an orbital quantum number, $l = 0,1,2, \ldots, n-1$.

To determine the equation for a radial wave, the solution of equation (5) is used with, $\rho = r^{(2l+1)} e^{-2\epsilon r}$

$$y_{n_r}(r) = \frac{C_{n_r}}{\rho(r)} \frac{d^{n_r}}{dr^{n_r}} \left(\sigma^{n_r}(r) \rho(r) \right), \quad (26)$$

$$y_{n_r}(r) = C_{n_r} r^{-(2l+1)} e^{2\epsilon r} \frac{d^{n_r}}{dr^{n_r}} ((r)^{(2l+1)+n_r} e^{-2\epsilon r}). \quad (27)$$

Equation (26) shows the Rodrigues equation with the Laguerre polynomial associated,

$$y_{n_r}(r) = L_{n_r}^{2l+1}(r),$$
$$R(r) = \phi y_{n_r}(r) = B_n r^l e^{-\epsilon r} L_{n-l-1}^{2l+1}(r). \quad (28)$$

Completion of the Angular Schrodinger equation

From the Schrodinger equation the angular part in equation (17b) can be rewritten as,

$$\frac{\partial^2 \Upsilon(\theta)}{\partial \theta^2} + cot\theta \frac{\partial \Upsilon(\theta)}{\partial \theta} - \left(\frac{\nu(\nu+1)+m^2}{sin^2\theta} - 2\mu cot\theta\right)\Upsilon(\theta) + \lambda \Upsilon(\theta) = 0, \quad (29)$$

with variable substitution $\cot\theta = x$ in equation (29), obtained:

$$(1+x^2)\frac{\partial^2 \Upsilon}{\partial x^2} + x\frac{\partial \Upsilon}{\partial x} - \left\{(\nu(\nu+1)+m^2) - \frac{2\mu x}{(1+x^2)} - \frac{\lambda}{(1+x^2)}\right\}\Upsilon = 0. \quad (30)$$

By comparing equations (30) and (9), general solutions are obtained:

$$\Upsilon(\theta) = g_{n_l}(x) = (1+x^2)^{\frac{\beta}{2}} e^{\frac{-\alpha}{2} tan^{-1} x} R_{n_l}^{(-\beta,-\alpha)}(x). \quad (31)$$

If equation (31) is substituted to equation (13), it is obtained,

$$(1+x^2)\frac{\partial^2 \Upsilon}{\partial x^2} + \{x(2\beta+1) - \alpha\}\frac{\partial \Upsilon}{\partial x} - \{\nu(\nu+1)+m^2 - \beta^2 - \beta + \beta\}\Upsilon = 0. \quad (32)$$

Equation (31) is then compared with equation (4), obtained,

$$\alpha_{n_l} = \frac{2\mu}{\beta - \frac{1}{2}} = -\frac{2\mu}{\sqrt{\nu(\nu+1)+m^2} + n_l + \frac{1}{2}}, \quad (33)$$

$$\beta_{n_l} = -\sqrt{\nu(\nu+1)+m^2} - n_l, \quad (34)$$

$$l + \frac{1}{2} = \sqrt{\left(\sqrt{\nu(\nu+1)+m^2} + n_l + \frac{1}{2}\right)^2 - \frac{\mu^2}{\left(\sqrt{\nu(\nu+1)+m^2} + n_l + \frac{1}{2}\right)^2}}, \quad (35)$$

from equations (33), (34) and (35), the angular wave function can be obtained through equation (31), and obtained general equations,

$$w^{(-\beta,-\alpha)} = (1+x^2)^{\beta n_l} \cdot e^{-\alpha n_l \tan^{-1} x}, \quad (36)$$

$$R_{n_l}^{(-\beta,-\alpha)} = \frac{1}{(1+x^2)^{\beta n_l} \cdot e^{-\alpha n_l \tan^{-1} x}} \frac{d^{n_l}}{dx^{n_l}}\left[(1+x^2)^{n_l + \beta n_l} \cdot e^{-\alpha n_l \tan^{-1} x}\right], \quad (37)$$

$$\Upsilon_l^m = g(x) = (1+x^2)^{\frac{\beta n_l}{2}} \cdot e^{-\frac{\alpha n_l}{2}\tan^{-1} x} \cdot R_{n_l}^{(-\beta,-\alpha)}. \quad (38)$$

Result and Discussion

The following is the result of the wave function s equation of the wave function Coulombic Rosen Morse's non-central potential with value $n_r = 1, n_l = 0, m = 0$.

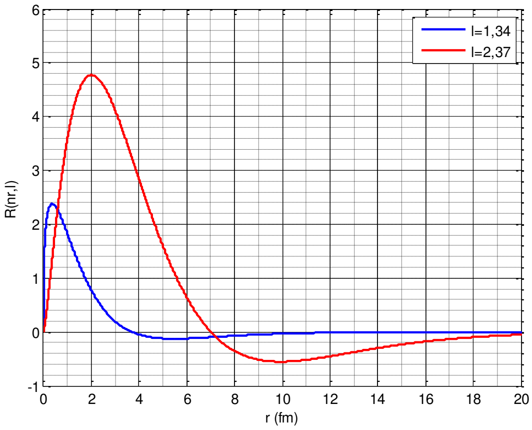

Figure 1. Radial Wave Function.

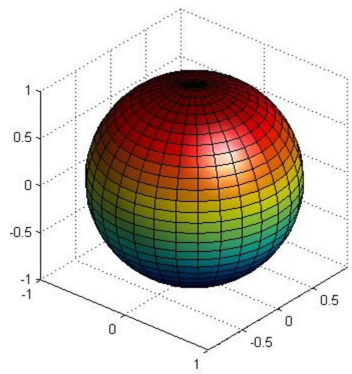

Figure 2. The function of the Angular Wave is not disturbed for $n_l = 0, m = 0$.

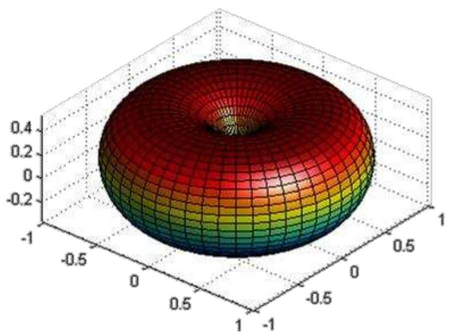

Figure 3. Interrupted Angular Wave Function $\nu = \mu = 1$ untuk $n_l = 0, m = 0$.

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In accordance with equations (16), (17a) and (17b) the wave function of the Schrodinger equation is a combination of radial and angular wave functions. The radial wave function shown in Figure 1 proves that for a system not disturbed, between the distances of $0 < r < 3$ fm there is the largest wave amplitude which means that there is an isolated electron in that area. Whereas the angular wave function shown in Figure 2 proves that the isolated electrons make a circular motion around the point $r = 0$ each time with the probability of rotation in the form of a spherical space. Electrons have a negative charge isolated in the area of $0 < r < 3$ fm and doing circular motion this means that there is an equality of forces between two opposing forces on the electron. The force that can work on electrons that are electrically charged is the electromagnetic force so that the other particles that affect it should be charged particles. Angular wave function shows that to produce circular motion of electrons around the point $r = 0$, then at point $r = 0$ there must be charged particles which are attracting electrons to point $r = 0$, then the particle must have a mass much greater than the electron and positively charged. These symptoms are events of atomic formation or dzarrah.

The mention of the atom itself is already in the Qur'an, Allah says in surah As-Saba verse 3:

وَقَالَ الَّذِينَ كَفَرُوا لَا تَأْتِينَا السَّاعَةُ ۖ قُلْ بَلَىٰ وَرَبِّي لَتَأْتِيَنَّكُمْ
عَالِمِ الْغَيْبِ ۖ لَا يَعْزُبُ عَنْهُ مِثْقَالُ ذَرَّةٍ فِي السَّمَاوَاتِ وَلَا
فِي الْأَرْضِ وَلَا أَصْغَرُ مِنْ ذَٰلِكَ وَلَا أَكْبَرُ إِلَّا فِي كِتَابٍ
مُبِينٍ

Meaning: "*And those who disbelieve say: 'The day to rise will not come to us'. Say: 'It must come, for the sake of my Lord who knows the unseen, surely the end will surely come to you. There is nothing hidden from Him that is in the heavens and on the earth, and there is no (also) smaller than that and greater, but it is in the real Book (Lauh Mahfuzh)* " (Anonim, 2004).

According to the interpretation of Quraish Shihab the word dzarrah in Arabic refers to a very small object, the size of an ant's child or fine dust. The phrase mitsqalu dzarrah in this verse means the weight of an atom. This interpretation implies the presence of a compound whose specific gravity is lighter than an atom. The development of knowledge especially in the field of science proves that atoms have certain elements.

Conclusion

Promising research in quantum physics continues with a variety of problem solving approaches. The results of this study provide confirmation of the Qur'anic verses that what has been said in the Quran come down from the past does not necessarily conflict with the established research findings.

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