

# On The Optimization of Weighted Sum Rate for MIMO Broadcast Channels

Vien Nguyen-Duy-Nhat\*, Mai T. P. Le, Hung Nguyen-Le

**Abstract**—In this paper, we propose a novel algorithm to optimize the weighted total system rate (WSR) for a broadcast communication system using multiple input-multiple output (MIMO) antenna technology based on the Harris Hawking Optimization (HHO) algorithm, called Algorithm 1, using the Minimum Mean Square Error (MMSE) filter at the mobile stations (MS). In addition, we also developed Algorithm 2 from Algorithm 1 to overcome the disadvantages of applying the traditional HHO algorithm to the MIMO system. Numerical results have been used to demonstrate the outperformance of the proposed algorithm, comparing with existing methods such as Block Diagonalization (BD) combined with Waterfilling (WF) and the Particle Swarm Optimization (PSO) method for any signal-to-noise ratio (SNR) value.

**Index Terms**—Harris Hawking Optimization (HHO); weighted sum rate (WSR); Broadcast MIMO; Particle Swarm Optimization (PSO); Minimum Mean Square Error (MMSE).



## 1. Introduction

IN addition to the ability to support extremely high data rates and capacity, broadcasting communications (BC) systems are anticipated to be heavily exploited for emerging communication models such as vehicle transport (V2X) owing to the system scalability with low latency [1]. Moreover, with the advantage of providing high reliability, low latency, BC system is becoming more and more important in the applications relying on Ultra-reliable and low-latency communication (URLLC), tactile Internet and Internet of Things (IoT), for e.g. in automated traffic, industrial control, and virtual reality [2], [3]. To meet these requirements, an indispensable technology is the multiple input multiple output antenna (MIMO) technology with the potential to significantly improve the spectrum efficiency and energy efficiency [4], [5]. In particular, MIMO techniques have been widely adopted in next-generation broadcast standards, including digital video broadcasting next generation handset (DVB-NGH) [6] and Advanced Television System Association (ATSC) 3.0 [7]. Therefore, recent research works have been investigated aiming to improve spectrum efficiency (SE) as well as weighted sum rate (WSR) for a multi-user MIMO broadcast network (BC-MIMO).

In this paper, we study an important scenario of a downlink BC system, in which a base station

(BS) equipped with multiple antennas sends different information streams to multiple mobile stations (MS), assuming that each MS is equipped with multiple antennas. Compared with classical point-to-point MIMO systems, the greatest challenge of a multi-user downlink BC system is how to eliminate co-channel interference at the MS.

In order to solve the problem of reducing co-channel interference and improving data transmission rate in the downlink multi-user BC-MIMO wireless network, a number of linear pre-coding algorithms have been proposed in recent studies such as: Dirty Paper Coding (DPC) [8-9], Block Diagonalization (BD) [10] or Particle Swarm Optimization (PSO) [11]. Although DPC has been proven to be the optimal method for multi-user BC-MIMO systems, this algorithm is very complex to implement in practice. On the other hand, the BD scheme combined with the classical Waterfilling (WF) algorithm [12] has been shown by the authors in [10] to be able to achieve suboptimal performance with lower complexity. Basically, BD is developed based on the Zero Forcing (ZF) precoding method, where the precoding vectors are designed to eliminate interference between users. Therefore, BD allows to increase the per MS rate and total rate by combining the advantages of space multiplexing and multi-user MIMO gain. However, for effective noise suppression, a constraint of the BD scheme is that the number of transmit antennas at the BS must be greater than the total number of receive antennas of all MSs [13]. This makes BD inapplicable to systems with many MSs or with BSs having a finite number of antennas. To maximize the sum rate of a BC-MIMO system, authors in [11] have applied the swarm optimization algorithm (PSO) for the optimization sum rate problem using the weighted sum rate (WSR). In particular, the PSO algorithm is used to find the precoding matrix at the BS while using

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the linear minimum mean error (MMSE) method to find the decoding matrix at the MS. Compared to the BD approach, the PSO method is more favorable in practice since it removes the constraint on the number BS antennas while improving the system performance.

Unlike the previous approaches, in this paper, we propose a new algorithm for solving the sum rate optimization problem of BC-MIMO system based on the advanced Harris Hawks Optimization (HHO) algorithm [14]. Since it first introduced by Heidari *et al.* [13], HHO has garnered a lot of attention from researchers due to its flexible structure and significant improved performance. The title of the HHO method originates from the algorithmic nature, where the idea of the algorithm is similar to the Harris hawk's chasing style in the wild with "surprise pounces" [14]. To evaluate the effectiveness of the proposed algorithm, we compare the performance of the BC MIMO system with the benchmark algorithms such as the WF algorithm combined with the BD method, called WF+BD, in [13] and the PSO algorithm in [11] via means of simulation. In the end, numerical results are used to demonstrate the superior performance of the proposed HHO-based algorithm over the existing approaches.

The rest of this paper is organized as follows. In Section II, we introduce the multi-user downlink BC-MIMO system model. Section III presents the WSR maximization approaches for BC-MIMO system. The proposed HHO-based algorithm is introduced in Section IV. In Section V, we apply the HHO algorithm to maximize the WSR for the proposed system. Numerical results are given via means of simulation in Section VI. Finally, Section VII concludes the paper.

*Notations:* We use the upper-case boldface letters for matrices and lower-case boldface letters for vectors.  $\mathbb{C}^{N \times M}$  represents the space of  $N \times M$  complex matrices and  $\mathbf{I}_M$  indicates an  $M \times M$  identity matrix.  $\mathbf{X}^H$ ,  $|\mathbf{X}|$ , and  $\text{tr}(\mathbf{X})$  stand for conjugate transpose, determinant and trace of a matrix  $\mathbf{A}$ , respectively.  $E[\mathbf{x}]$  denotes the expectation of vector  $\mathbf{x}$ . A complex Gaussian random vector variable  $\mathbf{z}$  with mean  $\mu$  and variance  $\sigma^2$  is represented as  $\mathbf{z} \sim \mathcal{CN}(\mu, \sigma^2)$ .

## 2. System model

Consider a multi-user BC-MIMO system model as depicted in Figure 1, where the BS is equipped with  $N_t$  transmitting antennas and serves  $K$  MS, each is equipped with  $N_r$  antennas. Let  $\mathbf{x}_k \in \mathbb{C}^{d \times 1}$  the signal transmitted by the BS to the  $k$ -th MS, where  $d$  is the number of transmitted data streams and  $E\{\mathbf{x}_k \mathbf{x}_k^H\} = \mathbf{I}_d$ . Prior to the transmission, the pre-coding process is carried out by multiplying the transmitted signal with the precoding matrix  $\mathbf{F}_k \in \mathbb{C}^{N_t \times d}$ . The BS transmit power is then determined as:

$$P = \mathbb{E} \left\{ \left\| \sum_{k=1}^K \mathbf{F}_k \mathbf{x}_k \right\|^2 \right\} = \sum_{k=1}^K \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) \leq P_{max}, \quad (1)$$

where  $P_{max}$  is the maximum BS transmit power.

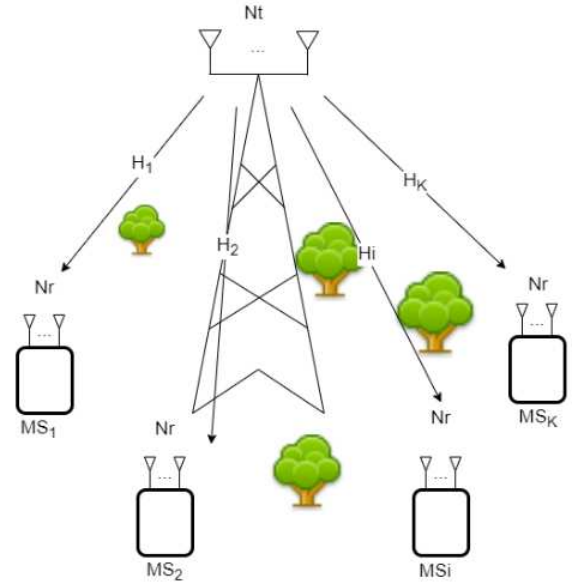


Fig. 1: Model of broadcast MIMO system.

Assume that the MIMO channels are flat fading, where the channel from the BS to the  $k$ -th MS is represented as  $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ . Since the BS transmits all signals to the MSs, each MS not only receives its desired signal, but also receives interference signals from other MSs. The received signal  $\mathbf{y}_k \in \mathbb{C}^{N_r \times 1}$  at the  $k$ -th MS can be expressed as follows:

$$\begin{aligned} \mathbf{y}_k &= \sum_{l=1}^K \mathbf{H}_k \mathbf{F}_l \mathbf{x}_l + \mathbf{n}_k \\ &= \mathbf{H}_k \mathbf{F}_k \mathbf{x}_k + \sum_{l=1, l \neq k}^K \mathbf{H}_k \mathbf{F}_l \mathbf{x}_l + \mathbf{n}_k \end{aligned}, \quad (2)$$

where the first component ( $\mathbf{H}_k \mathbf{F}_k \mathbf{x}_k$ ) is the desired signal of the  $k$ -th MS, while the second component ( $\sum_{l=1, l \neq k}^K \mathbf{H}_k \mathbf{F}_l \mathbf{x}_l$ ) is the inter-user interference (ISI), and  $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_k^2 \mathbf{I}_{N_r})$  is the additive Gaussian white noise vector with zero expectation and variance  $\sigma_k^2$ . In this paper, we adopt the block-fading channel model, where the channel response is divided into multiple coherence blocks, in which the channel response is considered stationary in a block and vary independently among blocks.

To recover the desired signal, the decoding operation is performed at the  $k$ -th MS by multiplying the received signal  $\mathbf{y}_k$  by the matrix  $\mathbf{W}_k \in \mathbb{C}^{N_r \times d}$ . The recovery signal is expressed as:

$$\begin{aligned} \hat{\mathbf{x}}_k &= \mathbf{W}_k^H \mathbf{y}_k \\ &= \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_k \mathbf{x}_k + \sum_{l=1, l \neq k}^K \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_l \mathbf{x}_l + \mathbf{W}_k^H \mathbf{n}_k \end{aligned}, \quad (3)$$

where the first component in the right-hand side ( $\mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_k \mathbf{x}_k$ ) is the desired signal after recovery, the second component ( $\sum_{l=1, l \neq k}^K \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_l \mathbf{x}_l$ ) is the ISI after recovery, and the third component ( $\mathbf{W}_k^H \mathbf{n}_k$ ) is the noise after recovery. The achievable rate of the  $k$ -th MS can be calculated as [8]:

$$R_k = \log_2 \left| \mathbf{I} + \mathbf{W}_k^H \mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{W}_k \mathbf{R}_{z_k}^{-1} \right|, \quad (4)$$

where  $\mathbf{R}_{zk} = \mathbf{W}_k^H \left( \sum_{l=1, l \neq k}^K \mathbf{H}_k \mathbf{F}_l \mathbf{F}_l^H \mathbf{H}_k^H + \sigma_k^2 \mathbf{I} \right) \mathbf{W}_k$  is the correlation matrix of interference and noise in formula (3). Thus, the total WSR is computed as:

$$R = \sum_{k=1}^K \omega_k R_k, \quad (5)$$

where  $\omega_k$  is the weighting factor, representing the power transmission priority for the  $k$ -th MS. The important task is how to design the pre- and post-coding matrices to get the maximum WSR.

### 3. WSR maximization approaches for BC-MIMO system

#### 3.1. Eliminate Inter-User Interference Using Block Diagonalization

An effective method to suppress ISI in a downlink MIMO system is Block Diagonalization (BD) [10]. The idea of this technique is to find the pre- and post-coding matrices which suppress the downlink ISI and thus maximize the channel capacity. More particular, the pre-coding matrix is designed to locate in the null-space of the equivalent channel matrix of the interference channels, that is:

$$\mathbf{H}_k \mathbf{F}_l = \mathbf{0}, \quad \forall k, \forall l \neq k. \quad (6)$$

Let the equivalent channel matrix of the channels that interfere with the  $k$ -th MS be:

$$\mathcal{H}_k = [\mathbf{H}_1^T, \dots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \dots, \mathbf{H}_K^T], \quad (7)$$

with  $\mathcal{H}_k \in C^{(K-1)N_r \times N_t}$ . Now, (6) becomes:

$$\mathcal{H}_k \mathbf{F}_k = \mathbf{0}, \quad \forall k. \quad (8)$$

This is equivalent to having the precoding matrix  $\mathbf{F}_k$  in the space of  $\mathcal{H}_k$ . Let  $\tilde{L}_k = \text{rank}(\mathcal{H}_k) \leq (K-1)N_r$ . Using the Singular Value Decomposition (SVD) analysis on  $\mathcal{H}_k$ , we obtain

$$\mathcal{H}_k = \mathbf{U}_k \Sigma_k \mathbf{V}_k^H, \quad (9)$$

where  $\Sigma_k$  is a diagonal matrix whose components on the diagonal are eigenvalues of  $\mathcal{H}_k$ ,  $\mathbf{U}_k$  and  $\mathbf{V}_k$  are unitary matrices. Let  $\mathbf{B}_k \in C^{N_t \times d}$  be the matrix consisting of  $d$  last columns in the matrix  $\mathbf{V}_k$ , then  $\mathbf{B}_k$  is in the nullspace of  $\mathcal{H}_k$ . This means if  $\mathbf{F}_k$  contains  $\mathbf{B}_k$ , the ISI will be eliminated. Let  $\bar{L}_k = \text{rank}(\mathbf{H}_k \mathbf{B}_k)$ , the following condition is satisfied:  $L_k + \tilde{L}_k - N_t \leq \bar{L}_k \leq \min\{L_k, \tilde{L}_k\}$  [10], with  $L_k = \text{rank}(\mathbf{H}_k)$ . Since then, the condition for existence of  $\mathbf{F}_k$  is  $N_t > (K-1)N_r$ . Continuing to apply SVD to the matrix  $\mathbf{H}_k \mathbf{B}_k$ , we achieve:

$$\mathbf{H}_k \mathbf{B}_k = \tilde{\mathbf{U}}_k \Lambda_k \tilde{\mathbf{V}}_k^H, \quad (10)$$

where  $\Lambda_k$  is a diagonal matrix containing the eigenvalues of  $\mathbf{H}_k \mathbf{B}_k$ . Let  $\mathbf{D}_k \in C^{d \times d}$  be the matrix containing  $d$  first columns  $\tilde{\mathbf{V}}_k$ . At this point, the precoding matrix is designed as such:

$$\mathbf{F}_k = \mathbf{B}_k \mathbf{D}_k \mathbf{P}_k^{1/2}, \quad (11)$$

where  $\mathbf{P}_k = \text{diag}(p_{k,1}, p_{k,2}, \dots, p_{k,N_r})$  is the power allocation diagonal matrix.

The post-encoding matrix  $\mathbf{W}_k$  is designed by selecting the first  $d$  columns of  $\tilde{\mathbf{U}}_k$ . Due to the orthogonal nature of the matrix, the achievable rate of the  $k$ -th MS from (4) is computed as:

$$R_{k, BD} = \sum_{i=1}^d \log_2 \left( 1 + \frac{\Lambda_k^2(i, i)}{\sigma_k^2} p_{k,i} \right), \quad (12)$$

where  $\Lambda_k(i, i)$  is the  $i$ -th component on the diagonal of the matrix  $\Lambda_k$  in (12).

#### 3.2. Maximize WSR using Waterfilling technique:

To maximize the WSR, it is necessary to find the power factors in the power distribution matrix  $\mathbf{P}_k$  by solving the following optimization problem:

$$P1: \quad \max_{p_{k,i}} \sum_{k=1}^K \sum_{i=1}^d \omega_k \log_2 \left( 1 + \frac{\Lambda_k^2(i, i)}{\sigma_k^2} p_{k,i} \right) \\ \text{s.t.} \quad \sum_{k=1}^K \sum_{i=1}^d p_{k,i} \leq P_{max} \quad (13)$$

We formulate Lagrangian function for P1 as:

$$\mathcal{L}(p_{k,i}, \lambda) = - \sum_{k=1}^K \sum_{i=1}^d \omega_k \log_2 \left( 1 + \frac{\Lambda_k^2(i, i)}{\sigma_k^2} p_{k,i} \right) \\ + \lambda \left( \sum_{k=1}^K \sum_{i=1}^d p_{k,i} - P_{max} \right), \quad (14)$$

where  $\lambda$  is the Lagrangian multiplier associated with the maximum transmit power constraint at the BS. The optimal solutions in (14) must satisfy the set of conditions KKT (Karush–Kuhn–Tucker) as follows:

$$- \frac{\omega_k \frac{\Lambda_k^2(i, i)}{\sigma_k^2}}{\left( 1 + \frac{\Lambda_k^2(i, i)}{\sigma_k^2} p_{k,i} \right) \ln(2)} + \lambda = 0 \\ \sum_{k=1}^K \sum_{i=1}^d p_{k,i} = P_{max} \\ \lambda \geq 0 \quad (15)$$

Solving the above conditions, we get the optimal power for each MS as:

$$p_{k,i} = \max \left( 0, \frac{\omega_k}{\lambda \ln 2} - \frac{\sigma_k^2}{\Lambda_k^2(i, i)} \right). \quad (16)$$

Finally, the optimal value  $\lambda$  can be found using the classical Waterfilling algorithm.

### 4. Harris Hawks Optimization (HHO) Algorithm

HHO is known as a gradient-free, swarm-based optimization algorithm with several active and time-varying phases of exploration and mining. HHO recently has increasingly attracted the interest of researchers thanks to its flexible structure, high performance and high quality results. The main idea of the designed HHO method is based on the cooperative behavior and chasing style of Harris hawks in the wild known as the "surprise pounce" [14]. The HHO algorithm can be briefly presented as follows:

## 4.1. Exploration phase

In this phase, the Harris hawk randomly perches on several locations and waits for prey (rabbit) to be detected based on two strategies. If we consider equal opportunities  $q$  for each strategy, we have:

$$X(t+1) = \begin{cases} X_1, q \geq 0.5 \\ X_2, q < 0.5, \end{cases} \quad (17)$$

where  $X_1 = X_{rand}(t) - r_1|X_{rand}(t) - 2r_2X(t)|$ ,  $X_2 = X_{rabbit}(t) - X_m(t) - r_3(LB + r_4(UB - LB))$ ,  $X(t+1)$  is the position vector of the hawk in the next iteration of  $t$ ,  $X_{rabbit}(t)$  is the position of the rabbit,  $X(t)$  is the vector of the current position of the hawk,  $r_1, r_2, r_3, r_4$  and  $q$  are random numbers in the range (0,1) and are updated in each iteration,  $LB$  and  $UB$  are the upper and lower bounds of the variables,  $X_{rand}(t)$  is the position of the randomly selected hawk from the current swarm and  $X_m$  is the average position of the present hawks

$$X_m(t) = \frac{1}{N} \sum_{i=1}^N X_i(t), \quad (18)$$

where  $X_i(t)$  is the position of the  $i$ -th hawk in the iteration  $t$  and  $N$  represents the total number of hawks in the swarm.

## 4.2. From exploration to mining

During this phase, the energy of the prey is computed by:

$$E = 2E_0 \left(1 - \frac{t}{T}\right), \quad (19)$$

where  $T$  is the maximum number of iterations and  $E_0$  is the initial energy of the prey.

## 4.3. Mining phase

### 4.3.1. Soft Enclosure

This behavior is modeled as:

$$X(t+1) = \Delta X(t) - E|JX_{rabbit}(t) - X(t)|. \quad (20)$$

$$\Delta X(t) = X_{rabbit}(t) - X(t), \quad (21)$$

where  $\Delta X(t)$  is the difference between the position vector of the prey and the current position in the iteration  $t$ ,  $J = 2(1 - r_5)$  with  $r_5$  being a random number in the range (0,1) which represents the magnitude of the random jump of the prey during the escape. The values of  $J$  change randomly during each iteration to simulate the nature of the rabbit's movements.

### 4.3.2. Hard Enclosure

During this period, the current position of the hawk is updated as follows:

$$X(t+1) = X_{rabbit}(t) - E|\Delta X(t)|. \quad (22)$$

### 4.3.3. Soft encirclement with surprise pounces :

To conduct a soft siege, hawks decide their next move based on the following rule:

$$Y = X_{rabbit}(t) - E|JX_{rabbit}(t) - X(t)|. \quad (23)$$

Assume that the hawk pounces on a jaw-based bait  $LF$  using the following rule

$$Z = Y + S \times LF(D), \quad (24)$$

where  $D$  is the size of the problem,  $S$  is a random vector of size  $1 \times D$  and  $LF$  is a levy flight function:

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{1/\beta}}, \quad \sigma = \left( \frac{\Gamma(1+\beta) \times \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2^{\frac{\beta-1}{2}}} \right)^{1/\beta}, \quad (25)$$

where  $u, v$  are random values in the range (0,1),  $\beta$  is a constant with the value of 1.5.

Therefore, the strategy to update the hawk's position during the soft encirclement phase can be implemented by

$$X(t+1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases}, \quad (26)$$

with  $Y, Z$  obtained from (25)-(26), while  $F(\cdot)$  is the objective function of the prey.

### 4.3.4. Hard siege with surprise pounces :

The following rule is to be followed in hard enclosure conditions:

$$X(t+1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases}, \quad (27)$$

with  $Y, Z$  obtained from (28) and (29).

To conduct a soft siege, hawks decide their next move based on (25) with the following rules:

$$Y = X_{rabbit}(t) - E|JX_{rabbit}(t) - X_m(t)| \quad (28)$$

$$Z = Y + S \times LF(D). \quad (29)$$

## 5. Application of HHO in maximizing WSR in multi-user downlink BC-MIMO system

For simplicity, we use a Wiener MMSE filter at each MS to recover the signal:

$$\mathbf{W}_k = \left( \sum_{l=1}^K \mathbf{H}_k \mathbf{F}_l \mathbf{F}_l^H \mathbf{H}_k^H + \sigma_k^2 I_{N_r} \right)^{-1} \mathbf{H}_k \mathbf{F}_k. \quad (30)$$

To find  $\mathbf{F}_k$ , we need to solve the following optimization problem:

$$P2 : \quad \max_{\{\mathbf{F}_k\}_{k=1}^K} \sum_{k=1}^K \omega_k R_k \quad (31)$$

$$s.t. \quad \sum_{k=1}^K \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) \leq P_{max}$$

$P2$  is a non-convex NP-hard optimization problem, which is typically difficult to solve. In this section, we will apply the HHO algorithm to find the optimal solution for  $P2$ .

Let  $\mathbf{X}_k = \mathbf{F}_k$  be the precoding matrix to be found using the HHO algorithm. This matrix can be computed based on the cost function of  $P2$ , where the optimal precoding matrix of the  $k$ -th MS is the position of the

$k$ -th rabbit  $\mathbf{X}_{rabbit_k}$ . We use the projection method [15] to solve the optimization problem with constraint P2 by defining the feasible region  $\mathcal{F}$  as follows:

$$\mathcal{F} = \left\{ \mathbf{X}_k = \{\mathbf{F}_k\}_{k=1}^K : \sum_{k=1}^K \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) \leq P_{max} \right\}. \quad (32)$$

If the rabbit's position  $\mathbf{X}_k$  is not in the feasible area  $\mathcal{F}$ , we project  $\mathbf{X}_k$  into  $\mathcal{F}$  follows:

$$\hat{\mathbf{X}}_k = \Pi(\mathbf{X}_k), \quad (33)$$

with the projection  $\Pi(\cdot)$  represented as:

$$\Pi(\{\mathbf{F}_k\}_{k=1}^K) = \arg \min_{\{\hat{\mathbf{F}}_k\}_{k=1}^K \in \mathcal{F}} \sum_{k=1}^K \|\hat{\mathbf{F}}_k - \mathbf{F}_k\|_F, \quad (34)$$

where  $\{\hat{\mathbf{F}}_k\}_{k=1}^K$  is the position belonging  $\mathcal{F}$  that is near  $\mathbf{X}_k$ . This is equivalent to the statement:

$$P3: \min_{\{\hat{\mathbf{F}}_k\}_{k=1}^K} \sum_{k=1}^K \text{tr} \left( (\hat{\mathbf{F}}_k - \mathbf{F}_k) (\hat{\mathbf{F}}_k - \mathbf{F}_k)^H \right) \quad (35)$$

*s.t.*  $\sum_{k=1}^K \text{tr}(\hat{\mathbf{F}}_k \hat{\mathbf{F}}_k^H) \leq P_{max}$

P3 is a convex optimization problem, so it can be solved by Lagrange multiplier method. Here the Lagrangian function is set up as follows:

$$\mathcal{L}(\{\hat{\mathbf{F}}_k\}_{k=1}^K, \mu) = \sum_{k=1}^K \text{tr} \left( (\hat{\mathbf{F}}_k - \mathbf{F}_k) (\hat{\mathbf{F}}_k - \mathbf{F}_k)^H \right) + \mu \left( \sum_{k=1}^K \text{tr}(\hat{\mathbf{F}}_k \hat{\mathbf{F}}_k^H) - P_{max} \right), \quad (36)$$

wherein the Lagrangian multiplier  $\mu$  associated with the maximum transmit power constraint at the BS. The optimal solution of P3 must satisfy the set of KKT conditions as follows:

$$\begin{aligned} (1 + \mu) \hat{\mathbf{F}}_k - \mathbf{F}_k &= 0 \\ \sum_{k=1}^K \text{tr}(\hat{\mathbf{F}}_k \hat{\mathbf{F}}_k^H) &= P_{max}, \quad k = 1, \dots, K. \\ \mu &\geq 0 \end{aligned} \quad (37)$$

Solving (37), we get the optimal solution for P3 with:

$$\hat{\mathbf{F}}_k = \sqrt{\frac{P_{max}}{\sum_{k=1}^K \text{tr}(\mathbf{F}_k \mathbf{F}_k^H)}} \mathbf{F}_k, \quad k = 1, \dots, K. \quad (38)$$

In summary, Algorithm 1 describes the application of the HHO algorithm to find the optimal pre-post-coder matrices WSR for the proposed multi-user BC-MIMO system.

In Algorithm 1, there is total six phases: two exploration phases and four mining phases. To find the optimal solution  $\mathbf{F}$ , Algorithm 1 executes randomly one of six phases. The existence of random variables in the exploration and mining phases reduces the convergence rate of HHO and reduces the ability to find the global optimal solution in the trials. Even if Algorithm 1 returns a successful result, the existence of random variables in the steps of exploration and mining phases can cause it to fall into the local optimization trap [16].

**Algorithm 1** Apply HHO to optimize WSR for multi-user BC-MIMO system

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1: Input:  $N, T, P_{max}, \mathbf{H}_k, \mathbf{v}_k, \omega_k$ , where  $\mathbf{v}_k$  is the
   precoding matrix computed based on BD+WF
2: Output: Precoding Matrix:  $\mathbf{W}, \mathbf{F}$ 
3: Initialization:  $t = 0, \mathbf{X}_k = \mathbf{v}_k$ 
4: while ( $t < T$ ) do
5:   Calculate  $Fitness1 = WSR$  of total hawks by (5)
6:   Assign  $\mathbf{F} = \mathbf{X}$ 
7:   for each  $k$ -th hawk do
8:     Update  $E_0$  and  $J$ 
9:     Update  $E$  using (19)
10:    if  $|E| \geq 1$  then
11:      Update  $\mathbf{X}_k$  by (17) combining with (38)
12:    else
13:      if ( $r \geq 0.5$ ) then
14:        if ( $|E| \geq 0.5$ ) then
15:          Update  $\mathbf{X}_k$  by (20) combining with
16:          (38)
17:        else
18:          Update  $\mathbf{X}_k$  by (22) combining with
19:          (38)
20:        end if
21:      else
22:        if ( $|E| \geq 0.5$ ) then
23:          Update  $\mathbf{X}_k$  by (26) combining with
24:          (38)
25:        else
26:          Update  $\mathbf{X}_k$  by (27) combining with
27:          (38)
28:        end if
29:      end if
30:    end for
31:    Calculate  $Fitness2 = WSR$  according to (5)
32:    if  $Fitness2 \geq Fitness1$  then
33:      Update  $\mathbf{F} = \mathbf{X}$ 
34:    end if
35:     $t = t + 1$ 
36:  end while
37: Return  $\mathbf{X}_{rabbit}$ 
38:  $\mathbf{F} = \mathbf{X}_{rabbit}$ 
39: Calculate  $\mathbf{W}$  according to (30)

```

---

To control the random variables of HHO, we rely on the Chaotic Harris hawks algorithm Optimization (CHHO) which is suggested in [16]. In the CHHO algorithm, 10 chaos maps are Chebyshev (CHHO1), Circle (CHHO2), Gauss (CHHO3), Iterative (CHHO4), Logistic (CHHO5), Piecewise (CHHO6), Sine (CHHO7), Singer (CHHO8), Sinusoidal (CHHO9), and Tent (CHHO10), which are incorporated into the HHO algorithm to tune the probe mechanism instead of using random variables.

In this work, we propose the following corrections while applying the CHHO algorithm:

$$E = 2C_1 E_0 \left( 1 - \frac{t}{T} \right), \quad (39)$$

$$X(t+1) = \begin{cases} CX_1, & q \geq 0.5 \\ CX_2, & q < 0.5 \end{cases} \quad (40)$$

where  $CX_1 = C_2 X_{rand}(t) - C_3 |X_{rand}(t) - 2C_4 X(t)|$ ,  $CX_2 = X_{rabbit}(t) - X_m(t) - C_5 (LB + C_6 (UB - LB))$ , with  $C_i, (i = 1, \dots, 5)$

being chaos maps according to one of the 10 maps mentioned above.

As such, chaos map are applied in the exploration phase to find the hawk locations. Meanwhile, the mining phase still follows the original HHO algorithm. In addition, the comparison of *Fitness2* and *Fitness1* to update *F* is done in the “**For**” loop instead of outside the “**For**” loop as in the original HHO algorithm. The proposed CHHO algorithm is presented in Algorithm 2.

---

**Algorithm 2** Applying CHHO to optimize WSR for multi-user multi-antenna broadcast system

---

```

1: Input:  $N, T, P_{max}, \mathbf{H}_k, \mathbf{v}_k, \omega_k$ , where  $\mathbf{v}_k$  is the
   precoding matrix in terms of BD+WF
2: Output: Pre- and Pos-Coding Matrix:  $\mathbf{W}, \mathbf{F}$ 
3: Initialization:  $t = 0, \mathbf{X}_k = \mathbf{v}_k$ 
4: while ( $t < T$ ) do
5:   Calculate  $Fitness1 = WSR$  of total hawks by (5)
6:   Assign  $\mathbf{F} = \mathbf{X}$ 
7:   for each  $k$ -th hawk do
8:     Create a chaos map  $C_i, i = 1, \dots, 5$ .
9:     Update  $E_0$  and  $J$ 
10:    Update  $E$  using (39)
11:    if ( $|E| \geq 1$ ) then
12:      Update  $\mathbf{X}_k$  by (40) combining with (38)
13:    else
14:      if ( $r \geq 0.5$ ) then
15:        if ( $|E| \geq 0.5$ ) then
16:          Update  $\mathbf{X}_k$  by (20) combining with (38)
17:        else
18:          Update  $\mathbf{X}_k$  by (22) combining with
19:          (38)
20:        end if
21:      else
22:        if ( $|E| \geq 0.5$ ) then
23:          Update  $\mathbf{X}_k$  by (26) combining with
24:          (38)
25:        else
26:          Update  $\mathbf{X}_k$  by (27) combining with
27:          (38)
28:        end if
29:      end if
30:      Calculate  $Fitness2 = WSR$  according to (5)
31:      if ( $Fitness2 > Fitness1$ ) then
32:        Update  $\mathbf{F} = \mathbf{X}$ 
33:      end if
34:    end for
35:     $t = t + 1$ 
36: end while
37: Return  $\mathbf{X}_{rabbit}$ 
38:  $\mathbf{F} = \mathbf{X}_{rabbit}$ 
39: Calculate  $\mathbf{W}$  based on (30)

```

---

### Complexity Analysis

We now evaluate the complexity of the proposed algorithms via big- $\mathcal{O}$  analysis. Since Algorithm 1 has three main phases, its complexity is computed as follows:

$$\mathcal{O}_1 = \mathcal{O}(N \times (T + T \times D + 1)). \quad (41)$$

In particular,

- The complexity of the initialization phase is  $\mathcal{O}(N)$ .
- The complexity of updating the positions of all hawks is  $\mathcal{O}(T \times N \times D)$ .

- The complexity of determining the best position is  $\mathcal{O}(T \times N)$ .

Herein,  $T$  is the maximum number of iterations,  $N$  is the size of the hawks, and  $D$  is the dimension of the problem.

Theoretically, the computational complexity  $\mathcal{O}_2$  of Algorithm 2 is of the same magnitude as that of Algorithm 1. Since only the control parameters are added as chaos maps instead of generating random values, this process does not add complexity to the algorithm, i.e.  $\mathcal{O}_2 = \mathcal{O}_1$ .

## 6. Numerical Results

In this section, numerical simulation results are provided to evaluate the performance of the proposed HHO algorithm for the design of pre-post-processing matrices in downlink multi-user BC-MIMO system. The performance of the proposed method is compared with the BD method combined with Waterfilling (BD+WF) and the optimal method based on PSO in [11] in terms of total WSR. The WSR results of the methods were averaged over 500 iterations.

Simulation parameters of the system are described in Table 1 with the two scenarios, in which Algorithm 2 is executed using CHHO7 chaos map in both scenarios. The channels from the BS to the MSs are complex Gaussian random variables with zero expectation and unit variance, which are randomly generated in each iteration. As shown in Table 1, the noise variance at each normalized MS is equal to each other and equal to 1. The Rayleigh channel coefficients are generated according to a complex Gaussian distribution  $\mathcal{CN}(0, 1)$ . As such,  $\text{SNR} = P_{max}$ .

TABLE 1: Simulation Parameters

	Parameter	Value
Scenario 1	K	2
	Nr	2
	Nt	4
	$\sigma^2$	first
	$\omega = (\omega_1, \dots, \omega_K)$	(0.4, 0.6)
Scenario 2	K	3
	Nr	2
	Nt	6
	$\sigma^2$	1
	$\omega = (\omega_1, \dots, \omega_K)$	(0.1, 0.2, 0.7)
Chaos Map	$C_i$	CHHO7

To compare the WSR of the proposed algorithm based on HHO with that of the BD+WF algorithm, we consider the downlink BC-MIMO system with respect to Scenario 1 and Scenario 2. First, the efficiency of Algorithm 1 and the BD+WF algorithm are compared in terms of WSR as shown in Fig. 2.

In this figure, we see that the proposed approach may significantly improve the total WSR compared to the BD+WF method, especially when the number of

MSs is limited, for e.g.  $K = 2$  (Scenario 1). As the number of MS increases (Scenario 2), the system performance also increases. Moreover, the scheme BD+WF gradually asymptotes with the system performance using HHO, the WSR gap of the two methods becomes decreased in the high SNR region. The reason is that at high SNR, the system has less noise, so the BD+WF scheme approaches the optimal solution because it can effectively remove noise.

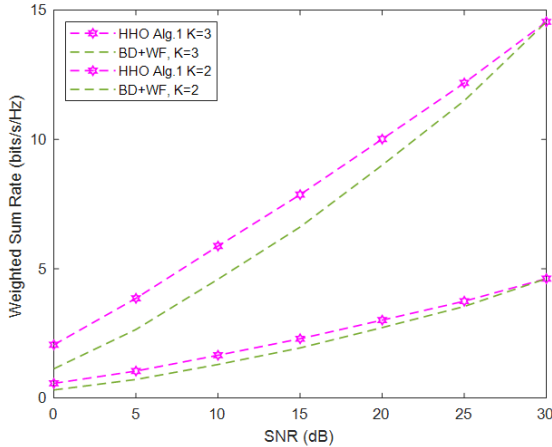


Fig. 2: WSR of the system using Algorithm 1 and BD+WF as a function of SNR (dB).

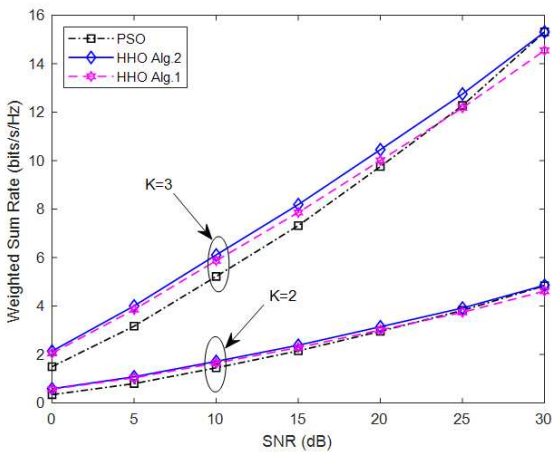


Fig. 3: WSR of the system using the proposed algorithm with Algorithm 1, 2 and PSO as a function of SNR (dB).

In Fig. 3, we compare the system performance using the PSO method with empirical coefficients obtaining from [11] and the proposed method. In this figure, we see that the PSO algorithm has better performance than Algorithm 1 in the high SNR domain. Since Algorithm 1 has many phases, these phases are decided by random variables, which makes it difficult for HHO to converge, and performance comparisons (*Fitness1* and *Fitness2*) are made outside “For” loop so it is possible that the updated hawk position is outdated and suboptimal. Algorithm 2 is proposed to overcome these disadvantages of Algorithm 1, that is, applying chaotic map to control the transition between phases and put

the update part in the “For” loop to update the locations more efficient.

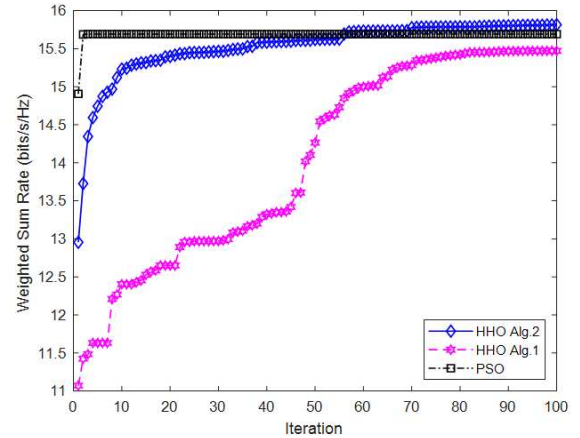


Fig. 4: Convergence of Algorithms 1, 2 and PSO.

Fig. 4 shows the convergence of Algorithms 1, 2 and PSO, considering the Scenario 2 with  $K = 3$  UEs, each UE is equipped with 2 antennas, while the number of BS antennas transmitting is 6. As shown in the figure, we see that the PSO algorithm reaches the convergence first among the three schemes, however, since it falls into the local optimal region, the saturate value remains constant in 10 iterations. Meanwhile, Algorithm 1 has several times that falls into local optimal regions (horizontal segments) but then it exits to fall into other local optimal regions. HHO algorithm may overcome the local optimal regions because it is divided into many phases while PSO algorithm with only one phase. On the other hand, although the convergence speed of Algorithm 2 is slower than that of PSO, it may reach towards global optimization. This happens due to the application of the chaos map, where it quickly gets out of the local optimal regions compared to Algorithm 1. Furthermore, due to the reasonable hawk position update, Algorithm 2, in the end, can perform more effective than Algorithm 1.

## 7. Conclusion

This paper studied the problem of WSR optimization in a multi-user BC-MIMO system, in which a HHO-based algorithm was proposed to design pre-post-coding matrices. In particular, Algorithm 1 is developed based on the original HHO algorithm for the multi-user BC-MIMO system, wherein the initial stage of random variable causes the degradation of system performance using HHO (Algorithm 1) lower than the existing schemes in high SNR region. To overcome these disadvantages, we propose another approach, called Algorithm 2, which improves the system performance for any SNR value. Simulation results showed that HHO offers superior performance compared to current popular algorithms such as BD+WF and PSO, opening a new direction to help solve optimization problems, particularly for the next generation wireless communication.



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