Superposition Principle-based Method for Power Loss Allocation in Power Systems

Duong Duy Long, Nguyen Tuan Anh, Pham Nang Van*

Abstract—The paper presents a novel method for power loss allocation of power systems to demands. This loss allocation technique is based on the contribution of individual demands to the branch power loss of power grids. Results of power flow analysis, precise power system modeling and superposition theorem are deployed in this proposed method. The advantage of the method described in the paper is that it is accurate and ensures fairness among network users. In addition, this allocation method is simple and can be effectively exploited to both meshed and radial electrical networks. A six-node meshed transmission system and an IEEE 33-bus radial distribution grid are utilized to implement power loss allocation based on the proposed method. The influence of demands' power factor on the power loss allocated is also taken into account using the six-node meshed transmission network.

Index Terms—Transmission systems; distribution systems; power flow (PF); power loss allocation; superposition theorem.

---✦---

1. Introduction

THE deregulation of the electric industry has resulted in a complete revolution of the tariff structure for grid users. The tariff method of network users has to be suitably designed to allocate power transmission and distribution costs. Except for other grid use charges, calculating the charges pertaining to power loss allocation is particularly difficult due to the fact that the relationship between power loss and branch power flow is nonlinear. Fairness and consistency are essential characteristics when allocating the power loss regarding generators, demands and grid operators. Recently, a wide range of approaches to cope with the problem of power loss allocation has been available in the technical literature. Authors [1] addressed allocated power loss in electrical networks integrated with distributed generation (DG) units using the power summation method. However, this method is only applicable to radial distribution systems. In addition, the heuristic formulations representing the quadratic relationship between branch power flows and power loss are deployed; therefore, obtained results from this approach cannot be fair and consistent. The energy summation method was put for-

ward in [2] to allocate energy loss in radial electrical grids with DG units. Nevertheless, the main drawback of the paper [3] is that nodal voltages were calculated by using demands' average power and generators' average generating output. In [3], the authors developed the technique based on circuit theory for allocating power loss in active distribution systems. The technique presented in [3] includes two steps. The first step is the decomposition of cross-terms of power loss by deploying a branch-oriented method. The superposition principle is applied to achieve the current contribution of DG units in the second step. Nonetheless, loss allocation factors to decompose cross-terms are still based on heuristics and are only appropriate for radial power distribution systems. The cooperative game technique based on Shapley value was presented in [4] to address the problem of power loss allocation for radial and weakly meshed distribution systems. The work [5] proposed the current decomposition procedure aiming to allocate power loss in power distribution systems considering the power factor of customers. A procedure of allocating power loss based on the graph for transactive energy markets in distribution systems was demonstrated in [6]. However, the studies in [5]–[6] only took into account the radiality structure of electrical distribution grids. Several approaches for power loss allocation were suggested in [7]–[8] and [9]–[10]. The paper [11] proposed a power loss procedure with two steps. The first step is implementing the power flow problem using similar methods as described in [12]–[13]. The second step is representing demands by equivalent admittance and generators by equivalent current sources or equivalent current sources combined with the admittance. However, the work [11] only considered the allocation of power loss to each generator. Until the present time, the main technical challenge existing in all techniques is that the nonlinear

Duong Duy Long is a student at Department of Electrical Engineering, School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Hanoi, Vietnam.

Nguyen Tuan Anh is a student at Department of Electrical Engineering, School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Hanoi, Vietnam.

Pham Nang Van is with Department of Electrical Engineering, School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Ha Noi, Vietnam (e-mail: van.phamnang@hust.edu.vn).

*Corresponding author: Pham Nang Van (e-mail: van.phamnang@hust.edu.vn)

relationship between branch loss and power flow is not addressed satisfactorily. This paper complements the work in [11] for some aspects, including:

- A procedure that allocates power loss to each load is proposed;
- The impact of the power factor of demands on power loss allocation is analyzed and compared;
- A proposed approach for allocating loss is validated using both meshed transmission networks and radial distribution systems.

It is foregrounded that compared to [11], this paper develops a procedure for allocating the power loss to demands rather than generators. Then, this procedure is adopted to evaluate the influence of the demand's power factor on the power loss allocated. Furthermore, compared to [11], the study in this paper is especially suitable for the power loss allocation in the electrical distribution systems that usually have one power supply source. Moreover, our proposed methodology is further validated using an IEEE 33-bus electrical distribution grid whose topology is radial

2. Proposed power loss allocation procedure

A procedure for allocating power loss is proposed assuming that power systems are considered to be balanced and symmetrical. This procedure consists of three main steps: (1) Power flow analysis; (2) Power system model and (3) superposition principle application.

2.1. Power flow analysis

Power flow analysis aims to obtain the voltage magnitude, phase angle at buses, active power and reactive power produced by generators. The input data for power flow analysis comprises:

- The complex voltage at the swing bus;
- The active power and reactive power of demands;
- The real power and voltage magnitude for voltage-controlled generators or the real power and reactive power for generators without voltage control;
- Equivalent circuit's parameters for line and transformer branches in the network.

Let N be the number of the total buses in the power system. The bus admittance matrix Y_{bus} is written as follows:

$$Y_{\text{bus}} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} & \ldots & \dot{Y}_{1k} & \ldots & \dot{Y}_{1N} \\ \dot{Y}_{21} & \dot{Y}_{22} & \ldots & \dot{Y}_{2k} & \ldots & \dot{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \dot{Y}_{i1} & \dot{Y}_{i2} & \ldots & \dot{Y}_{ik} & \ldots & \dot{Y}_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \dot{Y}_{N1} & \dot{Y}_{N2} & \cdots & \dot{Y}_{Nk} & \cdots & \dot{Y}_{NN} \end{bmatrix} \quad (1)$$

where

- $\dot{Y}_{ii}\ \left(\forall i = \overline{1, N}\right)$ is the self-admittance of bus i;
- $\dot{Y}_{ik}\ (\forall i, \forall k,\ i \neq k)$ is the transfer admittance between bus i and k.

Nonlinear power balance equations describing the power system in steady-state conditions are represented below:

$$\begin{cases} P_i = U_i \sum_{k=1}^{N} U_k \left(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}\right); \ i = 1, ..., N \\ Q_i = U_i \sum_{k=1}^{N} U_k \left(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}\right); \ i = 1, ..., N \end{cases}$$
$$(2)$$

where:

- P_i and Q_i are active and reactive powers injected to bus i, respectively;
- U_i and U_k stand for respective voltage magnitude at buses i and k;
- G_{ik} and B_{ik} are the real and imaginary parts of element ik in the bus admittance matrix;
- δ_i and δ_k denote voltage angles at bus i and k, respectively.

From (2), we achieve two nonlinear equations and four unknown variables for each bus i. Therefore, to solve the power flow equations, we have to specify two variables for each node. Three types of buses can be defined as follows:

- PQ nodes: Active and reactive powers injected to node i are specified:

$$P_i = P_i^{\text{sp}} = -P_{\text{D}i}^{\text{sp}}; \quad Q_i = Q_i^{\text{sp}} = -Q_{\text{D}i}^{\text{sp}} \quad (3)$$

where $P_{\text{D}i}^{\text{sp}}$ and $Q_{\text{D}i}^{\text{sp}}$ denote the active ans reactive power consumed by demands at bus i, respectively. The two unknown variables are the voltage magnitude (U_i) and voltage phase angle (δ_i).

- PV nodes: The voltage magnitude and the real power produced by generators at bus i are set to a specified value:

$$U_i = U_i^{\text{sp}}; \quad P_i = P_i^{\text{sp}} = P_{\text{G}i}^{\text{sp}} - P_{\text{D}i}^{\text{sp}} \quad (4)$$

where $P_{\text{G}i}^{\text{sp}}$ represents the active power generated by generators at bus i. The two unknown variables are the voltage magnitude (Q_i) and voltage phase angle (δ_i).

- Slack node: The voltage magnitude (U_i) and phase angle (δ_i) are known in advance. The unspecified variables are the active power (P_i) and reactive power (Q_i) injected to node i.

The Newton-Raphson method [14] is considered to be the most common approach to solve nonlinear equation systems (2). According to this approach, at rth iteration, we need to solve the linear equations as follows:

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}^{(r)} = \begin{bmatrix} \mathbf{H} & \mathbf{N} \\ \mathbf{M} & \mathbf{L} \end{bmatrix}^{(r)} \begin{bmatrix} \Delta\delta \\ \Delta\mathbf{U}/\mathbf{U} \end{bmatrix}^{(r)} \quad (5)$$

where $\mathbf{H}$, $\mathbf{N}$, $\mathbf{M}$ and $\mathbf{L}$ are four blocks of the Jacobian matrix.

$$[\mathbf{\Delta P}|\mathbf{\Delta Q}]^{\text{T}}$$
$$= \left[\Delta P_1, \Delta P_2, \ldots, \Delta P_{N-1} | \Delta Q_1, \Delta Q_2, \ldots, \Delta Q_{N_{\text{D}}}\right]^{\text{T}}$$
$$(6)$$

$$\Delta P_i = P_i^{\text{sp}} - U_i \sum_{k=1}^{N} U_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik});$$
$$i = \overline{1, N-1}$$
(7)
$$\Delta Q_i = Q_i^{\text{sp}} - U_i \sum_{k=1}^{N} U_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik});$$
$$i = \overline{1, N_{\text{D}}}$$

where N_D is the number of PQ buses in the power system; P_i^{sp} and Q_i^{sp} denote the active power and reactive power specified in advance at bus i, respectively. Updated values of voltage magnitude and voltage phase angle at $(r+1)$th iteration are computed below.

$$\begin{bmatrix} \delta \\ \mathbf{U} \end{bmatrix}^{(r+1)} = \begin{bmatrix} \delta \\ \mathbf{U} \end{bmatrix}^{(r)} + \begin{bmatrix} \mathbf{\Delta \delta} \\ \mathbf{\Delta U} \end{bmatrix}^{(r)}$$
(8)

2.2. Power system model

The method for allocating power loss proposed in this paper solely allocates the total power loss of the power network to demands. We model demand at bus k by an equivalent current source:

$$\dot{J}_{Dk} = -\frac{(P_{Dk} + jQ_{Dk})^*}{U_k^*}$$
(9)

where, P_{Dk}, Q_{Dk} and U_k^* denote the active power, reactive power of the load and the conjugate of complex voltage at bus k. It is highlighted the fact that we represent the load by the equivalent current source as (9) in order that the contribution of each individual load to the power loss of the electrical grid can be determined using the superposition theorem. Moreover, the value of this equivalent current source has to be computed from the voltage magnitude U_k^* that is calculated on the basis of solving the system of nonlinear equations (2) using the Newton-Raphson approach described in subsection 2.1. Therefore, according to the proposed method, we do not adopt the quadratic expressions of power loss with a view to allocating the power loss to each demand, which contributes to non-discrimination between power grid users. Furthermore, the aggregation of the power losses allocated is precisely equal to the total power loss of the electrical grid attained from power flow analysis. As a result, the developed technique in this paper is very appropriate for power system operation in the deregulation environment. Furthermore, each generator is represented by an equivalent admittance as follows:

$$\dot{y}_{Gi} = -\frac{(P_{Gi} + jQ_{Gi})^*}{U_i^2}$$
(10)

where P_{Gi}, Q_{Gi} and U_i are the real power, reactive power of the generator and the voltage magnitude at bus i. Then, the equivalent admittance of each generator $\dot{y}_{Gi}$ is added to the diagonal element (i, i) of the bus admittance matrix to modify this matrix. The modified bus admittance matrix is defined as $\mathbf{Y}_{\text{bus}}^{\text{mod}}$. The inverse of $\mathbf{Y}_{\text{bus}}^{\text{mod}}$ is called the modified bus impedance matrix below.

$$\mathbf{Z}_{\text{bus}}^{\text{mod}} = \left(\mathbf{Y}_{\text{bus}}^{\text{mod}} \right)^{-1}$$
(11)

It is emphasized that the computationally efficient method for building the modified bus impedance matrix for large-scale power systems is to determine the individual columns of this matrix instead of determining the whole matrix [15]. According to this approach, the modified bus admittance matrix $\mathbf{Y}_{\text{bus}}^{\text{mod}}$ is decomposed into the product of the lower triangular matrix ($\mathbf{L}$) and the upper triangular matrix ($\mathbf{U}$). Then, the forward and backward operations are implemented to calculate these individual columns of the matrix $\mathbf{Z}_{\text{bus}}^{\text{mod}}$

2.3. Superposition principle application

It is assumed that the considered power system has N_D demands and N_D is the first buses. A $(N_D \times N_D)$ diagonal matrix J is defined as follows:

$$\mathbf{J} = \begin{bmatrix} \dot{J}_1 & 0 & \dots & 0 \\ 0 & \dot{J}_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dot{J}_{N_{\text{D}}} \end{bmatrix}$$
(12)

A $(N \times N_D)$ matrix $\mathbf{Z}$ is built by deploying the N_D first columns of the matrix $\mathbf{Z}_{\text{bus}}^{\text{mod}}$. By multiplying two matrices Z and J, we obtain a $(N \times N_D)$ voltage matrix $\mathbf{U}$ below:

$$\begin{bmatrix} \dot{U}_{1,1} & \dot{U}_{1,2} & \dots & \dot{U}_{1,N_{\text{D}}} \\ \dot{U}_{2,1} & \dot{U}_{2,2} & \dots & \dot{U}_{2,N_{\text{D}}} \\ \dots & \dots & \dots & \dots \\ \dot{U}_{N,1} & \dot{U}_{N,2} & \dots & \dot{U}_{N,N_{\text{D}}} \end{bmatrix} = \mathbf{Z} \begin{bmatrix} \dot{J}_1 & 0 & \dots & 0 \\ 0 & \dot{J}_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dot{J}_{N_{\text{D}}} \end{bmatrix}$$
(13)

where the element $\dot{U}_{i,k}$ of the matrix U denotes the contribution of the current source $\dot{J}_k$ to the complex nodal voltage at bus i $\left(\dot{U}_i \right)$. It is noted that each column of matrix $\mathbf{U}$ can be constructed using the superposition theorem when only an equivalent current source is switched on. Furthermore, the complex voltage at bus i can be determined from the ith row of matrix $\mathbf{U}$ as follows:

$$\dot{U}_i = \dot{U}_{i,1} + \dot{U}_{i,2} + \dots + \dot{U}_{i,N_{\text{D}}}$$
(14)

The contribution of $\dot{J}_k$ the current source to the current of branch mn at bus m is calculated as follows:

$$\dot{I}_{mn,k} = \left(\dot{U}_{m,k} - \dot{U}_{n,k} \right) \dot{y}_{mn} + \dot{U}_{m,k} \dot{y}_{mn}^{\text{sh}}$$
(15)

where

- $\dot{y}_{mn}$ is the series admittance of branch mn.
- $\dot{y}_{mn}^{\text{sh}}$ is the shunt admittance of branch mn connected at bus m.

Similarly, the calculation of the contribution of the current source $\dot{J}_k$ to the current of branch mn at bus n is described below:

$$\dot{I}_{nm,k} = \left(\dot{U}_{n,k} - \dot{U}_{m,k} \right) \dot{y}_{mn} + \dot{U}_{n,k} \dot{y}_{nm}^{\text{sh}}$$
(16)

where $\dot{y}_{nm}^{\text{sh}}$ is the shunt admittance of branch mn connected at bus n. The contribution of the current source $\dot{J}_k$ to the power loss of branch mn is determined as follows:

$$\Delta \dot{S}_{mn,k} = \dot{U}_m . I_{mn,k}^* + \dot{U}_n . I_{nm,k}^*$$
(17)

The total power loss of branch mn is written as

$$\Delta \dot{S}_{mn} = \sum_{k=1}^{N_D} \Delta \dot{S}_{mn,k} = \sum_{k=1}^{N_D} \left(\dot{U}_m . I^*_{mn,k} + \dot{U}_n . I^*_{nm,k} \right)$$

$$= \dot{U}_m . \sum_{k=1}^{N_D} I^*_{mn,k} + \dot{U}_n . \sum_{k=1}^{N_D} I^*_{nm,k}$$

(18)

The contribution of each demand to the total power loss of the electrical network is achieved as

$$\Delta \dot{S}_k = \sum_{\forall \text{ branch } mn} \Delta \dot{S}_{mn,k} \qquad (19)$$

3. Result and discussions

In this section, the proposed method is validated using the six-bus meshed transmission system [11] and the IEEE 33-bus radial distribution system [16]. The power flow analysis is implemented using the POWER-WORLD software [17], and the proposed technique for allocating power loss is carried out using the MATLAB software [18].

3.1. Six-bus transmission system

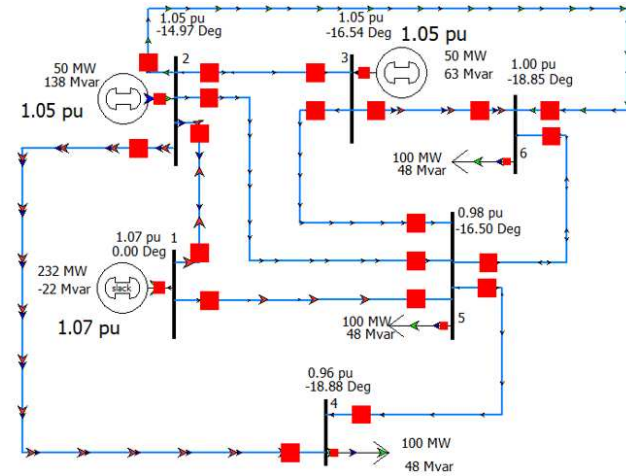The single line diagram of the six-bus transmission system is shown in Fig. 1.

Fig. 1: A six-bus transmission system

Data that correspond to this system, on a 100 MVA and 230 kV base, are listed in Table 1 and Table 2. Data for demands of this transmission system is modified to facilitate analyzing the effect of demands' power factor. The power factor of all demands is set to 0.9. Results of nodal voltages and complex power generated by generators in the steady-state condition are depicted in Table 3.

TABLE 1: A six-bus transmission system

Bus	Bus type	U pu	P_D MW	Q_D MVAr	P_G MW	Q_G MVAr
1	Slack	1.07	0	0	—	—
2	PV	1.05	0	0	50	—
3	PV	1.05	0	0	50	—
4	PQ	—	100	48.43	0	0
5	PQ	—	100	48.43	0	0
6	PQ	—	100	48.43	0	0

TABLE 2: Branch data for six-bus transmission system

From bus	To bus	R (pu)	X (pu)	B (pu)
1	2	0.10	0.20	0.04
1	5	0.08	0.30	0.06
2	3	0.05	0.25	0.06
2	4	0.05	0.10	0.02
2	5	0.10	0.30	0.04
2	6	0.07	0.20	0.05
3	5	0.12	0.26	0.05
3	6	0.02	0.10	0.02
4	5	0.20	0.40	0.08
5	6	0.10	0.30	0.06

TABLE 3: Results of power flow analysis for six-bus system

Bus	$\dot{U}_i$(pu)	$\dot{S}_{Gi}$(pu)
1	1.07	$2.32287 - j0.21917$
2	$1.01435 - j0.27125$	$0.5 + j1.37817$
3	$1.00657 - j0.29885$	$0.5 + j0.62714$
4	$0.90427 - j0.30919$	0
5	$0.94145 - j0.27888$	0
6	$0.94497 - j0.32264$	0

TABLE 4: Generator admittances and load currents

Bus	$\dot{I}_D$(pu)	$\dot{y}_G$(pu)
1	0	$-2.02888 - j0.19143$
2	0	$-0.45351 + j1.25004$
3	0	$-0.45354 + j0.56883$
4	$-0.82613 + j0.81806$	0
5	$-0.83640 + j0.76220$	0
6	$-0.79103 + j0.78261$	0

According to the developed method, generators are represented by the admittances, and demands are modelled as equivalent current sources. Values of the generator admittance and the load current are sketched in Table 4.

Matrices **J**, **Z** and **U** are described as follows:

$$\mathbf{J} = \begin{bmatrix} -0.8261+j0.8181 & 0 & 0 \\ 0 & -0.8364+j0.7622 & 0 \\ 0 & 0 & -0.7910+j0.7826 \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} -0.2295-j0.2172 & -0.2164-j0.2108 & -0.2326-j0.2263 \\ -0.2666-j0.1351 & -0.2678-j0.1616 & -0.2755-j0.1514 \\ -0.2797-j0.1577 & -0.2712-j0.1502 & -0.2703-j0.1167 \\ -0.2269-j0.0579 & -0.2631-j0.1501 & -0.2749-j0.1501 \\ -0.2631-j0.1501 & -0.2376-j0.1038 & -0.2657-j0.1448 \\ -0.2749-j0.1501 & -0.2376-j0.1448 & -0.2564-j0.0763 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 0.3672-j0.0083 & 0.3417+j0.0113 & 0.3611-j0.0030 \\ 0.3308-j0.1065 & 0.3472-j0.0689 & 0.3364-j0.0958 \\ 0.3600-j0.0985 & 0.3414-j0.0811 & 0.3052-j0.1192 \\ 0.2348-j0.1378 & 0.3345-j0.0750 & 0.3350-j0.0964 \\ 0.3402-j0.0912 & 0.2778-j0.0943 & 0.3235-j0.0934 \\ 0.3499-j0.1009 & 0.3326-j0.0814 & 0.2625-j0.1403 \end{bmatrix}$$

The power loss allocation of each branch to demands is presented in Table 5. Calculated results from Table 5 show that although the complex power of all demands in the six-bus transmission system is similar, the allocated power loss to demand 4 is the highest, and the power loss allocated to demand 6 is the lowest. In addition, demands contribute to the negative real power loss to some branches of the network, which means demands decrease the power loss of these branches. The total power loss allocated to all demands equals the total power loss of the power system.

TABLE 5: Allocated loss of each branch to demands

Line	Demand		
	4	5	6
1–2	4.0151+j10.8378	6.6041+j5.9224	4.7162+ j9.4160
1–5	3.1779+j5.8048	1.8007+j10.1117	2.8474+j7.1039
2–3	0.2822–j2.4980	–0.1302–j2.0644	–0.0888–j1.7364
2–4	4.6179+j8.8565	0.7364+j0.5311	0.0683–j0.5590
2–5	–0.3741–j1.5574	0.8940+j0.1267	0.0174–j1.0906
2–6	0.0157–j2.6321	0.1728–j1.0576	0.9739+j1.7613
3–5	0.1412–j1.3759	1.2181–j0.7211	–0.6804–j1.5985
3–6	0.0122–j0.0838	–0.1272–j0.1432	0.9538+j2.3202
4–5	0.4127–j1.1907	–0.1663–j3.0926	–0.0163–j2.7664
5–6	0.0835–j2.0287	0.6381–j2.2396	–0.5288–j1.0372
Total	12.3842+j14.1326	11.6405+j7.3735	8.2626+j11.8134
Total	32.2873+j33.3195		

The impact of demand's power factor on the power loss allocation is illustrated in Fig. 2. From Fig. 2, it can be seen that the allocated power loss to demand 4 reduces from 13.42 MW to 11.54 MW when the power factor increases from 0.85 to 0.95. At the same time, there is a slight reduction in the power loss allocated to demand 6.
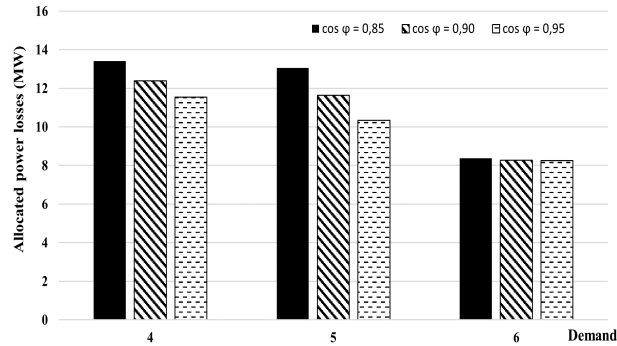

Fig. 2: Impact of power factor on allocated power loss

3.2. IEEE 33-node distribution system

The single line diagram of the IEEE 33-node distribution system is shown in Fig. 3. It is assumed that the power of loads is increased by 150%, and the voltage at the root node is equal to 1.05 pu (on 12.66 kV base). Table 6 reveals the results of node voltage calculated

from power flow analysis for the IEEE 33-bus distribution system. The power loss allocated to each load for this system is depicted in Fig. 4. From Fig. 4, we can see that the allocated power loss to demand at bus 30 is the highest (49.48 kW), followed by demand at bus 32 (43.29 kW). The lowest power loss is allocated to demand at bus 2 (0.65 kW).

TABLE 6: Results of nodal voltage for IEEE 33-node system

Bus	Voltage (pu)	Angle (degree)
1	1.05	0
2	1.04566162	0.02034334
3	1.02500612	0.13556729
4	1.01397946	0.22878057
5	1.00306598	0.32415894
6	0.97589849	0.19046869
7	0.97075256	-0.13958085
8	0.96360215	-0.08710648
9	0.95432377	-0.19202395
10	0.94571291	-0.28228497
11	0.94443645	-0.27199226
12	0.94221455	-0.25508713
13	0.93314768	-0.38788028
14	0.92978431	-0.5029788
15	0.92768777	-0.5581806
16	0.92565667	-0.59227672
17	0.92264563	-0.70583503
18	0.92174396	-0.71995289
19	1.04490467	0.00556434
20	1.0397776	-0.08596003
21	1.03876791	-0.11245467
22	1.03785429	-0.14031754
23	1.01982471	0.09282993
24	1.0101819	-0.03030331
25	1.00537295	-0.09118892
26	0.97304469	0.24696401
27	0.96925034	0.32753063
28	0.9523186	0.44605595
29	0.94015185	0.55854238
30	0.93488298	0.71193114
31	0.92871374	0.58853252
32	0.92735649	0.55475937
33	0.9269359	0.54342387

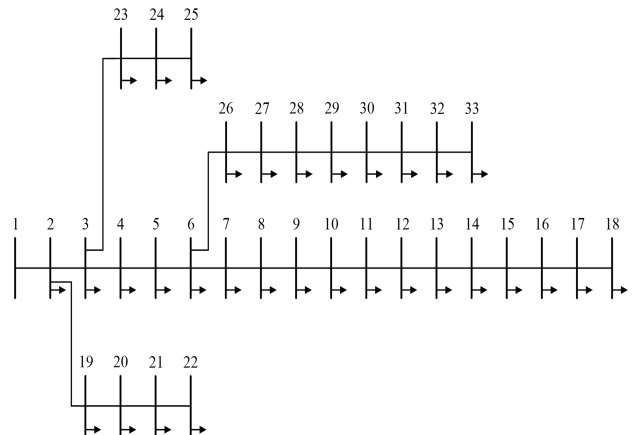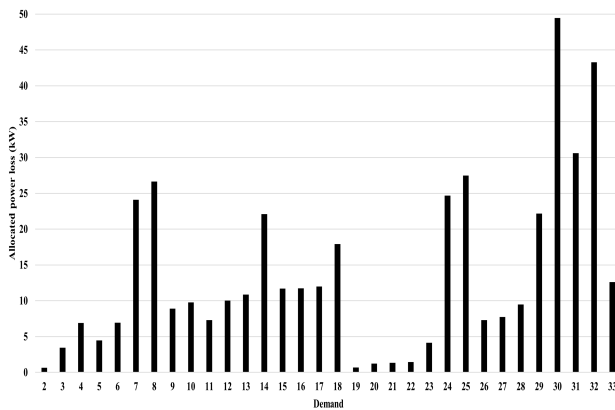
Fig. 3: IEEE 33-bus distribution system

Fig. 4: Power loss allocation to demands for IEEE 33-bus distribution system

4. Conclusion

A novel method for allocating power loss to the demands of power systems is proposed in this paper. This developed loss allocation approach is based on the contribution of individual demands to the branch power loss of power networks. Due to the deployment of results of power flow analysis, precise power system model and superposition principle, the method described in the paper is accurate and fair among network users. In addition, this power loss allocation method is simple and can be applied to both meshed transmission and radial distribution systems. The validation of the proposed method is executed using a six-node meshed transmission system and an IEEE 33-bus radial distribution grid. Further study is to develop a methodology with the aim of coincidentally allocating the power loss to generators and demands in power systems.

References

[1]   M. Atanasovski and R. Taleski, "Power Summation Method for Loss Allocation in Radial Distribution Networks With DG," *IEEE Trans. Power Syst.*, vol. 26, no. 4, Nov. 2011.

[2]   M. Atanasovski and R. Taleski., "Energy Summation Method for Loss Allocation in Radial Distribution Networks With DG," *IEEE Trans. Power Syst.*, vol. 27, no. 3, Aug. 2012.

[3]   P. Kumar, N. Gupta, K. R. Niazi, and A. Swarnkar, "A Circuit Theory-Based Loss Allocation Method for Active Distribution Systems," *IEEE Trans. Smart Grid*, vol. 10, no. 1, Jan. 2019.

[4]   S. Sharma and A. Abhyankarm, "Loss Allocation for Weakly Meshed Distribution System using Analytical Formulation of Shapley Value," *IEEE Trans. Power Syst*, pp. 1–1, 2016.

[5]   P. Kumar, N. Gupta, K. R. Niazi, and A. Swarnkar, "Current Decomposition Method for Loss Allocation in Distribution Systems," *IET Generation, Transmission and Distribution*, vol. 11, no. 18, Dec. 2017.

[6]   A. I. Nikolaidis, C. A. Charalambous, and P. Mancarella, "A Graph-Based Loss Allocation Framework for Transactive Energy Markets in Unbalanced Radial Distribution Networks," *IEEE Transactions on Power Systems*, vol. 34, no. 5, May. 2018.

[7]   K. M. Jagtap and D. K. Khatod, "Novel Approach for Loss Allocation of Distribution Networks With DGs," *Electric Power Systems Research*, vol. 143, Feb. 2016.

[8]   K. S. Ahmed and S. P. Karthikeyan, "Comparison of Various Transmission Loss/Cost Allocation Methods — A Reviews," *Innovations in Power and Advanced Computing Technologies (i-PACT)*, Apr. 2017.

[9]   S. Zhang, F. Chen, X. Wang, and J. Zhong, "Study on Loss Allocation Method of Distribution Network with Considering of Distributed Generation," *IEEE/IAS Industrial and Commercial Power System Asia (I&CPS Asia)*, Jul. 2020.

[10]   P. N. Van, "A Comparative Study of PTDF based-Methods to Determine Transmission Usage Allocation for Bilateral Transactions in Power Markets," *TNUJST*, vol. 226, no. 06, May. 2021.

[11]   D. Rajicic and M. Todorovski, "Participation of Every Generator to Loads, Currents, and Power Losses," *IEEE Trans. Power Syst*, vol. 36, no. 2, Mar. 2021.

[12]   P. N. Van and D. Q. Duy, "Different Linear Power Flow Models for Radial Power Distribution Drids: A Comparison," *TNT Journal of Science and Technology*, vol. 226, no. 15, Aug. 2021.

[13]   P. N. Van, L. T. M. Chau, and D. Q. Duy, *Power System Modeling and Analysis*, 1st ed.   Bach Khoa Publishing House, Ha Noi, 2021.

[14]   S. M. L. Kabir, A. H. Chowdhury, M. Rahman, and J. Ala, "Inclusion of Slack Bus in Newton Raphson Load Flow Study," pp. 282–284, Dec 2014.

[15]   A. R. Bergen and V. Vittal, *Power System Analysis*, 2nd ed. Prentice Hall, 2000.

[16]   S. H. Dolatabadi, M. Ghorbanian, P. Siano, and N. D. Hatziargyriou, "An Enhanced IEEE 33 Bus Benchmark Test System for Distribution System Studies," *IEEE Trans. Power SystT*, vol. 36, no. 3, May. 2021.

[17]   POWERWORLD, Accessed August 2021. [Online]. Available: https://www.powerworld.com/

[18]   MATLAB, Accessed August 2021. [Online]. Available: https://www.mathworks.com/products/matlab.html

Duong Duy Long is a fourth-year student at Department of Electrical Engineering, School of Electrical and Electronic Engineering, Hanoi University of Science and Technology.

Nguyen Tuan Anh is a fourth-year student at Department of Electrical Engineering, School of Electrical and Electronic Engineering, Hanoi University of Science and Technology.

Pham Nang Van received a Master's Degree of Science in Electrical Engineering from Hanoi University of Science and Technology (HUST) in 2010. He is a lecturer at Hanoi University of Science and Technology. His current research topic is new business models in retail markets with the integration of distributed renewables.