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ANALYSIS OF FOREST EXISTENCE AGAINST POPULATION OF *Eos histrio* IN NORTH SULAWESI

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Abstract. Red List International Union for the Conservation of Nature and Natural Resource (IUCN) states there are more than 8000 animals are critically endangered and it will be increase. One of the Indonesias endemic animal that is included in the IUCN Red List is *Eos histrio* or Burung Nuri Talaud which lives in North Nusa Islands, North Sulawesi. Their natural habitat is a forest with tall trees as a place to live. Based on data from the Central Statistics Agency (BPS) the area of forest in the region is decreasing every year, causing *Eos histrio* to lose his home and causing a decrease in the population. Based on data from the IUCN, the population of *Eos histrio* in 1999 was at 8230 21400, while in 2016 it was in the range of 5500 14000. The number will continue to decrease due to the large amount of deforestation and the lack of protection for animals. By using the Pontryagin Maximum method, it is found that one way to protect the population of *Eos histrio* is to reduce the rate of deforestation in North Sulawesi by at least 10% so as to increase the population of *Eos histrio* within 4 year.

Key words and Phrases:*Eos histrio*, deforestation, Pontryagin Maximum method

1. INTRODUCTION

The International Union for the Conservation of Nature and Natural Resource (IUCN) Red List reported that there are over 8.000 critically endangered species and it seems likely to climb. One of the species included in the IUCN [1] Red List is *Eos histrio*, also known as the Red-and-blue lory, a bird endemic to Indonesia that inhabits Pulau Karakelang Island, Nusa Utara Islands, North Sulawesi. Adi Widyanto [2] said that Karakelang is a conservation area in North Sulawesi Province with an area of 24669 hectares as stipulated by the Decree of the Minister of Forestry and Plantation Number 97/Kpts-II/2000. More than just a lowland rain forest and

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mountains with karst soil, which is a unique habitat, the area also houses Talaud Archipelagos typical biodiversity, including the Red-and-blue lory. According to Adi Widyanto [2], the decline rate of vegetation cover on the Talaud Islands, which includes the three main islands of Karakelang, Kabaruan, and Salibabu, was 4457 hectares in 2001 2017. On the other hand, the forest cover increase rate was 1320 hectares in 2001 - 2012. The forest loss locations are found not just in plantation regions, but also in forest areas. This data confirms the reports stated that there are still forest encroachment activities to open a farm around the forest area.

From Diah Irawati [3] we know that *Eos histrio*'s roosting trees criterion are branch-free bole length, canopy diameter, canopy density and the number of trees around the roost tree on diameter 20 - 40 cm and up to 41 cm. *Eos histrio*s native environment is a forest with tall trees and focused in the primary forest that is a closed classification forest with low habitat disturbance. In this case, IUCN [1] reported that there are 8230 21400 Eos histrio in 1999, however, in 2016, it was only around 5500 14000 full-grown *Eos histrio*. Due to the many threats in the conservation area, such as illegal trading of the bird and severe deforestation encroachment for farming on the main forest and commercial logging, the bird population is expected to continue to decline. Based on Adi Widiyanto [2] that focus on community empowerment on forest protection near Karakelang wildlife sanctuary and Jon Riley [4] that focus on population size of *Eos histrio*, hence we will analyze the effect of forest existence on *Eos histrio* population growth on Karakelang Island, North Sulawesi.

2. MAIN RESULTS

The mathematical model that used to analysis of forest carrying capacity on *Eos histrio* is Lotka-Volterra (prey-predator) model from A. Panvilov [5] where prey is assumed to be the population of *Eos histrio* while predators are assumed to be the rate of deforestation, as follows:

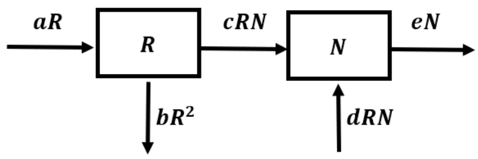


FIGURE 1. Flowchart of Lotka-Volterra

$$f = \frac{dR}{dt} = aR - bR^2 - cRN \tag{1}$$

$$g = \frac{dN}{dt} = dRN - eN. \tag{2}$$

From IUCN [1], Adi Widyanto [2], Jon Riley [4], and Birdlife International [6], we will find parameter values as follows: Because (1) and (2) is non-linear model, so we

TABLE 1. Description and parameter values

Parameter	Description	Value
R	*Eos histrio*'s population	–
N	Deforestation	–
a	Increased *Eos histrio*s population	0.49
b	Competition between *Eos histrio*	0.054
c	Number of bird who died due to deforestation	0.0089
d	Forest growth rate	0.054
e	Rate of deforestation	0.181

need to linearization that equation using Jacobi matrix around their equilibrium point. Fisrt, we will find the equilibrium point from equation (2), as follows:

$$\frac{dN}{dt} = 0$$
$$dRN - eN = 0$$
$$N(dR - e) = 0.$$

Hence the equilibrium point from (2) are

$$N = 0 \vee R = \frac{e}{d}. \tag{3}$$

Using equilibrium point from (2), we will find equilibrium point for (1), as follows :

$$\frac{dR}{dt} = aR - bR^2 - cRN = 0.$$

If $N = 0$, then

$$\frac{dN}{dt} = 0$$
$$aR - bR^2 = 0$$
$$R(a - bR) = 0$$
$$R = 0 \vee R = \frac{a}{b}. \tag{4}$$

When $R = \frac{e}{d}$ then

$$\frac{ae}{d} - b(\frac{e}{d})^2 - \frac{ce}{d}N = 0$$
$$N = \frac{ad - be}{cd}. \tag{5}$$

Thus there are three equilibrium points can be found from equation (3), (4), and (5), which are $(0, 0), (\frac{a}{b}, 0), (\frac{e}{d}, \frac{ad-be}{cd})$ and if we substitute the parameter values in Table 1, we get the results are (0,0), (90.74;0),(3.35;53).

STABILITY ANALYSIS : Perform the linearization process by forming the Jacobian matrix as followss as in A. Panvilov's Book [5]:

$$J = \begin{pmatrix} \partial_R f & \partial_R g \\ \partial_N f & \partial_N g \end{pmatrix} = \begin{pmatrix} a - 2bR - cN & -cR \\ dN & dR - e \end{pmatrix}. \tag{6}$$

Determine the stability around the equilibrium point by determining the eigen values using equation (6). For the equilibrium point $(0,0)$ then $|J - \lambda I| = 0$ was obtained :

$$\begin{vmatrix} a - \lambda & 0 \\ 0 & -e - \lambda \end{vmatrix} = 0$$

$$\lambda_1 = a \vee \lambda_2 = -e.$$

For the equilibrium point $(90.74, 0)$ then $|J - \lambda I| = 0$ was obtained :

$$\begin{vmatrix} -0.49 - \lambda & -0.81 \\ 0 & 4.72 - \lambda \end{vmatrix} = 0$$

$$\lambda_1 = -0.49 \vee \lambda_2 = 4.72.$$

For the equilibrium point $(0,0)$ then $|J - \lambda I| = 0$ was obtained :

$$\begin{vmatrix} -0.01788 - \lambda & -0.029815 \\ 2.862 & -0.0001 - \lambda \end{vmatrix} = 0$$

$$\lambda_1 = -0.00899 - 0.29i \vee \lambda_2 = -0.00899 + 0.29i.$$

From the above calculation the equilibrium point (3.35,53) is a stable equilibrium point based on A. Panvilov [5] and de Bour [7].

CONTROLLABILITY ANALYSIS : The objective function of this research is to maximize the population growth of *Eos histrio* by minimizing the rate of deforestation on Karakelang Island. The controllability of models (1) and (2) will be reviewed around the equilibrium point (3.35,53) by forming a control matrix.

$$J = \begin{pmatrix} \partial_e f \\ \partial_e g \end{pmatrix} = \begin{pmatrix} 0 \\ -e \end{pmatrix}. \tag{7}$$

Next step is find the matrix $M_c = (B|JB|J^2B|...|J^{n-1}B)$ based on Subiono's book [8].

$$M_c = \begin{pmatrix} 0 & cRe \\ -e & e^2 - edR \end{pmatrix}.$$

Because rank of matrix $M_c = J$ then the system is controlled [8].

DETERMINE THE OBJECTIVE FUNCTION : In this research, the objective function is to control forest damage so as to maximize the growth of the *Eos histrio* population on Karakelang Island. The objective function of the system is

$$H(e) = \int_{t_0}^{t_f} \left(R + \frac{1}{2} C_1 e^2(t) \right), dt; t \in [t_0, t_f]; e, R > 0.$$

OPTIMAL CONTROL SOLUTION : The optimal control problem for the system of equations (1) and (2) and the objective function above using the Pontryagin Maximum method with the following steps based on Naidu [9]:

Step 1: Determine the Hamiltonian function.

$$H(R, N, e, \lambda) = \left(R + \frac{1}{2}C_1 e^2(t)\right) + \lambda_1(aR - bR^2 - cRN) + \lambda_2(dRN - eN). \quad (8)$$

Step 2: Minimize H to $e(t)$.

$$\frac{\partial H}{\partial e} = 0 \quad (9)$$

$$C_1 e(t) - \lambda_2 N = 0$$

$$e(t) = \frac{\lambda_2 N}{C_1}.$$

Step 3 : Determine state and costate The formed of state equation is

$$\frac{\partial H}{\partial \lambda_1} = aR - bR^2 - cRN \quad (10)$$

$$\frac{\partial H}{\partial \lambda_2} = dRN - eN. \quad (11)$$

And the formed of costate equation is

$$-\frac{\partial H}{\partial R} = -(1 + \lambda_1(a - 2bR - cN)). \quad (12)$$

SIMULATION AND ANALYSIS : This section will be discussed about the results of numerical simulations before and after being subjected to control of the forest carrying capacity model on the population of *Eos histrio* where the control variable is deforestation rate in conservation areas on Karakelang Island and using the Runge Kutta method of order 4 based from Oruh [10]. The initial values that used in the simulation was the population density of *Eos histrio* per km^2 in primary forest is 15.2 28.1 individuals while in secondary habitat it is 4.8 19.3 individuals per km^2 and the rate of forest destruction is 44 hectares based on Adi Widyanto [2]. The numerical simulation will be given a control weight of 10% means that the rate of forest destruction will be reduced by 10% and the observation period is 4 years with the following results. In Figure 2, the red line shows the uncontrolled deforestation rate during 4-year observation period. After the control was adjusted to the deforestation rate during the 4-year observation period, a blue line shows the fluctuating damage rate, but the level is not as severe as when it has not been controlled. It means that the damage rate, although still relatively high, can still be controlled.

It has an impact on the *Eos histrio* population in the same observation period of four years as shown in Figure 3. The red line shows the *Eos histrio* population when the deforestation rate is not controlled. It can be seen it influences the population growth even to its lowest level of 0. Meanwhile, if the deforestation rate

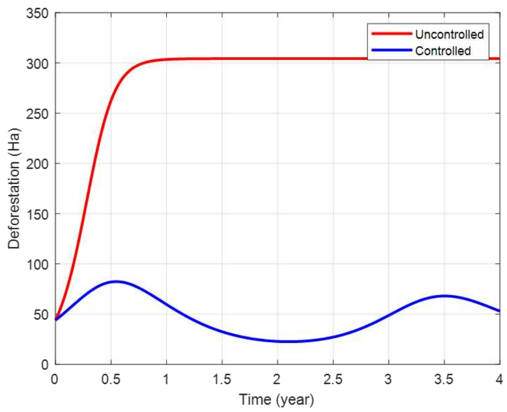


FIGURE 2. Comparison graph of the rate deforestation before and after given control.

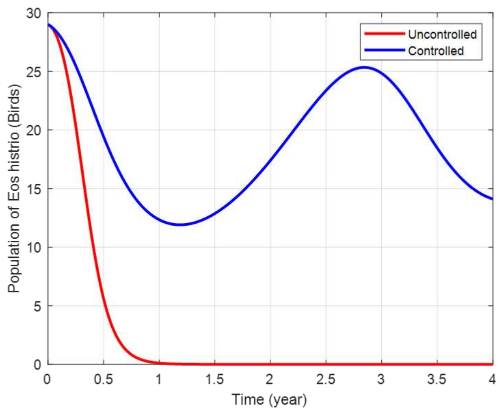
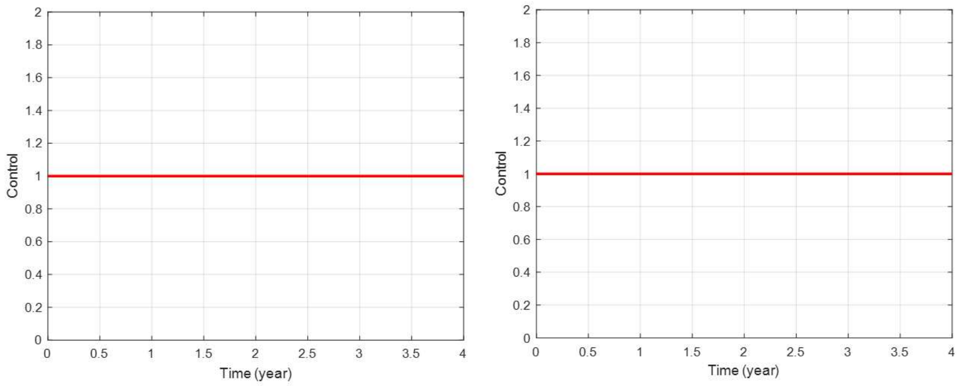


FIGURE 3. Comparison graph of the *Eos histrio* before and after given control.

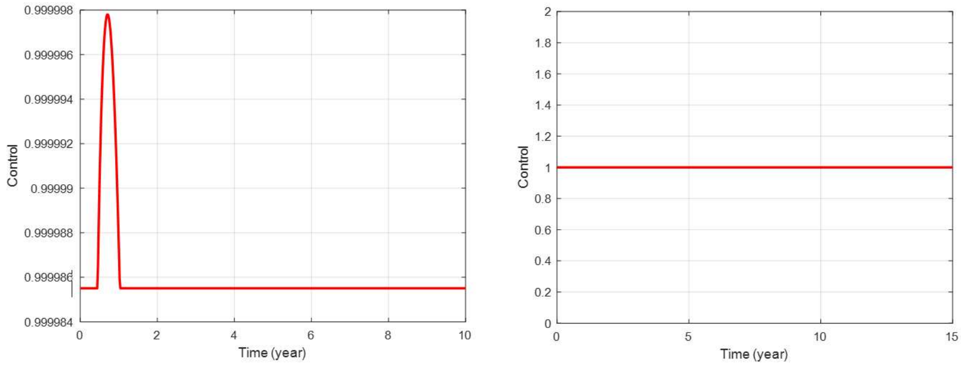
is controlled, the population of the bird is fluctuating. It correlates with the blue line in Figure 2. If the deforestation rate is controlled, the *Eos histrio* population could increase. On the other hand, if deforestation is uncontrolled, the *Eos histrio* population would be threatened.

Figure 4 shows deforestation rate has to be controlled by 10% for 4 years to optimally maintain the *Eos histrio* population. The value is constant so that the desired objective function can be achieved. It is indicated by the result of the objective function after the control value of 78.2973 was added, which is higher than



(a) Controlled by 10% for 4 years (b) Controlled by 5% for 4 years

FIGURE 4. Comparison of control that given to model for 4 years.



(a) Controlled by 5% for 10 years (b) Controlled by 10% for 15 years

FIGURE 5. Comparison of control that given to model for at least 10 years.

the objective function before the addition of the control value that is 0. Figure 4 show how if we compare deforestation rate has to be controlled by 5% and 10% for 4-years. The result is no different between them, although the value of objective function decrease into 67.9423. But, if we look for a long time, example in 10 years, there will have different in graphics control. In figure 5(a) there is graphic with controlled 5% for 10 years and the result is not constant. But, in figure 5(b) there is a graphic with controlled 10% for 15 years and more constant. So do in figure 6 there is a graphic with controlled 15% for 15 years and the result is constant. Therefore, deforestation rate has to be controlled at least by 10% to optimally maintain the *Eos histrio* population.

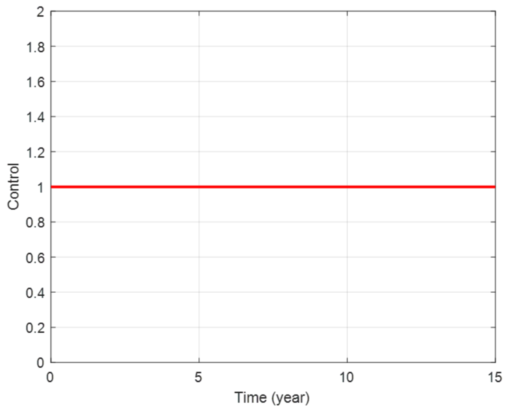


FIGURE 6. Deforestation rate controlled by 15% for 15 years.

3. CONCLUDING REMARKS

The existence of forest have an impact to *Eos histrio*s population. When the rate of deforestation can be controlled, the population of *Eos histrio* has the possibility to increase, and vice versa. With a forest destruction suppression rate at least 10%, the population of *Eos histrio* has the opportunity to increase when compared to no efforts to suppress the deforestation.

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