INTUITIONISTIC FUZZY SEMI δ-PREOPEN SETS AND INTUITIONISTIC FUZZY SEMI δ-PRECONTINUITY

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Abstract. The purpose of this paper is to introduce the concepts of fuzzy semi δ-preopen sets and fuzzy semi δ-precontinuous mappings in intuitionistic fuzzy topological spaces and obtain some of their properties and characterizations.

Key words and Phrases: Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy semi δ-preopen sets and Intuitionistic fuzzy semi δ-precontinuity

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [26]. Using the concept of fuzzy set Chang [2] introduced the concept of fuzzy topological spaces.

In 1986, Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy setss. In 1997, Coker [5] introduced the concepts of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. After the introduction of intuitionistic fuzzy topology by Coker [5], many mathematicians such as Eom and Lee [7], Hanafy [9, 10], Jeon [11], Coker and his associates [8, 3, 24], Thakur and his associates [17, 18, 19, 20, 21, 22] and Lupianez [13, 14, 15, 16] have been extended various fuzzy topological concepts in intuitionistic fuzzy topology.

In the present paper, we introduced the concept of intuitionistic fuzzy semi δ-preopen sets and intuitionistic fuzzy semi δ-precontinuous mappings and study some of the basic properties.

2. PRELIMINARIES

This section contains some basic denitions and preliminary results which will be needed in the sequel.

Definition 2.1. [1] *Let X be a nonempty xed set. An intuitionistic fuzzy set A is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ where the functions $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership namely $\mu_A(x)$ and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.*

Obviously, every fuzzy set A on a nonempty set X is an intuitionistic fuzzy set having the form $A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$.

For the basic properties of intuitionistic fuzzy set the reader should refer [1, 5].

Definition 2.2. [5] *Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping.*

(a): *If $B = \{< y, \mu_B(y), \nu_B(y) >: y \in Y\}$ is an intuitionistic fuzzy sets in Y, then the pereimage of B under f denoted and defined by*
$f^{-1}(B) = \{< f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) : x \in X\}$;

(b): *If $A = \{x, < \mu_A(x), \nu_A(x) >: x \in X\}$ is an intuitionistic fuzzy sets in X, then the image of A under f denoted and defined by*
$f(A) = \{< y, f(\mu_A)(y), f_-(\nu_A)(y) : y \in Y\}$;
where

$$f(\mu_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq 0 \\ 0, & otherwise, \end{cases} \quad (1)$$

and

$$f_-(\nu_A)(y) = 1 - f(1 - \nu_A)(y) = \begin{cases} inf_{x \in f^{-1}(y)} \nu_A(x), & f^{-1}(y) \neq 0 \\ 1, & otherwise. \end{cases} \quad (2)$$

Definition 2.3. [5] *An intuitionistic fuzzy topology on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfy the following axioms:*

(a): $\tilde{0}, \tilde{1} \in \tau$,
(b): $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,
(c): $\cup G_i \in \tau$ *for any arbitrary family* $\{G_i : i \in j\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X.

Definition 2.4. [5] *The Complement of A^c of an intuitionistic fuzzy open set A is an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy closed set in X.*

Definition 2.5. [5] *Let (X, τ) be an intuitionistic fuzzy topological space and let $A = < x, \mu_A(x), \nu_A(x) >$ be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are dened by*
$int(A) = \cup\{$ G $|$ G is an intuitionistic fuzzy open set in X and $G \subseteq A\}$,
$cl(A) = \cap\{$ K $|$ K is an intuitionistic fuzzy closed set in X and $A \subseteq K\}$.

Definition 2.6. [4] *Let $\alpha, \beta \in [0, 1]$ and $0 \leq \alpha + \beta \leq 1$. An intuitionistic fuzzy point $x_{(\alpha, \beta)}$ of X is an intuitionistic fuzzy set in X defined by*

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x \\ (0, 1) & \text{if } y \neq x \end{cases} \tag{3}$$

Definition 2.7. [4] *Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and $A = < X, \mu_A, \nu_A >$ is an intuitionistic fuzzy set in X. Then $x_{(\alpha,\beta)} \subseteq A$ if and only if $\alpha \subseteq \mu_A(x)$ and $1 - x_{1-\beta} \geq \nu_A$, or equivalently, $\alpha \subseteq \mu_A(x)$ and $\beta \supseteq \nu_A(x)$.*

Definition 2.8. [4] *Two intuitionistic fuzzy set A and B of X said to be q-coincident (denoted by A_qB) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.*

Lemma 2.9. [4] *For any two intuitionistic fuzzy sets A and B of X.*
$\rceil(A_qB) \Leftrightarrow A \subset B^c$.

Definition 2.10. [8] *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy regular open set if $A = int(cl(A))$.*

The family of all intuitionistic fuzzy regular open sets of an intuitionistic fuzzy topological space (X, τ) is denoted by $IFRO(X)$.

Definition 2.11. [8] *An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy intuitionistic fuzzy regular closed if $A^C \in IFRO(X)$.*

Remark 2.12. [8] *Every intuitionistic fuzzy regular open (resp. intuitionistic fuzzy regular closed) set is intuitionistic fuzzy open (resp. intuitionistic fuzzy closed)*

Definition 2.13. [25] *The δ-interior (denoted by δint) of an intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is the union of all intuitionistic fuzzy regular open sets contained in A.*

Definition 2.14. [25] *The δ-closure (denoted by δcl) of an intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is the intersection of all intuitionistic fuzzy regular closed sets containing A.*

Definition 2.15. [25] *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy δ-open if $A = \delta int(A)$. The complement of intuitionistic fuzzy δ-open set is called intuitionistic fuzzy δ-closed.*

Definition 2.16. *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called:*

- **(a):** *intuitionistic fuzzy semiopen if $A \subseteq cl(int(A))$. [8]*
- **(b):** *intuitionistic fuzzy preopen if $A \subseteq int(cl(A))$. [8]*
- **(c):** *intuitionistic fuzzy α-open if $A \subseteq int(cl(int(A)))$. [8]*
- **(d):** *intuitionistic fuzzy semi preopen if there exists an intuitionistic fuzzy preopen set O in X such that $O \subseteq A \subseteq (cl(O))$. [12]*
- **(e):** *intuitionistic fuzzy δ-preopen if $A \subseteq int(\delta cl(A))$. [25]*
- **(f):** *intuitionistic fuzzy δ-semiopen) if $A \subseteq cl(\delta int(A))$. [25]*
- **(g):** *intuitionistic fuzzy γ-open if $A \subseteq cl(int(A)) \cup int(cl(A))$. [10]*

The family of all intuitionistic fuzzy semiopen (resp. intuitionistic fuzzy preopen, intuitionistic fuzzy α-open, intuitionistic fuzzy semi preopen, intuitionistic fuzzy δ-preopen, intuitionistic fuzzy δ-semiopen, intuitionistic fuzzy γ-open) sets of an intuitionistic fuzzy topological space (X, τ) is denoted by $IFSO(X)$ (resp. $IFPO(X), IF\alpha O(X), IFSPO(X), IF\delta PO(X), IF\delta SO(X), IF\gamma O(X)$).

Definition 2.17. *An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy intuitionistic fuzzy semiclosed (resp. intuitionistic fuzzy preclosed, intuitionistic fuzzy α-closed, intuitionistic fuzzy semi preclosed, intuitionistic fuzzy δ-preclosed, intuitionistic fuzzy δ-semiclosed, intuitionistic fuzzy γ-closed) if $A^C \in IFSO(X)$ (resp. $IFPO(X), IF\alpha O(X), IFSPO(X), IF\delta PO(X), IF\delta SO(X), IF\gamma O(X)$).*

Remark 2.18. [6] *Every intuitionistic fuzzy δ-open (resp. intuitionistic fuzzy δ-closed) set is intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) but the converse may not be true.*

Remark 2.19. [8] *Every intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) set is intuitionistic fuzzy α-open (resp. intuitionistic fuzzy α-closed), and every intuitionistic fuzzy α-open (resp. intuitionistic fuzzy α-closed) set is intuitionistic fuzzy semiopen (resp. intuitionistic fuzzy semiclosed) as well as intuitionistic fuzzy preopen (resp. intuitionistic fuzzy preclosed). But the separate converses may not be true.*

Remark 2.20. [12] *Every intuitionistic fuzzy semiopen (resp. intuitionistic fuzzy semiclosed) set and every intuitionistic fuzzy preopen (resp. intuitionistic fuzzy preclosed) set is intuitionistic fuzzy semi-preopen (resp. intuitionistic fuzzy semi-preclosed). But the separate converses may not be true.*

Remark 2.21. [23] *Every intuitionistic fuzzy preopen (resp. intuitionistic fuzzy preclosed) set is intuitionistic fuzzy δ-preopen (resp. intuitionistic fuzzy δ-preclosed) but the converse may not be true.*

Remark 2.22. [10] *Every intuitionistic fuzzy semiopen (resp. intuitionistic fuzzy preopen) set is intuitionistic fuzzy γ-open and intuitionistic fuzzy γ-open set is intuitionistic fuzzy semi-preopen but the separate converses may not be true.*

Definition 2.23. [25] *The δ-pre interior of an intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is the union of all intuitionistic fuzzy δ-pre open sets contained in A and it is denoted by $\delta pint(A)$).*

Definition 2.24. [25] *The δ-pre closure of an intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is the intersection of all intuitionistic fuzzy δ-pre closed sets which contain A and it is denoted by $\delta pcl(A)$).*

Remark 2.25. [25] *If A be an intuitionistic fuzzy set in (X, τ) then $A \subseteq \delta pcl(A) \subseteq cl(A)$.*

Definition 2.26. *A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called:*

- **(a):** *intuitionistic fuzzy semi continuous if $f^{-1}(A) \in IFSO(X)$ for each open set A of Y.* [8]
- **(b):** *intuitionistic fuzzy pre continuous if $f^{-1}(A) \in IFPO(X)$ for each open set A of Y.* [8]
- **(c):** *intuitionistic fuzzy α-continuous if $f^{-1}(A) \in IF\alpha O(X)$ for each open set A of Y.* [11]
- **(d):** *intuitionistic fuzzy semi pre continuous if $f^{-1}(A) \in IFSPO(X)$ for each open set A of Y.* [12]
- **(e):** *intuitionistic fuzzy δ-pre continuous if $f^{-1}(A) \in IF\delta PO(X)$ for each open set A of Y.* [23]
- **(g):** *intuitionistic fuzzy γ-continuous if $f^{-1}(A) \in IF\gamma O(X)$ for every intuitionistic fuzzy set $A \in IF\gamma O(Y)$.* [10]

Remark 2.27. [8] *Every intuitionistic fuzzy continuous mappings is intuitionistic fuzzy α-continuous, Every intuitionistic fuzzy α-continuous mapping is intuitionistic fuzzy semi continuous and intuitionistic fuzzy pre continuous and every intuitionistic fuzzy semi continuous (resp. intuitionistic fuzzy pre continuous) mapping is intuitionistic fuzzy semi pre continuous. But the converse may not be true.*

Remark 2.28. [10] *Every intuitionistic fuzzy semi continuous (resp. intuitionistic fuzzy pre continuous) mapping is intuitionistic fuzzy γ-continuous and intuitionistic fuzzy γ-continuous mapping is intuitionistic fuzzy semi pre continuous but the separate converses may not be true.*

Remark 2.29. [23] *Every intuitionistic fuzzy pre continuous mapping is intuitionistic fuzzy δ-pre continuous but the converse may not be true.*

Remark 2.30. [11] *The concepts of intuitionistic fuzzy semi continuous and intuitionistic fuzzy pre continuous mappings are independent.*

3. INTUITIONISTIC FUZZY SEMI δ-PREOPEN SETS

In this section, we introduce the concept of intuitionistic fuzzy semi δ-preopen set and study some of their properties in intuitionistic fuzzy topological spaces.

Definition 3.1. *An Intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called:*

(a): *intuitionistic fuzzy semi δ-preopen if there exists an intuitionistic fuzzy δ-preopen set O such that $O \subseteq A \subseteq \delta cl(O)$.*
(b): *intuitionistic fuzzy semi δ-preclosed if there exists an intuitionistic fuzzy δ-preclosed set F such that $\delta int(F) \subseteq A \subseteq F$.*

The family of all intuitionistic fuzzy semi δ-preopen (resp. intuitionistic fuzzy semi δ-preclosed) sets of an intuitionistic fuzzy topological space (X, τ) is denoted by $IFS\delta PO(X)$ (resp. $IFS\delta PC(X)$).

Remark 3.2. *Every intuitionistic fuzzy δ-semiopen (resp. intuitionistic fuzzy δ-semiclosed)set is intuitionistic fuzzy semiopen (resp. intuitionistic fuzzy semiclosed) but the converse may not be true.*

Example 3.3. *Let $X = \{a, b\}$ and intuitionistic fuzzy sets A, B, O are defined as follows:*

$$A = \{< a, 0.3, 0.7 >, < b, 0.4, 0.6 >\}$$

$$B = \{< a, 0.4, 0.6 >, < b, 0.3, 0.7 >\}$$

$$O = \{< a, 0.5, 0.5 >, < b, 0.6, 0.4 >\}$$

let $\tau = \{\tilde{0}, A, B, A \cup B, A \cap B, \tilde{1}\}$ be the intuitionistic fuzzy topology on (X, τ). Then O is intuitionistic fuzzy semiopen but not intuitionistic fuzzy δ-semiopen.

Remark 3.4. *Every intuitionistic fuzzy δ-open (resp. intuitionistic fuzzy δ-closed) set is intuitionistic fuzzy δ-semiopen (resp. intuitionistic fuzzy δ-semiclosed) but the converse may not be true. For, in the intuitionistic fuzzy topological space (X, τ) of example (3.3), the intuitionistic fuzzy set A is intuitionistic fuzzy δ-semiopen but not intuitionistic fuzzy δ-open.*

Remark 3.5. *The concepts of intuitionistic fuzzy δ-semiopen and intuitionistic fuzzy open sets are independent. For, in the intuitionistic fuzzy topological space (X, τ) of example (3.3), the intuitionistic fuzzy set O is intuitionistic fuzzy δ-semiopen but not intuitionistic fuzzy open and intuitionistic fuzzy A is intuitionistic fuzzy open but not intuitionistic fuzzy δ-semiopen.*

Theorem 3.6. *An intuitionistic fuzzy set $A \in IFS\delta PC(X)$ if and only if $A^C \in IFS\delta PO(X)$.*

Remark 3.7. *Every intuitionistic fuzzy semi preopen (resp. intuitionistic fuzzy semi preclosed) set and Every intuitionistic fuzzy δ-preopen (resp. intuitionistic fuzzy δ-preclosed) set is intuitionistic fuzzy semi δ-preopen (resp. intuitionistic fuzzy semi δ-preclosed). But the separate converse may not be true.*

Example 3.8. *Let $X = \{a, b\}$ and intuitionistic fuzzy sets A, B, O, F are intuitionistic fuzzy sets defined as follows:*

$$A = \{< a, 0.5, 0.5 >, < b, 0.3, 0.7 >\}$$

$$B = \{< a, 0.5, 0.5 >, < b, 0.1, 0.9 >\}$$

$$O = \{< a, 0.5, 0.5 >, < b, 0.9, 0.1 >\}$$

$$F = \{< a, 0.5, 0.5 >, < b, 0.6, 0.4 >\}$$

let $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ be an intuitionistic fuzzy topology on (X, τ), Then

   (a): *O is intuitionistic fuzzy semi δ-preopen (resp. O^C intuitionistic fuzzy semi preclosed) but not intuitionistic fuzzy semi preopen (resp. intuitionistic fuzzy semi preclosed).*
   (b): *F is intuitionistic fuzzy semi δ-preopen (resp. F^C is intuitionistic fuzzy semi preclosed) but not intuitionistic fuzzy δ-preopen (resp. intuitionistic fuzzy δ-preclosed).*

Remark 3.9. *It is clear that from remark (2.12), (2.18), (2.19), (2.20), (2.21), (2.22), (3.2), (3.4), (3.5) and (3.7) that the following figure of implications is true.*
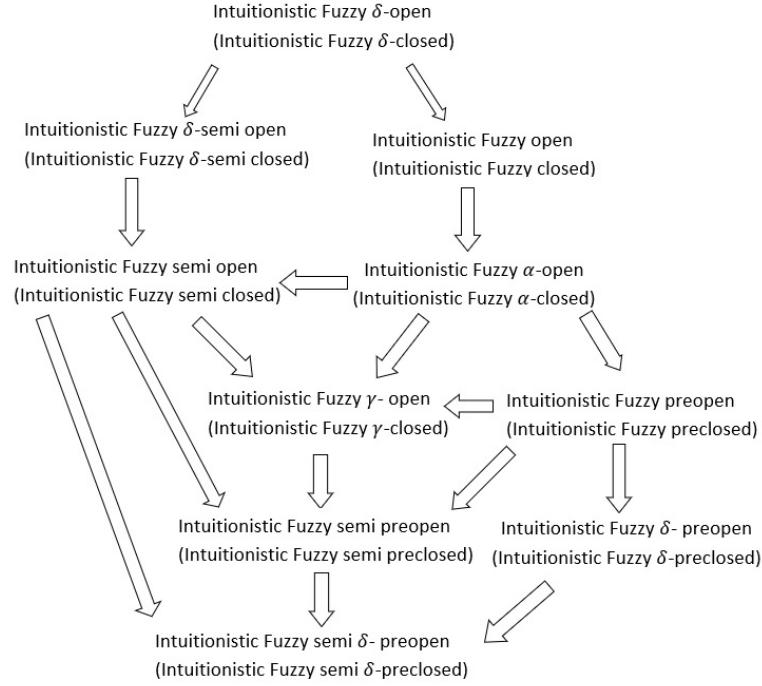
FIGURE 1

Theorem 3.10. *Let (X, τ) be an intuitionistic fuzzy topological space. Then*

(a): *Any union of intuitionistic fuzzy semi δ-preopen sets is intuitionistic fuzzy semi δ-preopen.*

(b): *Any intersection of intuitionistic fuzzy semi δ-preclosed sets is intuitionistic fuzzy semi δ-preclosed.*

PROOF. Obvious.

Theorem 3.11. *An Intuitionistic fuzzy set $A \in IFS\delta PO(X)$ if and only if for every intuitionistic fuzzy point $x_{(\alpha,\beta)} \in A$ there exists an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)} \in O \subseteq A$.*

PROOF. If $A \in IFS\delta PO(X)$ then we may take $O = A$ for every $x_{(\alpha,\beta)} \in A$.
Conversely. We have $A =_{x_{(\alpha,\beta)} \in A} \cup \{x_{(\alpha,\beta)}\} \subseteq_{x_{(\alpha,\beta)} \in A} \cup O \subseteq A$.

The result now follows from the fact that any union of intuitionistic fuzzy δ-preopen sets is intuitionistic fuzzy δ-preopen.

Theorem 3.12. *Let (X, τ) be an intuitionistic fuzzy topological space.*

  (a): *If $A \subseteq O \subseteq \delta cl(A)$ and $A \in IFS\delta PO(X)$ then $O \in IFS\delta PO(X)$,*
  (b): *If $\delta int(B) \subseteq F \subseteq B$ and $B \in IFS\delta PC(X)$ then $F \in IFS\delta PC(X)$*

PROOF. (a) Let $O_1 \in IF\delta PO(X)$ such that $O_1 \subseteq A \subseteq \delta cl(O_1)$.
Clearly $O_1 \subseteq O$ and $A \subseteq \delta cl(O_1)$ implies that $\delta cl(A) \subseteq \delta cl(O_1)$. Consequently,
$O_1 \subseteq O \subseteq \delta cl(O_1)$. Hence $O \in IFS\delta PO(X)$.
(b) Follows from (a).

Lemma 3.13. *An intuitionistic fuzzy set $A \in IF\delta PO(X)$ if and only if there exists an intuitionistic fuzzy set O such that $A \subseteq O \subseteq \delta cl(A)$.*

PROOF. **Necessity** If $A \in IF\delta PO(X)$ then $A \subseteq int(\delta cl(A))$.
Put $O = int(\delta cl(A))$ then O is an intuitionistic fuzzy open and $A \subseteq O \subseteq \delta cl(A)$.
Sufficiency Let O be an intuitionistic fuzzy open set such that $A \subseteq O \subseteq \delta cl(A)$,
then $A \subseteq int(O) \subseteq int(\delta cl(A))$. Hence $A \in IF\delta PO(X)$.

Lemma 3.14. *Let Y be an intuitionistic fuzzy subspace of intuitionistic fuzzy topological space (X, τ) and A be an intuitionistic fuzzy set in Y. If $A \in IF\delta PO(X)$ then $A \in IF\delta PO(Y)$.*

PROOF. Since $A \in IF\delta PO(X)$, By Lemma (3.13), there exists an intuitionistic fuzzy set O in (X, τ) such that $A \subseteq O \subseteq \delta cl(A)$. Therefore $A \cap Y \subseteq O \cap Y \subseteq \delta cl(A) \cap Y = \delta cl_Y(A)$. It follows that $A \subseteq O \subseteq \delta cl_Y(A)$. Hence by Lemma (3.13), $A \in IF\delta PO(Y)$.

Theorem 3.15. *Let Y be an intuitionistic fuzzy subspace of intuitionistic fuzzy topological space (X, τ) and A be an intuitionistic fuzzy set in Y. If $A \in IFS\delta PO(X)$ then $A \in IFS\delta PO(Y)$.*

PROOF. Let $O \in IF\delta PO(X)$ such that $O \subseteq A \subseteq cl(O)$. Then $O \cap Y \subseteq A \cap Y \subseteq cl(O) \cap Y$. It follows that $O \subseteq A \subseteq cl_Y(O)$. Now by Lemma (3.14), $O \in IF\delta PO(Y)$ and hence $A \in IFS\delta PO(Y)$.

Theorem 3.16. *Let X and Y be intuitionistic fuzzy topological space, such that X is product related to Y.*

  (a): *If $A \in IF\delta PO(X)$ and $O \in IF\delta PO(Y)$, then $A \times O \in IF\delta PO(X \times Y)$.*
  (b): *IF $A \in IFS\delta PO(X)$ and $O \in IFS\delta PO(Y)$, then $A \times O \in IFS\delta PO(X \times Y)$.*

PROOF. **(a)** IF $A \in IF\delta PO(X)$ and $O \in IF\delta PO(Y)$. Then $A \times O \subseteq int(\delta cl(A)) \times int(\delta cl(O)) = int(\delta cl(A \times O))$.
(b) Let $F \subseteq A \subseteq \delta cl(F)$ and $\psi \subseteq O \subseteq \delta cl(\psi)$, $F \in IF\delta PO(X)$ and $\psi \in IF\delta PO(Y)$. Then $F \times \psi \subseteq A \times O \subseteq \delta cl(F) \times \delta cl(\psi) = \delta cl(A \times \psi)$. Now the result follows from (a).

Definition 3.17. *Let (X, τ) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set of X. Then the intuitionistic fuzzy semi δ-preinterior (denoted by $s\delta pint$) and intuitionistic fuzzy semi δ-preclosure (denoted by $s\delta pcl$) of A respectively defined as follows:*
$s\delta pint(A) = \cup\{O : O \subseteq A; O \in IFS\delta PO(X)\}$,
$s\delta pcl(A) = \cap\{O : O \supseteq A; O \in IFS\delta PC(X)\}$.

The following theorem can be easily verified.

Theorem 3.18. *Let A and O be intuitionistic fuzzy sets in an intuitionistic fuzzy topological space (X, τ). Then:*

(a): $s\delta pcl(A) \subseteq cl(A)$
(b): $s\delta pcl(A)$ *is an intuitionistic fuzzy semi δ-preclosed.*
(c): $A \in IFS\delta PC(X) \Leftrightarrow A = s\delta pcl(A)$.
(d): $A \subseteq O \Rightarrow s\delta pcl(A) \subseteq s\delta pcl(O)$.
(e): $int(A) \subseteq s\delta pint(A)$.
(f): $s\delta pint(A)$ *is an intuitionistic fuzzy semi preopen.*
(g): $A \in IFS\delta PO(X) \Leftrightarrow A = s\delta pint(A)$.
(h): $A \subseteq O \Rightarrow s\delta pint(A) \subseteq s\delta pint(O)$.
(i): $s\delta pint(1 - A) = 1 - s\delta pcl(A)$.

Definition 3.19. *Let A be an intuitionistic fuzzy sets in an intuitionistic fuzzy topological space (X, τ) and $x_{(\alpha,\beta)}$ is an intuitionistic fuzzy point of X. Then A is called:*

(a): *intuitionistic fuzzy semi δ-preneighborhood of $x_{(\alpha,\beta)}$ if there exists an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)} \in O \subseteq A$.*
(b): *intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$ if there exists an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O \subseteq A$.*

Theorem 3.20. *An intuitionistic fuzzy set $A \in IFS\delta PO(X)$ if and only if for each intuitionistic fuzzy point $x_{(\alpha,\beta)} \in A$, A is an intuitionistic fuzzy semi δ-preneighborhood of $x_{(\alpha,\beta)}$.*

PROOF. Obvious

Theorem 3.21. *Let A be an intuitionistic fuzzy sets in an intuitionistic fuzzy topological space (X, τ). Then an intuitionistic fuzzy point $x_{(\alpha,\beta)} \in s\delta pcl(A)$, if and only if every intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$ is quasi-coincident with A.*

PROOF. **Necessity** Suppose $x_{(\alpha,\beta)} \in s\delta pcl(A)$ and if possible let there exists an intuitionistic fuzzy semi δ-pre Q-neighborhood O of $x_{(\alpha,\beta)}$ such that $\rceil(O_q A)$. Then there exists an intuitionistic fuzzy set $O_1 \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O_1 \subseteq O_1 \subseteq O$ which show that $\rceil(O_{1q}A)$ and hence $A \subseteq O_1^c$. As $O_1^c \in IFS\delta PC(X)$,

$s\delta pcl(A) \subseteq O_1^c$. Since $x_{(\alpha,\beta)} \in O_1^c$, we obtain that $x_{(\alpha,\beta)} \notin s\delta pcl(A)$ which is a contradiction.

Sufficiency Suppose every intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$ is quasi-coincident with A. If $x_{(\alpha,\beta)} \notin s\delta pcl(A)$ then there exists an intuitionistic fuzzy semi δ-preclosed set $O \supseteq A$ such that $x_{(\alpha,\beta)} \notin O$. So $O^c \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O^c$ and $\rceil(O_q^c A)$ a contradiction.

Definition 3.22. *A mappings $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy δ-preirresolute if $f^{-1}(A) \in IFS\delta PO(X)$ for every intuitionistic fuzzy set $A \in IF\delta PO(Y)$.*

Theorem 3.23. *If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy δ-pre irresolute and intuitionistic fuzzy open mapping, then $f^{-1}(A) \in IFS\delta PO(X)$, for every $A \in IFS\delta PO(Y)$.*

PROOF. Let $A \in IFS\delta PO(Y)$. Then there exists an intuitionistic fuzzy set $O \in IF\delta PO(X)$ such that $O \subseteq A \subseteq \delta cl(O)$. Therefore $f^{-1}(O) \subseteq f^{-1}(A) \subseteq f^{-1}(\delta cl(O))$ since f is intuitionistic fuzzy open and δ-pre irresolute. $f^{-1}(O) \subseteq f^{-1}(A) \subseteq f^{-1}(\delta cl(O)) \subseteq \delta cl(f^{-1}(O))$ and $f^{-1}(O) \in IFS\delta PO(X)$. Hence $f^{-1}(A) \in IFS\delta PO(X)$.

4. INTUITIONISTIC FUZZY SEMI δ-PRECONTINUOUS MAPPINGS

Definition 4.1. *A mappings $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy semi δ-precontinuous if $f^{-1}(A) \in IFS\delta PO(X)$ for every intuitionistic fuzzy open set A of Y.*

Remark 4.2. *Every intuitionistic fuzzy δ-pre continuous (resp. intuitionistic fuzzy semi precontinuous) mappings is intuitionistic fuzzy semi δ-precontinuous but the converse may not be true.*

Example 4.3. *Let $X = \{a, b\}$ and $Y = \{p, q\}$ and intuitionistic fuzzy sets A, B, O, F are defined as follows:*

$$A = \{< a, 0.5, 0.5 >, < b, 0.3, 0.7 >\}$$
$$B = \{< a, 0.5, 0.5 >, < b, 0.1, 0.9 >\}$$
$$O = \{< p, 0.5, 0.5 >, < q, 0.9, 0.1 >\}$$
$$F = \{< p, 0.5, 0.5 >, < q, 0.6, 0.4 >\}$$

let $\tau_1 = \{\tilde{0}, A, B, \tilde{1}\}$, $\tau_2 = \{\tilde{0}, O, \tilde{1}\}$ and $\tau_3 = \{\tilde{0}, F, \tilde{1}\}$. Then the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = p$, $f(b) = q$ is an intuitionistic fuzzy semi δ-pre continuous but not intuitionistic fuzzy semi precontinuous and the mapping $g : (X, \tau_1) \to (Y, \tau_3)$ defined by $g(a) = p$, $g(b) = q$ is an intuitionistic fuzzy semi δ-precontinuous but not intuitionistic fuzzy δ-pre continuous.

Remark 4.4. *Remark* (2.27), (2.28), (2.29) *and* (4.2) *reveals that the following figure of implications is true.*
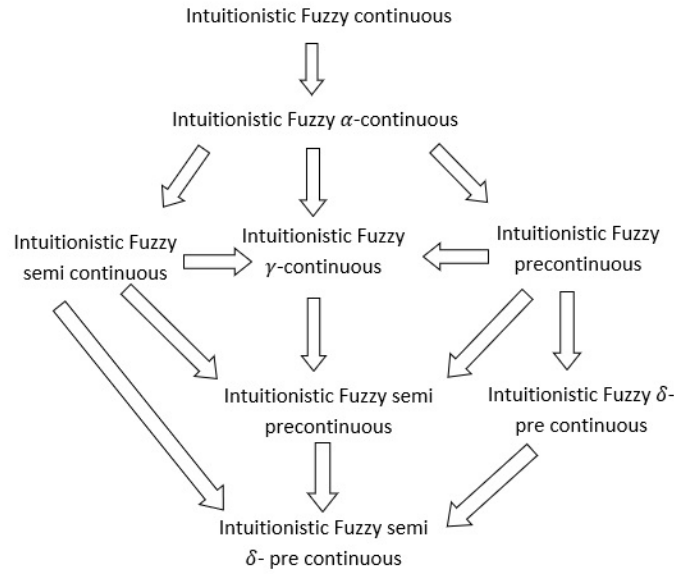

Intuitionistic Fuzzy continuous

Intuitionistic Fuzzy α-continuous

Intuitionistic Fuzzy semi continuous

Intuitionistic Fuzzy γ-continuous

Intuitionistic Fuzzy precontinuous

Intuitionistic Fuzzy semi precontinuous

Intuitionistic Fuzzy δ-pre continuous

Intuitionistic Fuzzy semi δ-pre continuous

FIGURE 2

Theorem 4.5. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a mapping from an intuitionistic fuzzy topological space* (X, τ) *to intuitionistic fuzzy topological space* (Y, σ). *Then the following statements are equivalent:*

(a): *f is intuitionistic fuzzy semi δ-precontinuous.*

(b): *for every intuitionistic fuzzy closed set A in Y,* $f^{-1}(A) \in IFS\delta PC(X)$.

(c): *for every intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ *in X and every intuitionistic fuzzy open set A such that* $f(x_{(\alpha,\beta)}) \in IFS\delta PO(X)$ *there is an intuitionistic fuzzy set* $O \in IFS\delta PO(X)$ *such that* $x_{(\alpha,\beta)} \in O$ *and* $f(O) \subseteq A$.

(d): *for every intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ *of X and every neighborhood A of* $f(x_{(\alpha,\beta)})$, $f^{-1}(A)$ *is an intuitionistic fuzzy semi δ-pre neighborhood of* $x_{(\alpha,\beta)}$.

(e): *for every intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ *of X and every neighborhood A of* $f(x_{(\alpha,\beta)})$, *there is an intuitionistic fuzzy semi δ-pre neighborhood U of* $x_{(\alpha,\beta)}$ *such that* $f(U) \subseteq A$.

(f): *for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ of X and every intuitionistic fuzzy open set A such that $f(x_{(\alpha,\beta)})_q A$, there is an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O$ and $f(O) \subseteq A$.*

(g): *for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ of X and every Q-neighborhood A of $f(x_{(\alpha,\beta)})$, $f^{-1}(A)$ is an intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$.*

(h): *for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ of X and every Q-neighborhood A of $f(x_{(\alpha,\beta)})$, there is an intuitionistic fuzzy semi pre Q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \subseteq A$.*

(i): $f(s\delta pcl(A)) \subseteq cl(f(A))$, *for every intuitionistic fuzzy set A of X.*

(j): $s\delta pcl(f^{-1}(O)) \subseteq f^{-1}(cl(O))$, *for every intuitionistic fuzzy set O of Y.*

(k): $f^{-1}(int(0)) \subseteq s\delta pint(f^{-1}(O))$, *for every intuitionistic fuzzy set O of Y.*

Proof. $(a) \Rightarrow (b)$ Obvious.

$(a) \Rightarrow (c)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be an intuitionistic fuzzy open set in Y such that $f(x_{(\alpha,\beta)}) \in A$. Put $O = f^{-1}(A)$, then by (a), $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)} \in O$ and $f(O) \subseteq A$.

$(c) \Rightarrow (a)$ Let A be an intuitionistic fuzzy open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(A)$. Then $f(x_{(\alpha,\beta)}) \in A$. Now by (c) there is an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)} \in O$ and $f(O) \subseteq A$. Then $x_{(\alpha,\beta)} \in O \subseteq f^{-1}(A)$. Hence by theorem (3.11), $f^{-1}(A) \in IFS\delta PO(X)$.

$(a) \Rightarrow (d)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be a neighborhood of $f(x_{(\alpha,\beta)})$. Then there is an intuitionistic fuzzy open set U such that $f(x_{(\alpha,\beta)}) \in U \subseteq A$. Now $F^{-1}(U) \in IFS\delta PO(X)$ and $x_{(\alpha,\beta)} \in f^{-1}(U) \subseteq f^{-1}(A)$. Thus $f^{-1}(A)$ is an intuitionistic fuzzy semi pre neighborhood of $x_{(\alpha,\beta)}$ in X.

$(d) \Rightarrow (e)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be a neighborhood of $f(x_{(\alpha,\beta)})$. Then $U = f^{-1}(A)$ is an intuitionistic fuzzy semi δ-pre neighborhood of $x_{(\alpha,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

$(e) \Rightarrow (c)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be an intuitionistic fuzzy open set such that $f(x_{(\alpha,\beta)}) \in A$. Then A is a neighborhood of $f(x_{(\alpha,\beta)})$. So there is intuitionistic fuzzy semi δ-pre neighborhood U of $x_{(\alpha,\beta)}$ in X such that $x_{(\alpha,\beta)} \in U$ and $f(U) \subseteq A$. Hence there is an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)} \in O \subseteq U$ and so $f(O) \subseteq f(U) \subseteq A$.

$(a) \Rightarrow (f)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be an intuitionistic fuzzy open set in Y such that $f(x_{(\alpha,\beta)})_q \in A$. Let $O = f^{-1}(A)$. Then $O \in IFS\delta PO(X)$, $x_{(\alpha,\beta)q}O$ and $f(O) = f(f^{-1}(A)) \subseteq A$.

$(f) \Rightarrow (a)$ Let A be an intuitionistic fuzzy open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(A)$

clearly $f(x_{(\alpha,\beta)}) \in A$, choose the intuitionistic fuzzy point $x^c_{(\alpha,\beta)}$ defined as

$$x^c_{(\alpha,\beta)}(z) = \begin{cases} (\beta, \alpha) & \text{if } z = x \\ (1,0) & \text{if } z \neq x \end{cases} \tag{4}$$

Then $f(x_{(\alpha,\beta)})_q A$ and so by (f), there exists an intuitionistic fuzzy set $O \in IFS\delta PO(X)$, such that $x_{(\alpha,\beta)q}O$ and $f(O) \subseteq A$.

Now $x^c_{(\alpha,\beta)q}O$ implies $x_{(\alpha,\beta)} \in O$.

Thus $x_{(\alpha,\beta)} \in O \subseteq f^{-1}(A)$. Hence by theorem (3.11) $f^{-1} \in IFS\delta PO(X)$.

$(f) \Rightarrow (g)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be a Q-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is an intuitionistic fuzzy open set A_1 in Y such that $f(x_{(\alpha,\beta)})_q A_1 \subseteq A$. By hypothesis there is an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O$ and $f(O) \subseteq A_1$. Thus $x_{(\alpha,\beta)q}O \subseteq f^{-1}(A_1) \subseteq f^{-1}(A)$.

Hence $f^{-1}(A)$ is an intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$.

$(g) \Rightarrow (h)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be a Q-neighborhood of $f(x_{(\alpha,\beta)})$. Then $U = f^{-1}(A)$ is an intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

$(g) \Rightarrow (f)$ Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X and A be an intuitionistic fuzzy open set such that $f(x_{(\alpha,\beta)})_q \in A$. Then A is Q-neighborhood of $f(x_{(\alpha,\beta)})$. So there is an intuitionistic fuzzy semi δ-pre Q-neighborhood δ of $x_{(\alpha,\beta)}$ such that $f(U) \subseteq A$. Now U being an intuitionistic fuzzy semi δ-pre Q-neighborhood of $x_{(\alpha,\beta)}$. Then there exists an intuitionistic fuzzy set $O \in IFS\delta PO(X)$ such that $x_{(\alpha,\beta)q}O \subseteq U$. Hence $x_{(\alpha,\beta)q}O$ and $f(O) \subseteq f(U) \subseteq A$.

$(b) \Leftrightarrow (i)$ Obvious.
$(h) \Leftrightarrow (j)$ Obvious.
$(j) \Leftrightarrow (k)$ Obvious.

Theorem 4.6. *Let X, X_1, X_2 be an intuitionistic fuzzy topological spaces and $p_i : X_1 \times X_2 \to X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ into X_i. Then if $f : X_1 \times X_2$ is an intuitionistic fuzzy semi δ- pre continuous mapping, it follows that $p_i o f$ is also an intuitionistic fuzzy semi δ- pre continuous mapping.*

Theorem 4.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. If the graph mapping $g : X \to X \times Y$ of f is an intuitionistic fuzzy semi δ-pre continuous, then f is an intuitionistic fuzzy semi δ-pre continuous.*

PROOF. Let O be an intuitionuistic fuzzy open set of Y. Then $\tilde{1} \times O$ is an intuitionistic fuzzy open in $X \times Y$. Since g is an intuitionistic fuzzy semi δ-pre continuous, $g^{-1}(\tilde{1} \times O) \in IFS\delta PO(X)$. But $f^{-1}(O) = \tilde{1} \cap f^{-1}(O) = g^{-1}(\tilde{1} \times O)$, $f^{-1}(O) \in IFS\delta PO(X)$. Hence f is an intuitionistic fuzzy semi δ-pre continuous.

Theorem 4.8. *Let X_i and X_i^* $(i = 1, 2)$ be an intuitionistic fuzzy topological spaces such that X_1 is product related to X_2. If $f_i : X_i \to X_i^*$ $(i = 1, 2)$ is an intuitionistic fuzzy semi δ-pre continuous, then $f_1 \times f_2 : X_1 \times X_2 \to X_i^* \times X_2^*$ is an intuitionistic fuzzy semi δ-pre continuous.*

PROOF. Let $A = \cup(A_\alpha \times O_\beta)$ where $A_\alpha's$ and $O_\beta's$ are intuitionistic fuzzy open sets of X_1^* and X_2^* respectively, be an intuitionistic fuzzy open sets of $X_1^* \times X_2^*$. We obtain $(f_1 \times f_2)^{-1}(A) = \cup\{f_1^{-1}(A_\alpha) \times f_2^{-1}(O_\beta)\}$. Since f_1 and f_2 are intuitionistic fuzzy semi δ-pre continuous, $f_1^{-1}(A_\alpha) \in IFS\delta PO(X_1)$ and $f_2^{-1}(O_\beta) \in IFS\delta PO(X_2)$. Therefore by theorem $(3.16)(b)$, $f_1^{-1}(A_\alpha) \times f_2^{-1}(O_\beta) \in IFS\delta PO(X_1 \times X_2)$. Hence by theorem (3.10), $(f_1 \times f_2)^{-1}(A) \in IFS\delta PO(X_1 \times X_2)$.

REFERENCES

[1] Atanassov, K.T., "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, **20** (1986), 87-96.

[2] Chang, C. L., "Fuzzy topological spaces", *J. Math. Anal. Appl.*, **24** (1968), 182-190.

[3] Coker, D. and Es, A. H., "On fuzzy compactness in intuitionistic fuzzy topological spaces", *J. Fuzzy Math*, **3** (1995), (4), 899-909.

[4] Coker, D. and Demirci, M., "On intuitionistic fuzzy points", *Notes on Intuitionistic Fuzzy Sets*, **2**, (1), (1995), 79-84.

[5] Coker, D., "An introduction to intuitionistic fuzzy topological space", *Fuzzy Sets and Systems*, **88** (1) (1997), 81-89.

[6] Eom, Y. S. and Lee, S. J., "Delta closure and delta interior in intuitionistic fuzzy topological spaces", *International Journal of Fuzzy Logic and Intelligent Systems*, **12** (4), (2012), 290-295.

[7] Eom, Y. S. and Lee, S. J., "Intuitionistic fuzzy δ-continuous functions", *International Journal of Fuzzy Logic and Intelligent Systems*, **13** (4) (2013), 336-344.

[8] Gurcay, H., Es, A. H., and Coker, D., "On fuzzy continuity in intuitionistic fuzzy topological spaces", *J. Fuzzy Math*, **5** (1997), (2), 365-378.

[9] Hanafy, I. M., "Intuitionistic fuzzy functions", *International Journal of Fuzzy Logic and Intelligent systems*, **3** (2), (2003), 200-205.

[10] Hanafy, I. M., "Intuitionistic fuzzy γ-continuity", *Canad. Math. Bull.*, **52** (4) (2009), 544-554.

[11] Jeon, J. K., Jun, Y. B. and Park, J. H., "Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity", *Int. J. Math. Math. Sci.*, **19** (2005), 3091-3101.

[12] Jun, Y. B. and Song, S., "Intuitionistic fuzzy semi-preopen sets and intuitionistic fuzzy semi-precontinuous mappings", *J. Appl. Math. & Computing*, **19** (2005), 1-2, 467-474.

[13] Lupianez, F. G., "Separation in intuitionistic fuzzy topological spaces", *Int. J. Pure Appl. Math.*, **17** (1) (2004), (b), 29-34.

[14] Lupianez, F. G., "Quasicoincidence for intuitionistic fuzzy points", *Int. J. Math. Math. Sci.*, **10** (2005), 1539-1542.

[15] Lupianez, F. G., "Nets and filters in intuitionistic fuzzy topological spaces", *Inform. Sci.*, **176** (2006), 2396-2404.

[16] Lupianez, F. G., "Covering properties in intuitionistic fuzzy topological spaces", *Kybernetes*, **36** (2007), 749-753.

[17] Thakur, S. S. and Chaturvedi, R., Generalized continuity in intuitionistic fuzzy topological Spaces, *Notes on Intuitionistic Fuzzy Sets*, **12** (1) (2006), 38-44.

[18] Thakur, S. S. and Chaturvedi, R., "Regular generalized closed sets in intuitionistic fuzzy topological Spaces", *Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau*, **16** (2006), 252-272.

[19] Thakur, S. S. and Chaturvedi, R., "Generalized closed sets in intuitionistic fuzzy topology", *The Journal of Fuzzy Mathematics*, **16** (2) (2008), 559-572.

[20] Thakur, S. S. and Chaturvedi, R., "Intuitionistic fuzzy *gc*-irresolute mappings", *Math. Notae. Ano XLV*, (2007-2008), 59-65.

[21] Thakur, S. S.and Bajpai, J. P., "Intuitionistic fuzzy *rw*-closed sets and intuitionistic fuzzy *rw*-continuity", *Notes on Intuitionistic fuzzy sets*, **17** (2) (2011), 82-96.

[22] Thakur, S. S.and Bajpai, J. P., "Intuitionistic fuzzy *gpr*-closed sets and intuitionistic fuzzy *gpr*-continuity", *Fuzzy Information and Engineering, Springer*, **4** (2012), 425-444.

[23] Thakur, S. S. and Rathor, C. P., "On δ-precontinuity in intuitionistic fuzzy topology", (submitted).

[24] Turnali, N. and Coker, D., "Fuzzy connectedness in intuitionistic fuzzy topological spaces", *Fuzzy Sets and Systems*, **116** (2000), 369-375.

[25] Chandrasekar, V., Durairaj S. and Vadivel, A., "On Fuzzy *e*-open Sets, Fuzzy *e*-continuity and Fuzzy *e*-compactness in Intuitionistic Fuzzy Topological Spaces", *Sahand Communications in Mathematical Analysis (SCMA)*, **12** (1) (2018), 131-153.

[26] Zadeh, L. A., "Fuzzy sets", *Information and Control*, **8** (1965), 338-353.