Profile of Student Algebraic Thinking with Polya's Problem-Solving Strategy: Study on Male Students with Field Independent Cognitive Style

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ABSTRACT
This study aims to describe the profile of students' algebraic thinking with Polya's problem solving strategy in completing a linear program conducted on male students with field independent cognitive style. This research is exploratory research with qualitative approaches. In accordance with the purpose of the study, the subject (single subject) were male student with field independent cognitive style. Single subject were screened from 119 participants (male students in grade X) at a high school in the city of Mataram, Indonesia. The criteria for determining a single subject are male students with field independent cognitive style, and having the highest math scores on linear algebra material. The research instrument consisted of the main instrument (researcher/human instrument) which interacted directly with the subject to explore the subject's algebraic thinking profile, and research aid instruments consisting of the GEFT test, mathematical ability test, linear programming tasks, and interview guidelines. Through Polya's problem solving, the students' algebraic thinking profile has been described, where the subject has represented mathematical ideas using algebraic expressions, the subject interprets algebraic expressions; the subject uses symbolic representations, formulations, and expressions of equations using algebraic conventions; and the subject can interpret the solution.

Keywords: Algebraic thinking profile; Polya's problem solving; Field independent cognitive style;

A. INTRODUCTION
During the last three decades, reforms in the field of mathematics education and teaching have continued (Niss et al., 2016), along with the development of Science, Technology, Engineering and Mathematics (STEM), and Mathematics is considered the "Queen of Knowledge" because almost all fields science is supported by mathematics (Evendi & Verawati, 2021). However, the achievement of mathematics teaching competence to date has become a major challenge (MacDonald, 2020). On the one hand, many students at the secondary school education level do not realize the importance of mathematics and its relevance to their work careers and their future education (Gijsbers et al., 2020). Previous studies found difficulties in teaching mathematics among students with low cognitive levels, because in the previous learning routine the teacher did not apply cognitive strategies effectively (Pendlington, 2005). Finally, updates on many aspects of mathematics teaching,
especially those oriented to basic needs in mathematical concepts, need to be investigated more intensively (Boesen et al., 2014).

One of the concepts of mathematics that is highly emphasized at all levels of education is algebra (Chimoni & Pitta-Pantazi, 2017). The realization of the need for algebra teaching is mainly based on the fact that algebra is related to the development, formation and communication of knowledge in all field or matter of mathematics (Midgett & Eddins, 2001). In addition, algebraic thinking has an important role in solving mathematical problems (Sukmawati et al., 2018). Early intervention in a well-planned way of learning and identifying the student’s way of thinking becomes a key element for initiating algebraic thinking, through this naturally increasing the translation of pure arithmetic situations into the symbolic language characteristics of algebraic thinking (Adamuz-Povedano et al., 2021). Therefore, an extensive study in mathematics was carried out to investigate algebraic concepts in elementary and middle school students, specifically focusing on the context of algebraic thinking (Cai & Knuth, 2011).

Previous studies have shown that algebraic thinking can develop starting in elementary and middle school age children by conducting an explicit approach (Pinnock, 2021), with a supportive learning environment (Radford, 2008). Algebraic thinking can also develop with the use of various forms of reasoning (Pedemonte, 2008). Algebraic thinking performance has also been investigated which is associated with various cognitive factors, such as information processing (Fuchs et al., 2012). Several recent studies have explored the process of algebraic thinking through Polya’s problem-solving steps (Muniroh et al., 2017; Nu, 2019). The problem-solving steps according to Polya (1945) are to understand the problem, develop a settlement plan, carry out a settlement plan, and re-examine the solution. Ways of solving problems and students’ thinking can be assessed through Polya’s problem-solving steps (Faulkner et al., 2021). Although there are many studies or research on algebraic thinking, it is acknowledged by Chimoni & Pitta-Pantazi (2017) that studies on how individuals use their cognition in algebraic thinking seem to need further exploration. In accordance with the research agenda of the National Council of Teachers of Mathematics (NCTM) which emphasizes future research for identification the mathematical concepts and cognitive reasoning processes that facilitate algebraic thinking (Danişman & Erginer, 2017).

Algebraic thinking is a way of thinking that involves mathematical thinking skills (generalization, abstraction, analytical thinking, dynamic thinking, modelling, and organization) which is very useful in solving mathematical problems, other fields of science, as well as problems in everyday life (Lew, 2004). The indicators of algebraic thinking according to NCTM are understanding the patterns, relationships, and functions of mathematic; the use of algebraic symbols to represent and analyze mathematical situations and structures; use mathematical models to represent and understand quantitative relationships; and analyzing change in various contexts (Midgett & Eddins, 2001). Hardiani et al. (2018) explores the opinions of several mathematicians about algebraic thinking, and suggests algebraic thinking as a thinking process that involves the representation of mathematical ideas including coefficients, constants, formulas, functions, variables using algebraic expressions, and interpreting the solutions.
In Indonesia, algebraic thinking in the mathematics learning curriculum at the elementary and secondary school levels is still implicit, so it is necessary to develop a comprehensive teaching design to make it easier for students to develop algebraic thinking (Pratiwi et al., 2019). Before this is realized (the development of mathematics teaching design is carried out), then as a foundation, information about the profile of algebraic thinking is needed for students. This profile is a prototype for developing mathematics learning designs to train students’ algebraic thinking. On the one hand, the results of previous studies show that algebraic thinking performance is related to individual cognitive regulation in information processing (Fuchs et al., 2012). Moving on from Bandura (1982) theory of self-efficacy, we believe that each student’s information processing is different, depending on their cognitive style. There is a strong relationship between self-efficacy and students' algebraic thinking skills, where students with high self-efficacy tend to have good algebraic thinking skills, and vice versa (Kurniawan & Mashuri, 2021). Studies in relation to cognitive style, the algebraic representations affiliated with students' cognitive styles (Nisak et al., 2020). Previous studies have shown that cognitive style is related to student acceptance of information processing (George et al., 2018), and information processing plays a role in individual acceptance of information and behavior (Armstrong et al., 2012).

Cognitive style is identified with the level of individual consistency in how to understand, organize, and process information (Rayner & Cools, 2011), and is believed to affect individual performance (Armstrong et al., 2012). In the aspect of cognitive regulation, cognitive style plays a role in mediating individual cognitive abilities in learning and receiving information (Viator et al., 2020). Individual cognitive styles are categorized into field dependent and field independent (Witkin, 1967; Witkin et al., 1977). Algebraic thinking has been investigated in relation to students' cognitive styles (Hardiani et al., 2018; Nisak et al., 2020). A previous study by Hardiani et al. (2018) has examined the algebraic thinking of female students with field independent cognitive style, the study results show three main results from the narrative questions presented; students are able to represent algebraic ideas by manipulating symbols and variables, building mathematical models that represent algebraic ideas, and interpreting solutions. As a comparison, a study needs to be conducted to explore the algebraic thinking of male students with field independent cognitive style. This study aims to describe the profile of students' algebraic thinking with Polya’s problem solving strategy in completing a linear program conducted on male students with field independent cognitive style.

B. METHODS

This research is exploratory research with qualitative approaches, specifically aims to describe the profile of algebraic thinking in male students with field independent cognitive style in completing linear programs. Participants in the study were 119 male students in grade X in a senior high school in the city of Mataram, Indonesia. The age range of the participants was 15-16 years, with 51 (43%) male and 68 (57%) female students. The research instrument consisted of the main instrument and research auxiliary instruments. The main instrument is the researcher (human instrument) who interacts directly with the subject to explore the profile of the subject's algebraic thinking. The research aid instruments
consist of 4 (four) instruments, namely; GEFT test (Group Embedded Figure Test), math ability test, linear programming task, and interview guide.

The research subjects were male students who met the criteria; a) field independent cognitive style (measured and determined by the GEFT instrument), and b) has the highest math score on linear algebra material (measured by the mathematical ability test instrument conducted on all participants). Based on these criteria, it was found that the research subject with GEFT results, mathematical ability score (MAS), and subject code as presented in Table 1.

The research subject was only 1 student (initial ZW) considering that based on the criteria, the maximum score was achieved by ZW with a MAS score of 76. As shown in Table 1.

<table>
<thead>
<tr>
<th>Initial</th>
<th>MAS</th>
<th>Cognitive style (GEFT) Score</th>
<th>Gender</th>
<th>Subjek code</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZW</td>
<td>76</td>
<td>17 field independent</td>
<td>Male</td>
<td>SIL (Subjek-Independent-Male)</td>
</tr>
</tbody>
</table>

Linear program task is given to the research subject. Narrative of linear program tasks as follows.

"PUJI" Flower Shop, sells 2 kinds of hand bouquets, cheap but interesting. The first set consisted of 4 roses, 2 dahlias, and 3 tulips, while the second set consisted of 2 roses, 3 dahlias, and 1 tulip. The florist currently only has 48 roses, 36 dahlias and 33 tulips in stock. If the profit in the first series is Rp. 15,000,- and the profit in the second series is Rp. 12,000,- then do the following calculation.

1. Create a mathematical model based on the data!
2. Draw a graph along with the solution area according to the mathematical model created!
3. From the graph, how many first and second series can be made so that the florist gets the maximum profit?

Interviews on research subjects were conducted using an interview guide instrument which contained a collection of questions posed by researchers to research subjects. Interview guides are needed to reveal a picture of the subject’s algebraic thinking. The interview transcript code for the subject used the SILSM code (SIL Math Problems/Linear Program Tasks), and for the researchers used the PSM code (Researcher). Completion of linear programming tasks follows Polya’s (1945) problem-solving steps, namely understanding the problem (P1), compiling a settlement plan (P2), implementing the completion plan (P3), and re-examining the solution (P4). Indicators of algebraic thinking measured on the subject are representing mathematical ideas using algebraic expressions (A1); interpret coefficients, variables, and constants in algebraic expressions (A2); use the symbolic representations using algebraic agreement (A3); solution interpretation (A4). The data from the research were then tabulated and described to obtain the algebraic thinking profile of the subject (SIL) in solving mathematical problems based on Polya's problem solving steps.
C. RESULT AND DISCUSSION

The data presented are the results of interviews and subject observations (SIL) in completing linear programming tasks based on Polya’s problem-solving steps. Interview transcripts were reduced by researchers, examples of interview excerpts as presented in Figure 1 to Figure 4 according to Polya’s problem-solving steps. Excerpts of the results of interviews and observations of researchers on Subject 1 (SILSM1) is presented in Figure 1, it is related to understanding the problem.

In the transcript of the conversation (interview) with the subject, the researcher noted that there were at least 50 conversational scripts that explored how the subject understood the problem. Based on Figure 1 to understand the problem, first SIL mentions all the important elements that are known from the problem using their own sentences by making a table. SIL writes $x$, $4x$, $2x$, $3x$, $y$, $2y$, $3y$, and so on until $z = 15,000x + 12,000y$ [SILSM1005] [SILSM1007]. SIL constructs the z equation of the gain of the first and second circuits as the objective function [SILSM1021] [SILSM1022]. SIL mentions that there are 3 questions in SM1 using their own sentences [SILSM1023].

After reading the questions, SIL understands the meaning of the questions by restating everything that is known on the questions using their own sentences. SIL explained in detail all the important information contained in the questions, for example, based on the question there were 2 flower arrangements, the first series consisted of 4 roses, 2 dahlias, and 3 tulips while in the second series there were 2 roses, 3 dahlias and 1 tulip. Each flower has a stock, for which 48 roses, 36 dahlias, and also 33 tulips. SIL formulates an objective function by formulating a linear equation $Z$ to find the maximum profit from the sale of 2 flower arrangements marked by writing $z = 15000x + 12000y$.

Based on Figure 1, SIL writes down the important elements to express algebraic expressions on questions that have been read with the help of tables because according to SIL, making tables will make it easier to solve problems. In conclusion, SIL finds important elements based on the problems presented and makes simple algebraic equation expressions, in this case SIL shows P1A1 algebraic thinking indicators. In addition, based on the algebraic expressions that have been raised, SIL interprets the understanding and compatibility between coefficients, variables, and constants in algebraic expressions (P1A2), as shown in Figure 1.
Figure 1. Excerpts from the researcher’s interview with subject 1 (SILSM1) at the phase of understanding the problem

Excerpts from interviews and observations of researchers on Subject 1 (SILSM1) at the stage of making a settlement plan are presented in Figure 2. At this stage (see Fig. 2), SIL said it would create a mathematical model in the form of inequalities to solve the first question based on the algebraic expressions [SILSM1024] [SILSM1025] [SILSM1026]. Based on these data, it can be concluded, SIL reveals a plan of steps to solve the problem starting from the idea of making a mathematical model in the form of inequality expressions based on algebraic expressions made in the table (P2A1). SIL said it would describe the solution area of the inequality on a Cartesian coordinate graph to answer the second question [SILSM1028], as shown in Figure 2.
Figure 2. Excerpts from the researcher’s interview with subject 1 (SILSM1) at the phase of the make a settlement plan.

Based on the idea of making a mathematical model in the form of inequalities, SIL depicts the solution area on the graph (Cartesian coordinates). Based on the graph of the settlement area, SIL already has a prediction (a value that can be found) as a solution to answer the next question (P2A4). SIL said it would calculate the maximum profit to answer the third question. Based on the settlement area, the intersection points will be searched to calculate the maximum profit. Using the function equation, SIL will calculate the maximum profit (P2A3).

Furthermore, excerpts from the results of interviews of researchers on Subject 1 (SILSM1) at the stage of implementing the completion plan are presented in Figure 3. The transcript of the conversation (interview) between the researcher and the subject at this phase was about 102 scripts. To answer the first question, SIL creates 3 inequalities for each flower, using the addition operation marked by writing 4x + 2y = 48 for roses, 2x + 3y = 36 for dahlias, and for tulips 3x + y = 33. SIL creates three inequalities based on the table that has been created. That is, SIL creates a suitable mathematical expression (inequality) to answer the question (P3A1). Expression of the answer to the second question, that of the three inequalities made, SIL assumes the variables x = 0 and y = 0 and performs substitutions on the inequalities and operates them to find the value of each axis. This shows that SIL interprets a variable in an inequality if one of them is made equal to 0 then the value of the other variable will be obtained by converting it to an equation first, and besides that, by assuming it is equal to 0, SIL interprets it has something to do with describing a point on the xy-axis (P3A2), as shown in Figure 3.
After assuming the variables $x = 0$ and $y = 0$ in each inequality, the step taken by SIL is to change the inequality by the substitution process, for example for the first equation to be $2y = 48$, we get $y = 24$, then the point becomes 0.24, and so on for equation other. At this stage, SIL uses number operations to obtain the value of $x$ and value of $y$ according to the applicable algebraic law so that a pair of points is obtained that can help answer the next question (P3A3). In the substitution process, finally found the value of the function $z$. In the end, SIL means traders get a maximum profit of 207,000 from the maximum point of the series. In conclusion, SIL explains the final solution obtained as a solution to the problem according to the context (P3A4). The last phase is re-checking the solution. Excerpts from interviews and observations of researchers on Subject 1 (SILSM1) at this stage are presented in Figure 4.
After SIL saw the question and matched it with the answer that had been written, SIL declared it was correct and correct [SILSM1082] [SILSM1084]. This stage is the determining stage, whether the solution obtained by SIL is the answer to the problem. SIL is given the opportunity to re-examine the solution that has been obtained as a whole. Beginning with SIL by looking at the questions with answers that have been written, then paying attention to the tables that already contain algebraic expressions. In conclusion, SIL states that the mathematical inequality model created is in accordance with algebraic expressions. In this case, SIL shows an indicator of algebraic thinking (P4A1). SIL looks for the intersection of the x-axis and y-axis, from the results of the image it gets the intersection point on 2 inequality lines so that by eliminating it finds the maximum point to get the maximum profit [SILSM1085] [SILSM1086]. SIL rechecks each step, until it finds the maximum point of intersection of the x and y axes on each inequality, paying attention to the image of the solution area that has been shaded and 3 points that have been bolded until it succeeds in getting the maximum benefit from the calculation of the equation of the objective function (P4A3). So SIL interprets the maximum point obtained in the form (x, y) as the value of the variable in question. The variables x and y are referred to as the first and second series, respectively. So that SIL gets the maximum benefit and is a solution to these problems. SIL also states that there is no other form or expression to solve this problem. This shows the SIL indicator of algebraic thinking (P4A4).

Based on the results of the SIL interview on the completion of linear programming tasks, it can be described the SIL algebraic thinking profile scheme in solving problems (linear program tasks) based on Polya’s problem solving steps, as presented in Figure 5. At the stage of understanding the problem, SIL finds important elements and components based on known problems, SIL can also identify the questions asked, and compile simple mathematical algebraic equations (algebraic expressions) based on problems. SIL defines the meaning and suitability between coefficients, variables, and constants (forms of algebraic expressions).
based on an agreed definition. At the stage of making a solution plan, SIL reveals a plan for the steps to solving the problem, starting from the idea of making a mathematical model (equations and inequalities) based on algebraic expressions. Based on this idea, SIL describes the solution area on the graph (coordinate of the axis), so that SIL has been able to predict that a value will be obtained as a solution. SIL has also been able to calculate the maximum profit using the objective function equation. These results are in line with the findings (Ashar et al., 2021) that students with high thinking skills are able to write and describe mathematical models, apply the properties of absolute value inequalities, and are able to get the right answer. Generally, if students are faced with math problems (questions) by 38% of students fail to understand the problem and 14% of students fail to develop a settlement plan (Nuryah et al., 2020). However, in the case of SIL which is faced with linear program problems, SIL is able to understand the problem and develop a problem-solving plan, as shown in Figure 5.

**Figure 5.** Schematic profile of SIL algebraic thinking in solving mathematical problems based on Polya’s problem solving steps
At the stage of implementing the completion plan which is the implementation stage of what has been planned by SIL. SIL makes a mathematical model in the form of inequalities that are appropriate to answer questions of three inequalities. In this inequality, SIL interprets a variable in an inequality if one of them is made equal to 0 then the value of the other variable will be obtained by converting it into an equation first. In addition, by assuming equal to 0, according to SIL it has to do with describing points on the x-axis and y-axis. Of the three inequalities, SIL uses the operations of multiplication, addition, subtraction, and division to obtain the value of x and value of y according to the applicable algebraic law in order to obtain a pair of points that can help answer the next question. From the pair of points obtained from the three inequalities, SIL describes the pair of points on the graph according to the applicable law so that the solution area is obtained. Based on the settlement area, SIL sees that there is a point that is suspected to be the maximum point. Then SIL looks for the point that intersects on 2 lines by elimination according to the applicable algebraic law. SIL uses multiplication, addition, subtraction, and division operations by substituting the value of x and value of y in an objective function equation based on applicable algebraic laws so that maximum profit is obtained which can help answer the next question. SIL explains the final solution obtained as a solution to the problem according to the context of the problem. Solving problems with the right number operating system is accuracy in the use of algebraic thinking (Tsaqifah, 2020).

Finally, the stage of re-checking the completion. At this stage SIL states that the inequality model that has been made is in accordance with the algebraic expressions in the table. Likewise, the equation of the objective function that has been written based on the story in the problem. SIL re-examines each step until it finds the maximum point from looking for the intersection of the x-axis and y-axis on each inequality, paying attention to the image of the solution area that has been shaded and 3 points that have been bolded until it succeeds in getting the maximum benefit from the calculation of the equation of the objective function. So SIL gets the maximum benefit and is the solution to the problem. SIL also stated that there is no other way to solve this problem. Until this phase, it seems almost the same as the findings of previous studies that students interpret the solution as a variable obtained from the intersection point in the solution area to get the maximum benefit (Hardiani et al., 2018).

D. CONCLUSION AND SUGGESTIONS

The results of the study have described the profile of students' algebraic thinking with Polya's problem solving strategy in completing a linear program carried out on male students with field independent cognitive style. Through Polya's problem solving, the profile of students' algebraic thinking has been described, where SIL has represented mathematical ideas using algebraic expressions in the form of equations, inequalities, or functions; SIL defines coefficients, variables, and constants in algebraic expressions; SIL uses equivalent symbolic representations to manipulate formulas, equation expressions, inequalities or functions using algebraic conventions; and SIL can interpret the solution. Based on the results of this study, we suggest further studies to be able to develop a prototype learning model that can train algebraic thinking intensively, of course this is based on the thinking profile that has been described.
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