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Mathematical Modeling: A Study of Corruption among Students of Nigeria Tertiary Institutions

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Abstract

Corruption is a slow poison damaging students and consequently societies and nations, virtually, all students of Nigerian tertiary institutions are exposed to corruption. In this study, an attempt is made to formulate the dynamics of corruption among students of Nigerian tertiary institutions. We describe mathematical modeling of corruption among students using an epidemiological compartment model. The population at risk of adopting corrupt ideology was divided into four compartments: $S(t)$ is the susceptible class, $E(t)$ is the Exposed class, $C(t)$ is the Corrupted class and $P(t)$ is the punished class. The positivity and boundedness of the model were established. The model possesses both corruption-free and endemic equilibrium. Likewise, the model exhibits threshold dynamics characterized by the basic reproduction number R_0. The numerical implementation of the model reveals that corruption will persist among Nigeria students if the root cause were not eradicated.

Keywords: Modeling, Endemic, Corruption, Compartment, Positivity.
2010 MSC: 92D30, 37M05, 34H05.

1. Introduction

Students' corruption, a form of corruption that exists among students in tertiary institutions is gaining universal attention among researchers because of its complex social and economic impact on the education industry in most countries. This form of corruption can be likened to a pest that feeds on the educational system of nations where it occurs and hinders higher education. Students' corruption takes many forms and ranges from admissions racketeering to sextortion and academic dishonesty [1]. Unabated students' corruption can bring the educational system of any nation to its knees. Corruption in the higher institution of learning is a universal problem often referred to as academia corruption. It occurs not only in Nigeria but also in other nations such as Sweden, Switzerland,

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and Japan; simply in different forms [2]. It includes all forms of corrupt practices taking place in academia which have a direct negative effect on the quality and standard of education [3]. A subset of academia corruption is the students' corruption, the focus of this study, which concentrates on corrupt practices perpetrated by students. Notable, while academia corruption is mostly attributed to management and staff of higher institutions of learning such as political manipulation and interference in university affairs by the government [1] and employment of unqualified staff, students' corruption are techniques used by students to engage in corrupt practices such as examination malpractices, bribery, sex for marks, the bugling of examination and records office, ingratiation, offering of unsolicited services to lecturers, all for undeserved grades for better job opportunities.

Though the concept of corruption is complex and vague in terms of description, this has resulted in several definitions of corruption depending on the context of use. Generally, corruption is defined as "the abuse of entrusted power for private gain" (Transparency International, n.d.). Also, it is commonly perceived as "the use of public office for private gain" (World Bank, n.d.). For the benefit of our study, corruption is "a form of anti-social behavior awarding improper privileges contrary to legal and moral norms". Some students' anti-social attitudes were averred by [4] such as sex for marks, cash for marks, sex/cash for grade alterations, examination malpractice, and the use of fake documents. Consequently, students' corruption jeopardizes the provision of qualitative education for the citizenry [5].

In Nigeria, corruption is a constant phenomenon that has plagued the existence of the country since its independence [6] thereby wreaking havoc on several sectors of the country including the education sector. Specifically, in the education sector, corruption exists at various levels in higher institutions in the country; management level, staff level, and student level, and these levels influence the complexity of students' corruption in an institution. Students' corruption is a complex problem because corruption aggravates inequality, poverty, social division and environmental crisis (Transparency International, n.d.) and undermines democratic institutions, slows economic development and contributes to instability (UNODC, n.d.). Despite the complexity, students' corruption can be easily identified by some characteristic traits when exhibited by the perpetrating students. Statistics show that Nigeria ranked 149th out of 180 countries in the 2020 Transparency International's Corruption Index; a worse circumstance to its 146th ranking in 2019[7]. A study conducted in some locations (Enugu, Kano, Kwara and Lagos) in Nigeria revealed that corruption is the most important challenge impeding the development of the country[8]. Corruption is much of a cultural attribute in Nigeria as many politicians, government workers and security agents get rich from engaging in corruption such as bribery and stealing from government coffers, hence, students also engage in corrupt practices following in the footsteps of their predecessors to cut corners in their educational pursuit.

Consequently, students' corruption can be said to be a learned behavior reinforced by the environment. The social learning theory posits that social behavior is learned by observing and imitating the behavior of others and the behavioral school of thought believes that punishment or rewards should be dished out immediately to reinforce behavior. Since many leaders are walking free of charges in Nigeria despite the public awareness of corruption charges against them, students' therefore engage in corrupt practices since it seems to be a culture in the larger society that has no upheld sanctions. Though the Students' Handbook, a manual that guides the conduct of students throughout their programme, and the Students' Disciplinary Committees which investigates any reported case against any students exist in most institutions, most cases of students' corruption are not reported except obvious examination malpractices.

A review of literature has shown that there are several socio-demographic factors associated with students' corruption and more social factors are gaining the attention of researchers on why students engage in corrupt practices. Evidence has shown that students' poor entry qualifications and poor study habits are the main causes of academic corruption in Nigeria [3]. The downward review of entry qualification into higher institutions permits students without the ability for higher education to gain admission, therefore, these students may find it difficult to cope and subsequently look for other ways to achieve academic success through "runs". [4] affirmed that students make use of the word "runs" as a superordinate code for diverse acts of corruption including sex for marks, cash for marks, sexcash for grade alterations, examination malpractice, and the use of fake documents. Similarly, [9] found out that cheating, gratification and bribery have significant relationships with corruption among students at Cross River University of Technology, Calabar, Nigeria.

[10] developed a mathematical model to evaluate the performance of Kenyan anti-corruption and disengagement programs. They discovered that the rate of corruption reproduction decreased when they used characteristics that represented effective constitutional anti-corruption policies, such as corruption awareness. This suggests that the interplay between corrupt, vulnerable, and corrupt persons transitioning to a political position influenced the number of corrupt individuals reproduced. Furthermore, [11] discovered that, while corruption is a two-way game between staff and students, student corruption decreases over time while academic corruption increases. As a result, efforts to combat corruption should be focused on staff rather than students.

To understand the nature of students corruption in Nigeria's educational system, this study formulated a mathematical model and compartmentalized students into four various compartments based on the following parameters; students who are susceptible to corruption, students who are exposed to corruption, students who are engaged in corrupt practices and students who have been reprimanded for their engagement in one corrupt practice or the other. The parameters were used as predictors of incidence of students' corruption over time and therefore, provide intervention to curb corruption among students.

2. Model Formulation

The modeling approach of Kermach and Mc Kendric ([12], [13], [14]) was used, we assumed the model has four different compartments, the Susceptible (S(t)), those individuals that have the potential of getting exposed to corruption, (E(t)) , are individuals that are exposed to corruption, the Corrupted(C(t)) are individuals who are embedded in indiscipline and are liable to be punished, Punished (R(t)) are individuals reprimanded as a result of been corrupt.

Model Assumptions

- The population is heterogeneous, hence the individual that makes up the population can be grouped into compartments according to its epidemiological state,
- The population size in the compartments is differentiable with respect to time and corruption is deterministic.
- People in compartments have an equal death rate,
- The only entry point is through a properly filled questionnaire, while the only exit is though death from natural/other causes.

Model Diagram

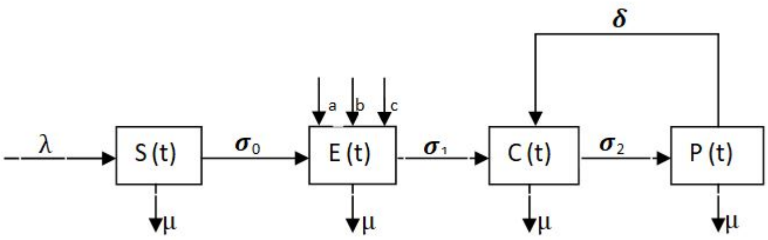

Figure 1: Systematic flow diagram of the Corruption Model

Governing equations

$$\frac{dS(t)}{dt} = \lambda - \mu S - \sigma_0 S,$$

$$\frac{dE(t)}{dt} = \sigma_0 S - \mu_0 E + (a + b + c) - \sigma_1 E,$$

$$\frac{dC(t)}{dt} = \sigma_1 E - \mu_0 C + \sigma_2 C + \delta P, \tag{2.1}$$

$$\frac{dP(t)}{dt} = \sigma_2 C - \delta P - \mu_0 P.$$

3. Model Analysis

3.1. Boundedness of the Model

The following theorem will ensure the boundedness of the model in equation (2.1) above

Theorem 3.1. *If the initial condition of the corruption model* (2.1) *are in* $\Omega = \left\{(S,E,C,P) \in \Re^4; N(t) \leqslant \frac{\lambda+a+b+c}{\mu}\right\}$ *then all solution of the system equations of the model enter and remain in* Ω.

Proof. Given the set S(t), E(t), C(t), P(t) with any solution of the system (2.1) and

$$N(t) = S(t) + E(t) + C(t) + P(t) \tag{3.1}$$

then we have

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dC(t)}{dt} + \frac{dP(t)}{dt} \tag{3.2}$$

$$\text{equation} \leqslant \lambda - \mu(S+E+C+P) + a + b + c\text{equation} \leqslant (\lambda + a + b + c) - \mu N$$

$$\frac{dN(t)}{dt} + \mu N \leqslant (\lambda + a + b + c) \tag{3.3}$$

and its solution is given as

$$N(t) \leqslant \frac{\lambda + a + b + c}{\mu}\left(1 - e^{-\mu t}\right) + N(0)e^{-\mu t}$$

$$\text{as } N(t) \to \infty$$

$$N(t) \leqslant \frac{\lambda + a + b + c}{\mu} \tag{3.4}$$

which implies that the solution is bounded for

$$0 \leqslant N(t) \leqslant \frac{\lambda + a + b + c}{\mu}$$

thus all solution of the model (1) are in Re^4_+, are restricted in the region.

$$\Omega = \left\{(S,E,C,P) \in \Re^4; N(t) \leqslant \frac{\lambda + a + b + c}{\mu} + \epsilon\right\}$$

for all $\epsilon > 0$ and $t \to \infty$. Therefore all the feasible solution for the model is positively invariant, epidemiological meaningful and mathematically well posed. □

3.2. Positivity of the Solution

It is necessary to prove that all solution of the system with positive initial data will remain positive for all time $t > 0$.

Theorem 3.2. *Let* $S(0) > 0$, $E(0) > 0$, $C(0) > 0$, $P(0) > 0$, *then all solution* $S(t)$, $E(t)$, $C(t)$, $P(t)$ *of the system* (2.1) *are positive for all* $t > 0$

Proof. From the first equation of the system

$$\frac{dS(t)}{dt} = \lambda = (\mu + \sigma_0)\, S$$

whose solution is

$$S(t) = \frac{\lambda}{\mu + \sigma_0}\left(1 - e^{-(\mu_0+\sigma_0)t}\right) + S_0(0)e^{-(\mu_0+\sigma_0)t}$$

as $t \to \infty$, we obtain

$$0 \leqslant S(t) \leqslant \frac{\lambda}{\mu + \sigma_0},$$

hence, feasible solution of the system (2.1) lies in the region $\Omega = \{S(t), E(t), C(t), P(t)\}$. Similar proof can be established for the positivity of the other solutions. □

3.3. *Corruption free Equilibrium*

In the absence of corruption, we assume $E(t) = C(t) = P(t) = 0$, therefore the corruption free equilibrium is given by

$$\lambda - (\mu + \sigma_0)\, S = 0$$

$$S = \frac{\lambda}{\mu + \sigma_0}$$

3.4. *Corruption Endemic Equilibrium*

We set

$$\lambda - \mu S - \sigma_0 S = 0$$

$$\sigma_0 S - \mu_0 E + (a + b + c) - \sigma_1 E = 0$$
$$\sigma_1 E - \mu_0 C + \sigma_2 C + \delta P = 0$$
$$\sigma_2 C - \delta P - \mu_0 P = 0$$

Solving For S, C, E and P, we obtain

$$S(t) = -\frac{\lambda}{\mu_0 - \sigma_0}$$

$$E(t) = \frac{a\mu_0 - a\sigma_0 + b\mu_0 - b\sigma_0 + c\mu_0 - c\sigma_0 - \lambda\sigma_0}{\mu^2 - \mu\sigma_0 + \mu\sigma_1 - \sigma_0\sigma_1}$$

$$C(t) = -\frac{((\delta + \mu_0)\,\sigma_1\,(a\mu_0 - a\sigma_0 + b\mu_0 - b\sigma_0 + c\sigma_0 - \lambda\sigma_0))}{\left(\mu_0\left(\delta\mu_0^2 - \delta\mu_0\sigma_0 + \delta\mu_0\sigma_1 - \delta\sigma_0\sigma_1 + \mu_0^3 - \mu_0^2\sigma_0 + \mu_0^2\sigma_1 - \mu_0^2\sigma_2 - \mu_0\sigma_0\sigma_1 + \mu_0\sigma_0\sigma_2 + \sigma_0\sigma_1\sigma_2\right)\right)}$$

$$P(t) = -\frac{(\sigma_0\sigma_1\,(a\mu_0 - a\sigma_0 + b\mu_0 - b\sigma_0 + c\mu_0 - c\sigma_0 - \lambda\sigma_0))}{\left(\mu_0\left(\delta\mu_0^2 - \delta\mu_0\sigma_0 + \delta\mu_0\sigma_1 - \delta\sigma_0\sigma_1 + \mu_0^3 - \mu_0^2\sigma_0 + \mu_0^2\sigma_1 - \mu_0^2\sigma_2 - \mu_0\sigma_0\sigma_1 + \mu_0\sigma_0\sigma_2 + \sigma_0\sigma_1\sigma_2\right)\right)}$$

4. Corruption Reproduction Number(CRN)

The computation of the basic reproduction number R_0 of the model is carried out using the next generation matrix approach [15].

Step 1: Write the model of the corrupted classes.

$$\frac{dE(t)}{dt} = \sigma_0 S - \mu_0 E + (a+b+c) - \sigma_1 E$$
$$\frac{dC(t)}{dt} = \sigma_1 E - \mu_0 C + \sigma_2 C + \delta P$$
$$\frac{dP(t)}{dt} = \sigma_2 C - \delta P - \mu_0 P$$

The operator f_i represent the rate at which new corruption arise and operator v_i reflects the rate at which compartment corresponding to corruption are exited.

$$f_i = \begin{pmatrix} \sigma_0 S + a + b + c \\ \sigma_1 E + \delta P \\ \delta_2 C \end{pmatrix}, v_i = \begin{pmatrix} \mu_0 E + \sigma_1 E \\ \mu_0 C + \sigma_2 C \\ \delta P + \mu_0 P \end{pmatrix},$$

$$f_i = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}. v_i = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

$$F = \begin{pmatrix} \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial P} \end{pmatrix}, V = \begin{pmatrix} \frac{\partial v_1}{\partial E} & \frac{\partial v_1}{\partial C} & \frac{\partial v_1}{\partial P} \\ \frac{\partial v_2}{\partial E} & \frac{\partial v_2}{\partial C} & \frac{\partial v_2}{\partial P} \\ \frac{\partial v_3}{\partial E} & \frac{\partial v_3}{\partial C} & \frac{\partial v_3}{\partial P} \end{pmatrix}$$

$$F = \begin{pmatrix} 0 & 0 & 0 \\ \sigma_1 & 0 & \delta \\ 0 & \sigma_2 & 0 \end{pmatrix}, V = \begin{pmatrix} \mu_1+\sigma_1 & 0 & 0 \\ 0 & \mu_2+\sigma_2 & 0 \\ 0 & 0 & \delta+\mu_1 \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{\sigma_1}{\mu_0+\sigma_1} & 0 & \frac{\delta}{\mu_0+\delta} \\ 0 & \frac{\sigma_2}{\mu_0+\sigma_2} & 0 \end{pmatrix}$$

The eigenvalue of FV^{-1} are 0, $\frac{\left(\sqrt{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}\right)\sigma_2\delta}{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}$, $-\frac{\left(\sqrt{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}\right)\sigma_2\delta}{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}$.

The corruption reproduction number is obtained by taking the largest eigenvalue of the model system (2.1).

$$R_0 = \frac{\left(\sqrt{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}\right)\sigma_2\delta}{\delta\mu_0+\delta\sigma_2+\mu^2+\mu_0\sigma_2}$$

The disease free equilibrium of the system (2.1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

5. Model Validation

To validate our developed model, a well structured and integrity-based questionnaire was administered to selected students in Nigeria higher institutions which spread across the 6 regions of the country.

The target population for the present study includes some randomly selected students of Nigerian tertiary on full time programme. The tertiary institutions are colleges of education, polytechnics, and universities. Eighteen tertiary institutions were randomly selected in which case, three tertiary institutions per state were selected from colleges of education, polytechnics, and universities from each region. The empirical investigation was conducted on 5000 students. The sample respondents were selected through the Random Probability sampling technique, where each students of the target population had the same probability of being chosen as the sample for the study.

5.1. Biographic Information

Figure 2 shows below that participant between age of 23 and 26 had the highest frequency, while age of frequency. Figure 3 a shows the females (55.9%) were more represented in the study. Likewise, 66.2% of the respondents were from the North-East part of the Country.

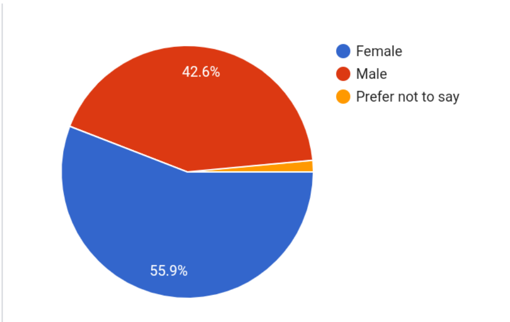

Figure 2: Age range of the Respondent

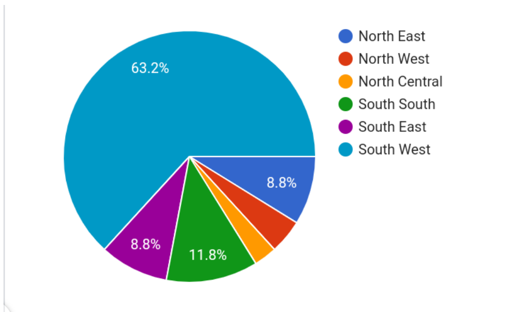

Figure 3: Region of the Respondents

The SECP model in equation (2.1) was solved numerically using Runge-Kutta of order four embedded inside Maple 17.

Table 1: The physical interpretation of the parameters and numerical values

Parameters	Physical description	Numerical value
σ_0	rate at which susceptible students are exposed to corruption	0.9222
σ_1	rate at which exposed to corruption students turn to been corrupted	0.6364
σ_2	rate at which corrupted students are punished	0.3241
δ	rate at which punished corrupted students return to corruption	0.0422
μ_0	natural death rate	$\frac{11.4}{1000}$
a	academic pressure rate	0.974
b	student family background	0.875
c	peer group influence	0.7443

These parameters above are used in the plotting of the graphs. The parameters determines whether corruption will persist or will be eradicated in due time. Figure 4, 5, 6 and 7 shows the Susceptible, Exposed and Corrupted and Punished Individuals graphs.

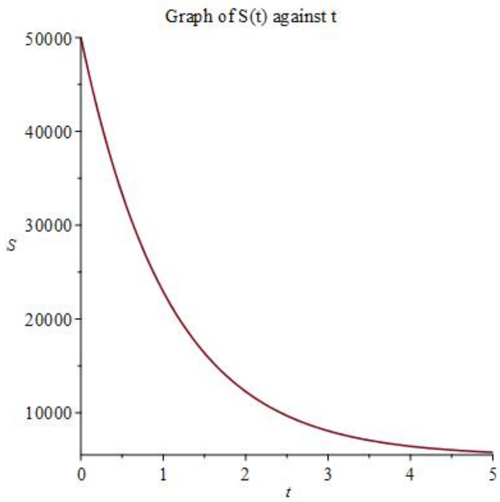

Figure 4: Susceptible

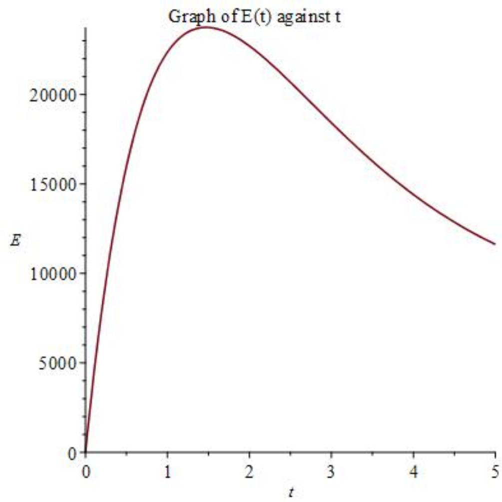

Figure 5: Exposed

Figure 4 shows the graph of susceptible individuals against time, there is a decrease in susceptible with respect to time, this signifies that the susceptible individuals will be reducing with respect to time due to being exposed to corruption. From the graph in Figure 5, it can be deduced that the exposed will reach it's peak before reducing with respect to time. Figure 6 shows that the corrupted individuals will be increasing in number till they reach a constant state with respect to time. Figure 7 shows that the punished individuals will be increasing with respect to time, thus corruption cannot be eradicated but controlled.

It is believed that this study will positively contribute to efforts by the Nigerian Government to comprehensively address the high prevalence of corruption in tertiary institutions across the country. The R_0 is directly affected by the parameters δ and σ_2 which measure the level of interaction between corruption and individuals that are punished. For instance, parameter σ_2 takes care of corrupt individuals that are punished, similarly, parameter δ accommodates events that those punished return to corrupted class,

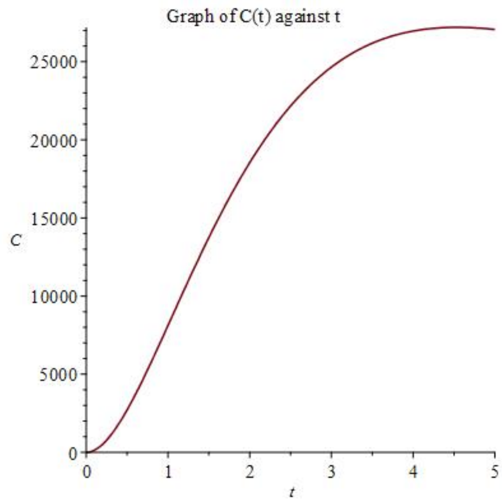

Figure 6: Corrupted

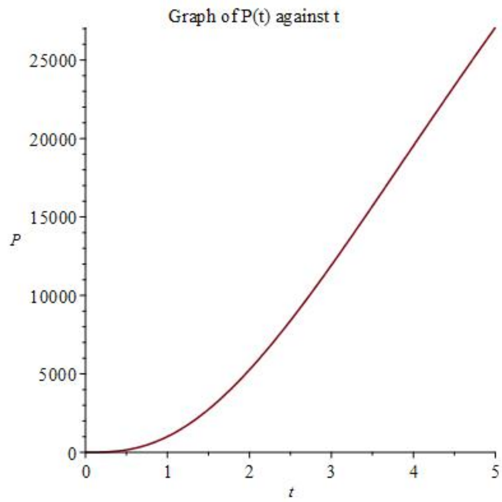

Figure 7: Punished

6. Conclusion

Corruption has lowered the standard of education in the country. Eliminating corruption will be of immense benefit to the increase in the standard of education thus leading to the growth and development of the nation.

Corruption continues to pose as one of the greatest challenges facing higher institutions in Nigeria. The youth of today, are the leaders of tomorrow, thus corruption among students has an adverse effect on our society as it continues to undermine good governance and distorts public policy, leading to mis-allocation of resources.

Good thinking, good product, but since our students are no longer interested in academic activities, this has contributed to slow economic growth as well as discouraged and frustrated both local and foreign investors due to inadequate properly trained local manpower. The poor management, excessive discretion in Government, appointments of people of dubious characters and political interference and lack of respect for professionalism have led to widespread corruption in Nigeria's tertiary institutions.

7. Recommendations

- The Government should declare a state of emergency in the education sector and organize a national education submit.
- Anti-corruption commission specifically designed for Education should be established.
- Integrity must be made a culture in Nigerian education system.
- Nepotism must be eradicated in Nigeria's education systems.
- Standard must not be compromised during the admission process.

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