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Bias Estimation of Linear Regression Model with Autoregressive Scheme using Simulation Study

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Abstract

In regression modeling, first-order auto correlated errors are often a problem, when the data also suffers from independent variables. Generalized Least Squares (GLS) estimation is no longer the best alternative to Ordinary Least Squares (OLS). The Monte Carlo simulation illustrates that regression estimation using data transformed according to the GLS method provides estimates of the regression coefficients which are superior to OLS estimates. In GLS, we observe that in sample size 200 and σ=3 with correlation level 0.90 the bias of GLS β_0 is -0.1737, which is less than all bias estimates, and in sample size 200 and $\sigma = 1$ with correlation level 0.90 the bias of GLS β_0 is 8.6802, which is maximum in all levels. Similarly minimum and maximum bias values of OLS and GLS of β_1 are -0.0816, -7.6101 and 0.1371, 0.1383 respectively. The average values of parameters of the OLS and GLS estimation with different size of sample and correlation levels are estimated. It is found that for large samples both methods give similar results but for small sample size GLS is best fitted as compared to OLS.

Keywords: OLS, GLS, Monte Carlo.

1. Introduction

We introduce about the linear regression estimation and the effect of serial correlation under first order autoregressive scheme by different scientists. The regression examination is a factual strategy broadly utilized in numerous fields, for example, financial aspects, innovation, sociologies and account. A direct relapse model is built to depict the connection between the needy variable and one or a few indicator factors. This regression model could be straightforward or different. Not with standing, in the direct relapse model, certain presumptions are made on how a dataset will be created by a hidden information producing measure.

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As indicated by these suspicions incorporate linearity, homoscedasticity, ordinariness and no autocorrelation between the blunder terms. Also, relapse model portrays the estimation of the needy variable as the amount of two sections, a deterministic part (illustrative factors) and the arbitrary part. The mistake term is basically an unsettling influence to a generally steady relationship and can catch the excess data in the reliant variable which couldn't be clarified by the autonomous factors.

Identifying with the supposition on the blunder term, on the off chance that the presumption of no relationship in the mistake term is disregarded, at that point, the fundamental model would be ruined with the standard mistakes of the boundaries getting one-sided. Besides, if the mistakes are related, the least squares assessors are wasteful and the assessed fluctuations are not suitable. By definition, autocorrelation is the slack connection of a given arrangement with itself, slacked by various time units. Hence, while applying relapse models to financial or the board information within the sight of autocorrelation, the normal least squares assessment technique stops to give productive assessors and suitable fluctuations .

Objective of the Study

The main objectives are:

- To apply the Monte Carlo method on linear regression model with First Order Auto-Regressive scheme.
- To compare the biases of OLS and GLS estimators in linear regression model with First Order Auto-Regressive scheme.

2. Literature review

Yale and Forsythe (1976) in [1] assessed a basic straight relapse model and contrast this strategy and OLS through relative productivity estimations got from Monte Carlo tests and they likewise apply this technique to a genuine informational collection. This method is likewise done by Rivest (1994) [2] for different slanted disseminations. Additionally, to eliminate the high impact of peripheral perceptions, Hoo *et. al.,* (2002) [3] built up this methodology, examine the idea of powerful measurements and present the techniques for strong multivariate exception separating.

Cheung (2007)[4] proposes an altered least-squares relapse approach un-weighted least squares relapse with a Huber-White powerful standard blunder for assessment of danger contrasts. Four adaptations of the powerful standard mistake are thought of. The binomial, common least squares and adjusted least-squares assessors are thought about diagnostically in a basic circumstance of one introduction variable. Multivariable relapse examinations are recreated to exhibit the convenience of the methodology. For test sizes of roughly 200 or less, a little example adaptation of the hearty standard mistake is suggested. The strategy is represented with information from a patient review in which the binomial relapse neglects to meet in the investigations of four out of five result factors.

Kiviet (2011) in [5] concentrated in econometric hypothesis are enhanced by Monte Carlo reenactment examinations. These show the properties of elective derivation strategies when applied to tests drawn from generally completely engineered information creating measures. They ought to give data on how strategies, which might be sound asymptotically, act in limited examples and afterward reveal the impacts of, model attributes

too complex to even consider analyzing systematically. Additionally the understanding of applied examinations ought to frequently profit when enhanced by a devoted recreation study, in light of a plan motivated by the hypothesized real observational information producing measure, which would verge on bootstrapping.

Ayinde et al., (2012) in [6] saw the exhibitions of assessors of direct relapse model with auto corresponded mistake term have been credited to the nature and particular of the illustrative factors. The infringement of presumption of the autonomy of the informative factors isn't remarkable particularly in business, monetary and sociologies, prompting the improvement of numerous assessors. In addition, expectation is one of the principle embodiments of relapse examination.

In [7] Suvarna and Ismail (2016) utilized the direct factual model, scientists face assortment of issues because of non trial nature for example vulnerability about the idea of the mistake cycle, model misspecifications, subordinate regressors and so on The marvel of connected mistakes in direct relapse models including time arrangement information is called autocorrelation. Infringement of the supposition of free regressors prompts multicollinearity. Henceforth, Ordinary edge gauges are loose to be very useful if there should arise an occurrence of auto related relapse model with the multicollinearity issue.

3. Material And Methods

3.1. Monte Carlo Simulation

Estimation of parameters with different correlation levels in linear regression model and constructed Monte Carlo (MC) Simulation with different sizes of sample. A Simulation is the impersonation of the activity of a genuine cycle or framework over the long run. Regardless of whether done by hand or on a PC, reenactment includes the age of a counterfeit history of a framework and the perception of that fake history to draw derivations concerning the working qualities of the genuine framework .

This simulation study is performed for sample sizes n=50, 100, 200, 300 and 500. The values of the model parameters of betas (1, 1) and first order autoregressive coefficients $(\rho = -0.9, -0.5, 0.5, 0.9)$ with error variances (1, 3). All experimental results reported are based on 5000 replications of sample sizes. For data generation and analysis, R programming and Minitab have been used.

3.2. Ordinary Least Squares

Ordinary Least Squares (OLS) regression is a measurable technique for investigation that appraises the connection between at least one free factors and a needy variable; the strategy gauges the relationship by limiting the amount of the squares in the contrast between the noticed and anticipated estimations of the reliant variable designed as a straight line . The simple linear regression model is:

$$Y = \beta_0 + \beta_1 X + e.$$

In matrix form

$$Y = X\beta + e.$$

Estimated model is

$$Y = X\beta + e.$$

$$Y - X\beta = eY = Y.$$

By minimizing the sum of squares of residuals that is

$$\sum e_i^2 = e^t e$$

$$e^t e = [Y - X\beta]^t [Y - X\beta]$$

$$e^t e = [Y^t - X^t \beta^t][Y - X\beta]$$

$$e^t e = Y^t Y - Y^t X\beta - X^t \beta^t Y + X^t \beta^t \beta X$$

Since $\beta^t X^t Y$ is scalar, therefore it is equal its transpose i.e. $\beta^t X^t Y = \beta X Y^t$

$$e^t e = Y^t Y - \beta^t X^t Y - \beta^t X^t Y + X^t \beta^t \beta X$$

$$e^t e = Y^t Y - 2\beta X^t Y + X^t \beta \beta X\beta = \beta^t.$$

Minimize with respect to β and equating zero.

$$\frac{\partial e^t e}{\partial \beta} = -2X^t Y + 2\beta X^t X$$

$$0 = -2X^t Y + 2\beta X^t X$$

$$2X^t Y = 2\beta X^t X$$

$$\frac{2X^t Y}{2X^t X} = \beta$$

$$\beta_{OLS} = (X^t X)^{-1} X^t Y$$

$$\text{Bias}(\beta_{OLS}) = \beta_{OLS} - \beta.$$

3.3. Generalized Least Squares

Generalized Least Squares (GLS) is a technique for fitting coefficients of illustrative factors that help to foresee the results of a reliant arbitrary variable. As its name recommends, GLS incorporates Ordinary Least Squares (OLS) as an uncommon case. GLS is likewise called "Aitken's assessor," after Aitken (1935) [8]. The simple linear regression model is:

$$Y = \beta_0 + \beta_1 X + e.$$

In matrix form

$$Y = X\beta + e.$$

Estimated model is

$$Y = X\beta + e.$$

$$Y = X\beta + eY = Y$$

with $E(e) = 0$, and $var(e) = \sigma^2\Omega$,where Ω is a known $n \times n$ matrix.$Given that$ $\sigma^2\Omega$ is a covariance matrix, we know that Ω must be symmetric and non singular. Therefore we can define:

$$\Omega = K^t K = KK$$

with K called the square root of Ω. Let we define:

$$\begin{cases} Y' = K^{-1}Y, \\ X' = K^{-1}X, \\ e' = K^{-1}e. \end{cases}$$

Note that:

$$E(e') = E(K^{-1}e) = K^{-1}E(e) = K^{-1}(0) = 0,$$

$$Var(e') = Var(K^{-1}e) = K^{-1}Var(e)K^{-1} = K^{-1}\sigma^2\Omega K^{-1} = \sigma^2 K^{-1}KKK^{-1} = \sigma^2.$$

By minimizing the sum of squares of residuals that is $\sum e_i^2 = e'^t e$

$$e'^t e = [Y' - X'\beta]^t [Y' - X'\beta],$$
$$e'^t e = [K^{-1}Y - K^{-1}X\beta]^t [K^{-1}Y - K^{-1}X\beta],$$
$$e'^t e = [K^{-1}(Y - X\beta)]^t [K^{-1}(Y - X\beta)],$$
$$e'^t e = (Y - X\beta)^t K^{-1}K^{-1}(Y - X\beta),$$

$$e'^{t}e = (Y - X\beta)^{t}\Omega^{-1}(Y - X\beta),$$
$$e'^{t}e = (Y^{t} - X^{t}\beta^{t})\Omega^{-1}(Y - X\beta),$$
$$e'^{t}e = Y^{t}\Omega^{-1}Y - Y^{t}\Omega^{-1}X\beta - X^{t}\beta^{t}\Omega^{-1}Y + X^{t}\beta^{t}\Omega^{-1}X\beta,$$
$$e'^{t}e = Y^{t}\Omega^{-1}Y - \beta^{t}X^{t}\Omega^{-1}Y - \beta^{t}X^{t}\Omega^{-1}Y + \beta^{t}X^{t}\Omega^{-1}X\beta,$$
$$e'^{t}e = Y^{t}\Omega^{-1}Y - 2\beta^{t}X^{t}\Omega^{-1}Y + \beta^{t}X^{t}\Omega^{-1}X\beta.$$

Minimize with respect to β and equating zero.

$$\frac{\partial e'^{t}e}{\partial \beta} = -2X^{t}\Omega^{-1}Y + 2X^{t}\Omega^{-1}X\beta,$$
$$0 = -2X^{t}\Omega^{-1}Y + 2\beta X^{t}\Omega^{-1}X,$$
$$2X^{t}\Omega^{-1}Y = 2\beta X^{t}\Omega^{-1}X,$$
$$\beta = \frac{X^{t}\Omega^{-1}Y}{X^{t}\Omega^{-1}X'},$$
$$\beta_{GLS} = (X^{t}\Omega^{-1}X)^{-1}X^{t}\Omega^{-1}Y,$$
$$\mathrm{Bias}(\beta_{GLS}) = \beta_{GLS} - \beta.$$

3.4. First Order Auto-Regressive (AR) Scheme

First order autocorrelation results from correlation between the error terms of adjacent time periods. If first order autocorrelation is present, the error for one time period e_t is a function of the error of the previous time period e_{t-1} as follow:

$$e_t = \rho e_{t-1} + u_t,$$

where $E(u_t) = 0, E(u_t^2) = \sigma_u^2$ and ρ is the parameter depicting the functional relationship among observations of the error term, e_t and u_t is stochastic error term which is iid.

4. Results and Discussion

We estimate the average values of parameters with different correlation structures for linear regression technique. For the generation of data set we use Monte Carlo Simulation method with linear regression model of OLS and GLS with first order autoregressive scheme. The Monte Carlo Simulation results are given in Table 4.1.

Fig. 4.1: Bias OLS β_0 when $\sigma = 1$

In Fig. 4.1 the Bias of OLS (β_0), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 1$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 500 with correlation level 0.90 the bias of OLS (β_0) is -0.0642 which is less than all others, and in sample size 200 with $\sigma = 1$the correlation level 0.90 have maximum value of bias OLS (β_0) is 0.0917.

Table 1: Monte Carlo Simulation Estimation Varying Sample Sizes and Correlations

	n=50	**n=100**	**n=200**	**n=300**	**n=500**
	β_o β_1	β_o β_1	β_o β_1	β_o β_1	β_o β_1
			rho=-0.9, var=1		
Bias OLS	0.0004 -0.0045	0.0006 -0.0197	0.0158 -0.0320	0.0098 0.0161	-0.0023 -0.0042
Bias GLS	0.0004 -0.0046	0.0006 -0.0199	0.0158 -0.0321	0.0099 0.0164	-0.0233 -0.0042
			rho=-0.5, var=1		
Bias OLS	0.0006 -0.0091	-0.0012 -0.0026	0.0200 -0.0199	-0.0006 -0.0236	-0.0090 -0.0045
Bias GLS	0.0006 -0.0091	-0.0012 -0.0026	0.0199 -0.0202	-0.0006 -0.0234	-0.0090 -0.0046
			rho=0.5, var=1		
Bias OLS	-0.0043 -0.0022	0.0059 -0.0019	0.0226 0.0083	0.0178 0.0001	0.0116 -0.0581
Bias GLS	-0.0044 -0.0011	0.0055 -0.0011	0.0218 0.0030	0.0179 -0.0001	0.0103 -0.0390
			rho=0.9, var=1		
Bias OLS	0.0022 0.0035	0.0313 0.0133	0.0917 0.0233	0.0127 0.0125	-0.064 -0.0105
Bias GLS	0.0085 -0.0029	0.0100 -0.0042	8.6802 -7.6101	0.0039 -0.0531	-0.1254 -0.0190
			rho=-0.9, var=3		
Bias OLS	-0.0024 -0.0181	-0.0028 0.0030	-0.0218 -0.0633	0.0143 0.1373	-0.0069 0.0682
Bias GLS	-0.0024 -0.0183	-0.0028 0.0029	-0.0218 -0.0640	0.0143 0.1383	-0.0070 0.0687
			rho=-0.5, var=3		
Bias OLS	0.0029 -0.0126	-0.0017 0.0077	0.0282 -0.0810	0.0374 0.0162	0.0128 0.0157
Bias GLS	0.0029 -0.0128	-0.0017 0.0077	0.0282 -0.0814	0.0374 0.0164	0.0128 0.0156
			rho=0.5, var=3		
Bias OLS	0.0062 0.0093	0.0265 0.0148	-0.0152 0.0078	0.0621 -0.0816	-0.0747 -0.0314
Bias GLS	0.0066 0.0047	0.0265 0.0121	-0.0150 0.0174	0.0600 -0.0177	-0.0732 -0.0148
			rho=0.9, var=3		
Bias OLS	0.0315 -0.0089	-0.0388 -0.0027	-0.1625 0.0475	0.2006 -0.0251	0.5300 0.0794
Bias GLS	0.3205 -0.0028	-0.053 -0.0086	-0.1737 -0.0123	0.1447 -0.0488	0.5223 0.0037

Table 2: Bias OLS β_0 when $\sigma = 1$

Sample Size	**Rho**			
	-0.90	**-0.50**	**0.50**	**0.90**
50	0.0004	0.0006	-0.0043	0.0022
100	0.0006	-0.0012	0.0059	0.0313
200	0.0158	0.0200	0.0226	0.0917
300	0.0098	-0.0006	0.0178	0.0127
500	-0.0023	0.0090	0.0116	-0.0641

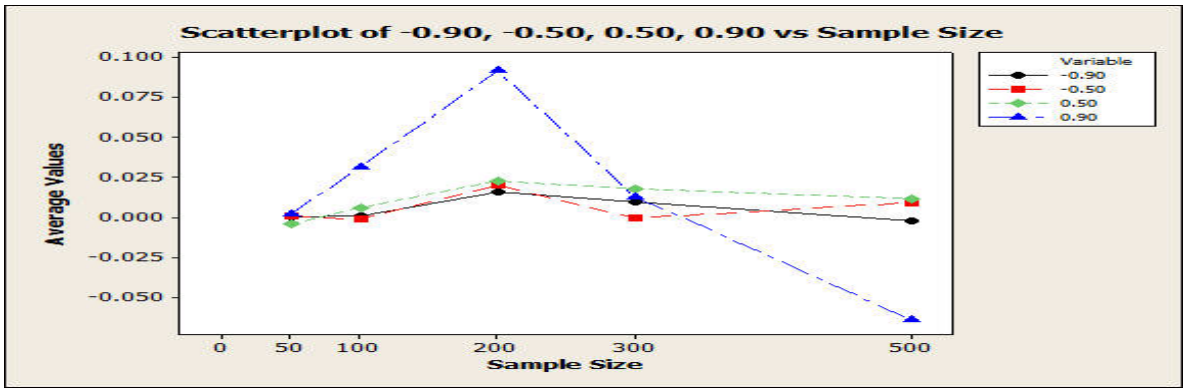


Table 3: Bias OLS β_1 when σ =1

Sample Size	**Rho**			
	-0.90	**-0.50**	**0.50**	**0.90**
50	-0.0045	-0.0091	-0.0022	0.0035
100	-0.0197	-0.0026	-0.0019	0.0133
200	-0.0320	-0.0199	0.0083	0.0233
300	0.0161	-0.0236	0.0001	0.0125
500	-0.0042	-0.0045	-0.0581	-0.0105

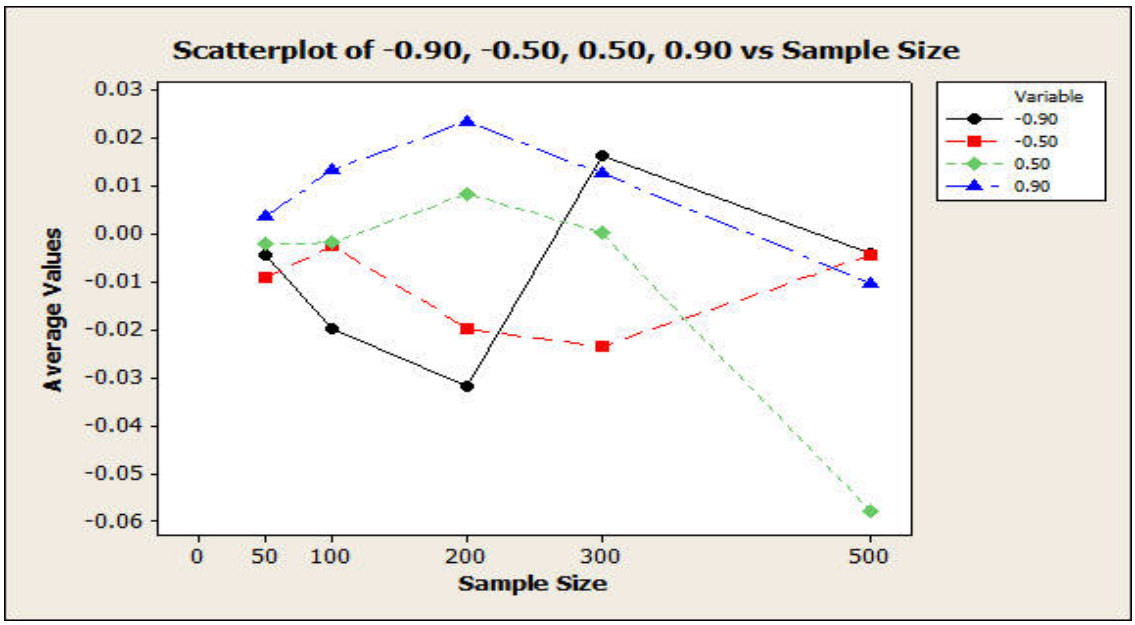


Fig. 4.2: Bias OLS β_1 when σ =1

In Fig. 4.2 the bias of OLS (β_1), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 1$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 500 with correlation level 0.50 the bias of OLS (β_1) is -0.0581 which is less than all others, and in sample size 200 with $\sigma = 1$ the correlation level 0.90 have maximum value of bias OLS (β_1) is 0.0233.

Table 4: Bias OLS β_0 when σ =3

Sample Size	**Rho**			
	-0.90	**-0.50**	**0.50**	**0.90**
50	-0.0024	0.0029	0.0062	0.0315
100	-0.0028	-0.0017	0.0265	-0.0388
200	-0.0218	0.0282	-0.0152	-0.1625
300	0.0143	0.0374	0.0621	0.2006
500	-0.0069	0.0128	-0.0747	0.5300

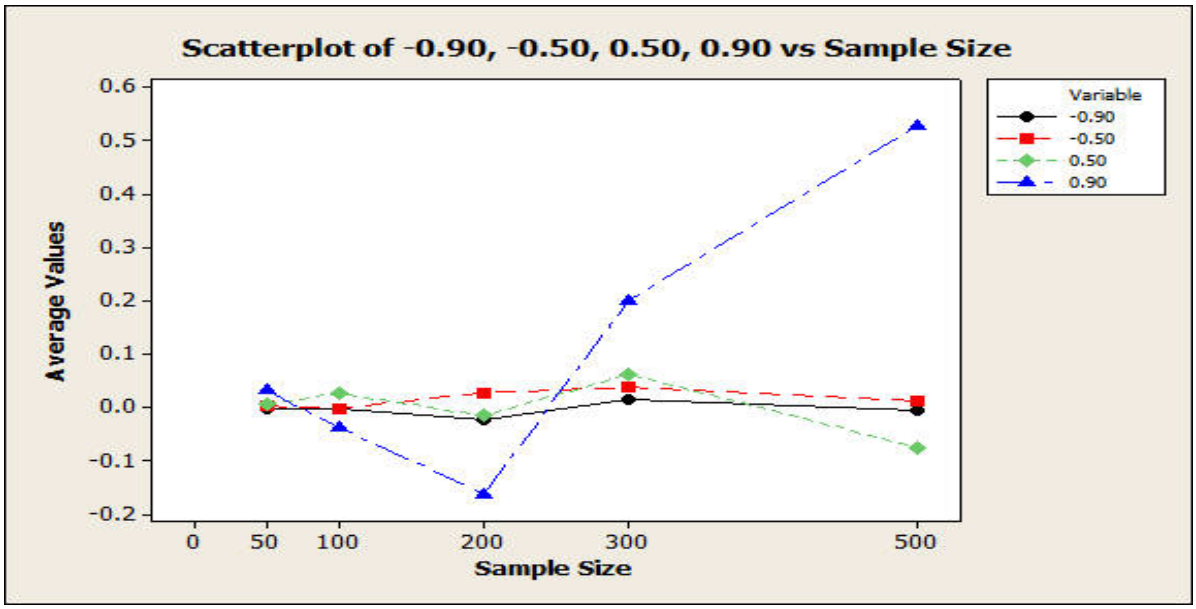


Fig. 4.3: Bias OLS β_0 when σ =3

In Fig. 4.3 the bias of OLS (β_0), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 3$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 200 with correlation level 0.90 the bias of OLS (β_0) is -0.1625, which is less than all others, and in sample size 500 with$\sigma = 3$, the correlation level 0.90 have maximum value of bias OLS (β_0) is 0.5300.

Fig. 4.4: Bias OLS β_1 when σ =3

In Fig. 4.4 the bias of OLS (β_1), we observe that in different sizes (50,100,200,300,500) and $\sigma = 3$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 300 with correlation level 0.50 the bias of OLS (β_1) is -0.0816, which is less than all others, and in sample size 300 with $\sigma = 3$ the correlation level -0.90 have maximum value of bias OLS (β_1) is 0.1373.

Fig. 4.5: Bias GLS β_0 when σ =1

Table 5: Bias OLS β_1 when σ =3

Sample Size			**Rho**	
	-0.90	**-0.50**	**0.50**	**0.90**
50	-0.0181	-0.0126	0.0093	-0.0089
100	0.0030	0.0077	0.0148	-0.0027
200	-0.0633	-0.0810	0.0078	0.0475
300	0.1373	0.0162	-0.0816	-0.0251
500	0.0682	0.0157	-0.0314	0.0794

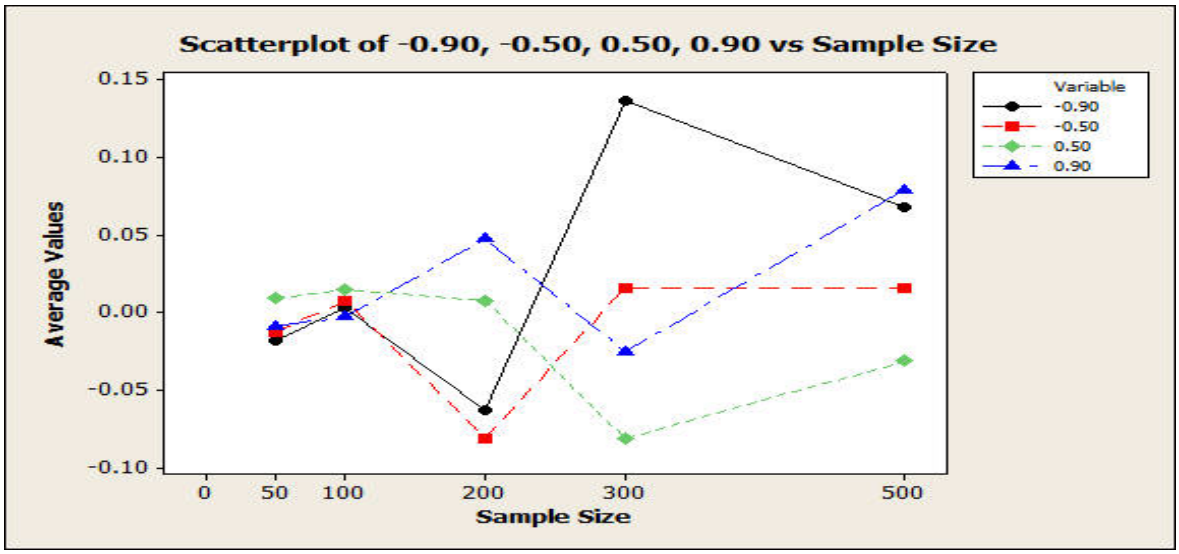


Table 6: Bias GLS β_0 when σ =1

Sample Size			**Rho**	
	-0.90	**-0.50**	**0.50**	**0.90**
50	0.0004	0.0006	-0.0044	0.0085
100	0.0006	-0.0012	0.0055	0.0100
200	0.0158	0.0199	0.0218	8.6802
300	0.0099	-0.0006	0.0179	0.0039
500	-0.0233	-0.0090	0.0103	-0.1254

In Fig. 4.5 the bias of GLS (β_0), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 1$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 500 with correlation level 0.90 the bias of GLS (β_0) is -0.1254, which is less than all others, and in sample size 200 with $\sigma = 1$ the correlation level 0.90 have maximum value of bias GLS (β_0) is 8.6802.

Fig. 4.6: Bias GLS β_1 when σ =1

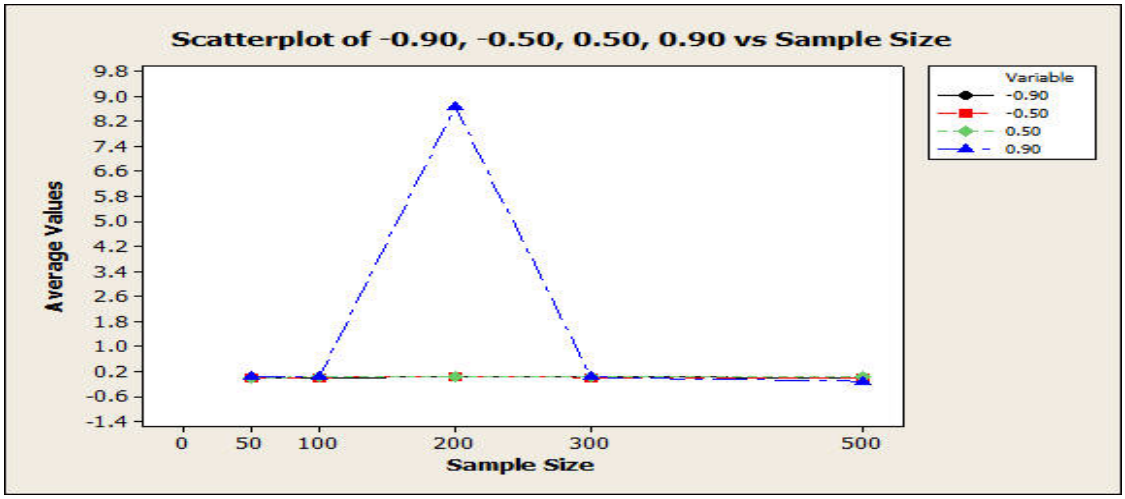


Table 7: Bias GLS β_1 when σ =1

Sample Size			**Rho**	
	-0.90	**-0.50**	**0.50**	**0.90**
50	-0.0046	-0.0091	-0.0011	-0.0029
100	-0.0199	-0.0026	-0.0011	-0.0042
200	-0.0321	-0.0202	0.0030	-7.6101
300	0.0164	-0.0234	-0.0001	-0.0531
500	-0.0042	-0.0046	-0.0390	-0.0190

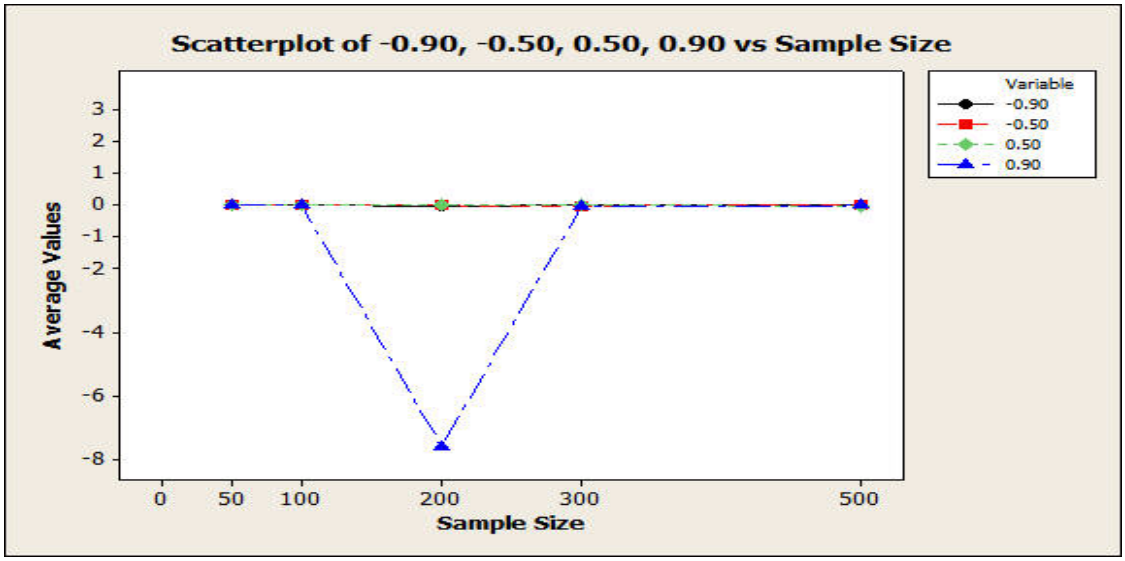
In Fig. 4.6 the bias of GLS (β_1), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 1$ with different correlation levels (-0.9,-0.50, 0.50, 0.90). In sample size 200 with correlation level 0.90 the bias of GLS (β_1) is -7.6101, which is less than all others, and in sample size 300 with $\sigma = 1$ the correlation level -0.90 have maximum value of bias GLS (β_1) is 0.0164.

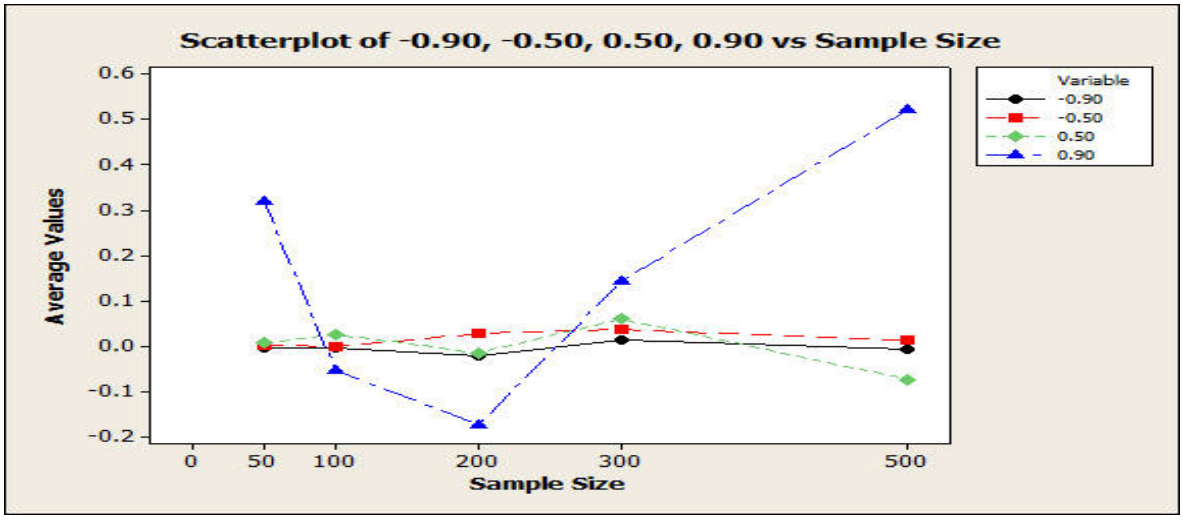
Table 8: Bias GLS β_0 when σ =3

Sample Size			**Rho**	
	-0.90	**-0.50**	**0.50**	**0.90**
50	-0.0024	0.0029	0.0066	0.3205
100	-0.0028	-0.0017	0.0265	-0.0534
200	-0.0218	0.0282	-0.0150	-0.1737
300	0.0143	0.0374	0.0600	0.1447
500	-0.0070	0.0128	-0.0732	0.5223

Fig. 4.7: Bias GLS β_0 when σ =3

In Fig. 4.7 the bias of GLS (β_0), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 3$ with different correlation (-0.90,-0.50, 0.50, 0.90). In sample size 200 with





correlation level 0.90 the bias of GLS (β_0) is -0.1737, which is less than all others, and in sample size 500 with $\sigma = 3$ the correlation level 0.90 have maximum value of bias GLS (β_0) is 0.5223.

Table 9: Bias GLS β_1 when σ =3

Sample Size			**Rho**	
	-0.90	**-0.50**	**0.50**	**-0.50**
50	-0.0183	-0.0128	0.0047	-0.0028
100	0.0029	0.0077	0.0121	-0.0086
200	-0.0640	-0.0814	0.0174	-0.0123
300	0.1383	0.0164	-0.0177	-0.0488
500	0.0687	0.0156	-0.0148	0.0037

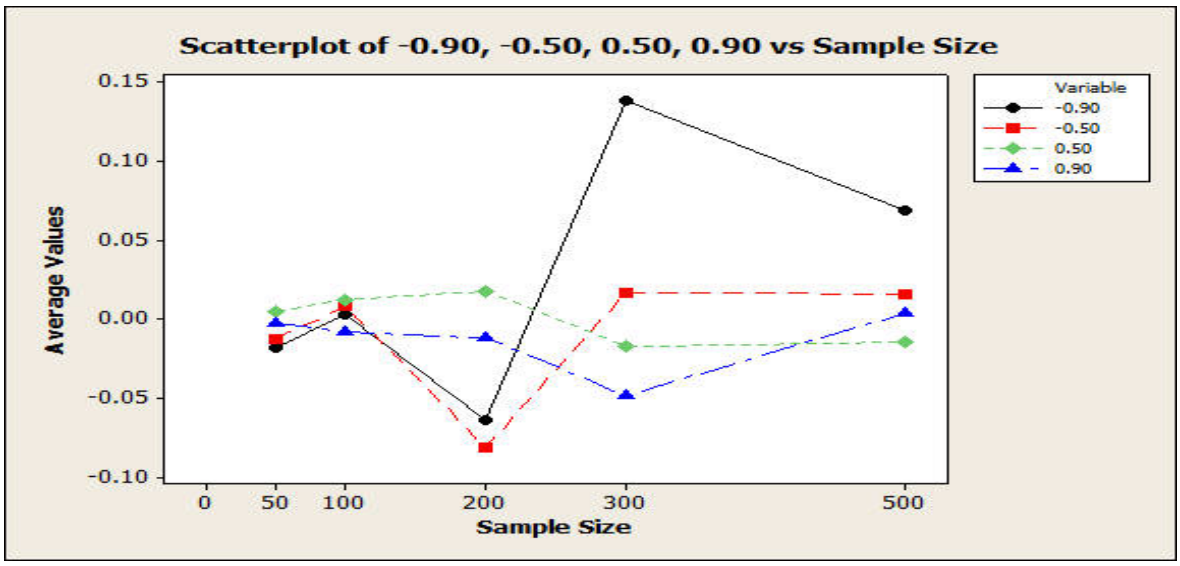


Fig. 4.8: Bias GLS β_1 when σ =3

In Fig. 4.8 the bias of GLS (β_1), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 3$ with different correlation levels (-0.9-,-0.50, 0.50, 0.90). In sample size 200 with correlation level -0.50 the bias of GLS (β_1) is -0.0814, which is less than all others, and in sample size 300 with $\sigma = 3$ the correlation level -0.90 have maximum value of bias GLS (β_1) is 0.1383.

5. conclusion

We have discussed the Ordinary Least Squares and Generalized Least Squares techniques and estimate with First Order Auto-Regressive (AR1) scheme from different correlation levels by using simple linear regression model. For this purpose, we use simulation of Monte Carlo study and the experiment is repeated 5000 times and performed for different sample sizes.

The average values of parameters of the Ordinary Least Squares and Generalized Least Squares estimation with different size of sample and correlation levels are estimated. When the bias values of Ordinary Least Squares and Generalized Least Squares is not normal with haphazard manner of average values.

Comparing the bias of OLS and GLS of (β_0), we observe that in different sample sizes (50,100,200,300,500) and $\sigma = 1,3$ with different correlation levels (-0.90,-0.50, 0.50, 0.90). In sample size 200 with correlation level 0.90 the bias of OLS (β_0) is -0.1625, which is less than all other estimates, and in sample size 500 with the correlation level 0.90 have maximum value of bias OLS (β_0) is 0.5300.

In GLS, we observe that in sample size 200 and $\sigma = 3$with correlation level 0.90 the bias of GLS (β_0) is -0.1737, which is less than all bias estimates, and in sample size 200 and $\sigma = 1$ with correlation level 0.90 the bias of GLS (β_0) is 8.6802, which is maximum in all levels. Similarly minimum and maximum bias values of OLS and GLS of (β_1) are -0.0816, -7.6101 and 0.1371, 0.1383 respectively. These result shows GLS is best in different sample sizes and correlation situations.

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