THE COMMON ERRORS IN THE LEARNING OF THE SIMULTANEOUS EQUATIONS

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\textbf{ABSTRACT}

The purpose of this study is to understand the causes of common errors and misconceptions in the learning attainment of simultaneous equations, specifically on linear and non-linear equations with two unknowns. The participants consisted of 30 Year 9 students in one of the elite government schools in Brunei Darussalam. Further analyses of their work led to the categorisation of four factors derived from the recurring patterns and occurrences. These four factors are complicating the subject, wrong substitution of the subject, mathematical error and irrational error in solving the question. These factors usually cause participants to make errors or simply misconceptions that usually led them to errors in solving simultaneous equations.

\textbf{Keywords:}
Simultaneous equations, Errors, Misconceptions, Secondary Mathematics, Brunei Darussalam

\textbf{1. INTRODUCTION}

Simultaneous equation is often perceived as a difficult and demanding topic to deal with requiring a lot of algebraic processes to find the solution (Ugboduma, 2012). The nature of it being heterogeneous and often vigorous is why most participants have little to no interest in studying or even attempting the question during their test or examination (Ugboduma, 2006). This is particularly true in Brunei Darussalam (hereinafter, referred to as Brunei) where rote memorisation has normally been the common way of teaching and learning Mathematics (Khalid, 2006), only to be used for passing certain tests or examinations (Shahrill, 2009, 2018; Salam & Shahrill, 2014; Shahrill & Clarke, 2014, 2019; Zakir, 2018). This prevents participants to utilise and relate any lessons learned from the class to the real-life situation. Moreover, the lack of understanding in learning Mathematics due to rote memorisation usually led participants to forget most of the knowledge taught after going through their said tests or examinations, which usually daunts them once they have to go through it again, due to the repeating nature of Mathematics (Matzin et al., 2013;
Shahrill et al., 2013). This is why for most students, they have negative attitude towards mathematics rendering it as one of the most challenging subjects for students in Brunei generally (Chua, et al., 2016; Khoo et al., 2016). Hence, an intervention to alleviate this negative trend is required.

Although there are some literature studies that investigate the matter of Simultaneous equations (Ugbodumua, 2006, 2012; Yunus et al., 2016; Nordin et al., 2017), this study in particular focuses on Linear and Non-Linear Equations in Two Unknowns. This makes the investigation of this material to be more significant, at least in the opinion of the researchers. This is because Simultaneous Equations is an integral part of Algebra which is needed in most mathematical topics or even other learning area of the 21st century such as Computer, Sciences or even Engineering to name a few. Nevertheless, Simultaneous equations are usually one of the challenging topics to be taught in school as participants usually struggle to understand the concepts and just prefer to memorise steps and methods for the sake of getting through tests or exams. Accordingly, this really creates a question whether the current method of teaching is ineffective and should a different method of teaching be required as an alternative of teaching Simultaneous equations, particularly of Linear and Non-Linear Equations in Two Unknowns in Brunei.

Yunus et al. (2016) also pointed out that most teachers in Brunei teaches simultaneous equation by telling which only provide participants with instrumental understanding in applying the rules of algebra in solving simultaneous equations, neglecting the relational understanding which is more helpful in participants’ understanding when also present. Yunus et al. (2016) then further mentioned that, because of this, participants usually interpret teacher’s instruction wrongly due to failure in understanding participants’ thought processing mechanism. Therefore, a learner-centred approach is recommended in order to minimise any misconceptions that might arise. This is in-line with the thoughts from Ugbodumua (2006, 2012) that mentioned good methodologies are required to help stimulate participants in enhancing their understanding of simultaneous equations. He further stated that a carefully designed methodology that is adopted by an adept teacher is a key factor for participants to improve their learning.

Jaggi (2006) clearly impart that a statement of equality is defined as an equation. We call it as an Identity when the statement of equality is true for all the unknown values involved, denoted by the symbol \( \equiv \), and we call it as conditional equation using the symbol \( = \), when the statement of equality is only true for certain values of the unknown qualities. Häggström (2008) defined an act of equaling by which a state is being equal as an equation. This formal statement of equivalence in terms of mathematical logical expressions is often denoted by the symbol of equal sign, \( = \). A mathematical statement that has two same values is an Equation. For example, \( 2+1=4-1 \). Häggström (2008) later explained that when two events are done, occurring or happening at the same time, it is then called Simultaneous. Therefore as stated by Ugbodumua (2012), if we have two or more equations that are true at one end, satisfying the same values of involved unknowns, then we can call it as Simultaneous equations.

For a straight-line equation that has two variables, the number of solutions will be infinite. If we denote the first variable as \( x \) and the other variable as \( y \), then any one of the solutions for \( x \) can be substituted into the straight-line equation giving its corresponding \( y \) value. However, if two of such equations are simultaneously calculated together then there might be only one set of solution of \( x \) and \( y \) that satisfy both equation simultaneously (Ugbodumua, 2012). For simultaneous equations of linear and non-linear equations in two unknowns, this amount of solution that can satisfy both equations simultaneously increase depending on the degree of the Non-Linear equations. For instance, if the non-linear part of
the equation is a quadratic equation, then the solutions should come in two sets or one repeating solution.

As mentioned by Yahya and Shahrill (2015), it should be easier to improve participants’ understanding in their future endeavours in solving algebraic problems, which is crucial for simultaneous equations, if the reasons of their workings can be identified. Consequently the purpose of this present study is to understand the causes of common errors and misconceptions made by participants in their attainment of simultaneous equations, particularly of linear and non-linear equations in two unknowns. This is so that an alternative method of teaching can be proposed to minimise these misconceptions and errors as much as possible by way of analysing and thinking.

Another reason is to investigate the causes of common errors and misconceptions that participants keep on committing in attaining the learning of simultaneous equations of linear and non-linear equations in two unknowns, especially for mid to low level ability participants. Although there are a lot of studies that cover on types common errors and misconceptions (Sarwadi & Shahrill, 2014), it is hoped that in identifying the causes of it may help participants in preventing in committing those common errors and misconceptions so participants can have a better understanding, attitude and mindset in the process of learning the topic.

Importantly, with the formation of this study, we hope that further contribution can be made on the literature concerning how Simultaneous equation is taught in Brunei. The authors also feel the necessity of the study since upon reviewing literature, particularly in simultaneous equations of linear and non-linear equations in two unknowns, almost none surfaced. Its instant existence in literature can be used as a doorway to pave for future studies in providing an alternative way of teaching simultaneous equations in mathematics lessons, particularly of linear and non-linear equations with two unknowns. As such, this present study is guided by the research question “What are the common errors and misconceptions made by participants in their learning of the Simultaneous Equations?”

2. METHOD

A total of 30 participants participated for this study taken from two Year 9 classes in one of the elite government schools in Brunei. The level of ability of both classes ranges from medium to low ability, mostly being medium. Both classes have the required algebraic and arithmetic skills to do simultaneous equations of linear and non-linear equations with two unknowns, such as linear equation manipulation and solving quadratic equations.

A test was administered to the participants that contained three item questions chosen from a pool of questions (refer to Table 1) validated by experienced mathematics teachers. The questions chosen should test participants in various ways on solving simultaneous equation of linear and non-linear equations in two unknowns, such as choosing a proper subject to be used for the substitution method or how they can avoid complications of simultaneous equations by simplifying equations further before solving. The validity was assessed through judgmental methods collected from comments and opinions of experienced Mathematics teachers from the school Mathematics department. It was also assessed to a specification method using the first four levels of Blooms’ taxonomy namely remembering, understanding, applying and analysing.
Table 1. List of questions for the test with item number

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y = xy + x^2 - 9 ) ( y = 3x - 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 4x + y = -8 ) ( x^2 + x - y = 2 )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{x - y}{\frac{3}{2}} + 3 = 0 ) ( \frac{3}{2} + \frac{1}{1} = 0 ) ( x + y = 2 )</td>
</tr>
</tbody>
</table>

3. RESULTS AND DISCUSSION

3.1. Results

The nature of Item 1 \((y = xy + x^2 - 9, y = 3x - 1)\) consisting of one simple linear equation and a quadratic equation is quite straightforward in relative to the other items. For the linear equation, the variable \( y \) has already been arranged as the subject. Both equations are not in a fraction form making it easier for algebraic manipulations. Ideally this should be a straightforward task of substitution, expansions, simplifications and quadratic equation solving, skills that all participants already acquired.

There are two common errors that were made most by participants for this item. The first one is where participants try to make \( x \) as the subject from the linear equation. While this is actually not a form of error in any ways, making \( x \) as the subject in this case will give a fractional subject of \( x = \frac{y + 1}{3} \) which usually will result in an error due to complication that it will produce. Typically, a lot of errors were usually made when the subject is made into a complicated fraction form (Low et al., 2020).

Figure 1 exhibits a sample of a Student 1’s work dealing with said fractional subject. This unnecessary step causes the question to become more complicated in a form of fractions, expanding fractions, and fractional algebra manipulation, to name a few. These unnecessary complications usually increase the risk of participants being careless as shown by this participant’s work, where he did not expand \( \left( \frac{y^2 + 1}{3} \right) \times \frac{3}{3} \) correctly which then leads him to get the wrong final quadratic equation.

One can assume that this is just an overlooked error made by the participant since the other mathematical part of his workings was done quite well. One mark was given for his correct although unnecessary substitution of subject \( x \), and another one mark is given for his valid attempt in solving the final quadratic equations.
Secondly, there are quite a number of participants who made an error of failing to substitute their subject \( y = 3x - 1 \) into both side of the quadratic equation \( y = xy + x^2 - 9 \). This will make their equation impossible to solve since both variable \( x \) and \( y \) still exist, defeating the purpose of substitution, which is to eliminate one out of the two variables. From the sample work by one participant shown in the Figure 2, the participant only substituted \( y = 3x - 1 \) into the right-hand side of \( y = xy + x^2 - 9 \) which in the end gives her the final equation of \( y = 4x^2 - x + 9 \).

Student 2 then forces her way through in solving the final quadratic equation even though both variables still exist. This then resulted a wrong answer with no marks, since there were no opportunities for the marker to give one throughout the workings.
For Item 2 \((4x + y = -8, x^2 + x - y = 2)\), almost half of the participants managed to get full marks, reflecting the easier nature of the question. The common error made by participants, who mostly scored 2 marks for this item, is very similar to the error made in Item 1. It is using the fractional subject for substitution, which as mentioned before is not an actual error, but usually leads to mathematical errors since it complicates workings.

**Figure 3** below shows another working of a participant where he makes the variable \(x\) as the subject, i.e. \(x = \frac{-8-y}{4}\). This subject is not only fractional in nature but also contains a lot of negatives sign, which usually can cause carelessness that leads to complications. However, the error made by Student 3 is the expansion of \((\frac{-8-y}{4})^2\), where the denominator 4 is not expanded.

This can be due to a simple misstep or lack of indices skills or knowledge. One can assume that these two factors can be easily eliminated if variable \(y\) was made as the subject, since the participant will then have a fraction-less subject leading to a straightforward expansion. This error then leads to a wrong solution. Student 3 was awarded with 2 marks for a correct substitution and a valid attempt on solving the final quadratic equation.
For Item 3 \( \left( \frac{x - \frac{y}{2}}{3} + 3 = 0, \frac{3}{x} + \frac{2}{y} - \frac{1}{2} = 0 \right) \), although unanimously, all of the participants were unable to score more than 1 mark, some attempts can be seen to have the correct idea in generally solving the simultaneous equations. However, most participants lack the skills in manipulating algebraic fractions that led to errors that hindered them to get the required final quadratic equations, resulting in the severe loss of marks. This also caused a lot of participants to quit trying after their working seems to get very complicated.

The sample work of Student 4 in Figure 4 reflects on this. After choosing the subject \( x = \frac{3y}{2} - 9 \), a correct substitution into the second equation yield her 1 mark. She then proceeds to simplify the equations by trying to get rid of the fraction. In doing so, she made an error by dividing the whole equation with 2 instead of multiplying. As the equation goes peculiar, she then stops trying. It can also be observed that she failed to see \( \frac{12y}{2} \) as \( 6y \), which can then make her equation simpler. One can perceive from this that lack of critical thinking was present when attempting the question.

![Figure 4. Sample of Student 4’s work for Item 3](image)

Then some attempts, especially from participants who managed to score 1 mark, were quite decent relative to the challenging nature of the question. Figure 5 shows an example of this. Student 5 can be seen to have a very good algebraic manipulation skills but made an error in expanding \((2 - \frac{1}{2}y)(-18 + 3y)\) which then made him lose 4 marks. The 1 mark was given for a valid attempt to find the solution from his wrong working. Errors were generally made when the subject is made into a complicated fraction form, which seems to be the case here. Once again one can assume that Student 5 may obtain more than 1 mark if the error can be avoided by getting the fraction in the first instance.

Item 3 can be categorised as a challenging due to the fact that both are in a fraction state of form. However, the difficulty can be lowered if participants can change the fraction nature to whole number by multiplying it with the LCM of the denominator. From there the question will then be on par as Items 1 and 2.
To achieve in-depth insights of the students’ work for further analysis, the following four categories of factors were derived from the recurring patterns and occurrences, which affects the students’ work. These factors usually cause participants to make errors or simply misconceptions that usually led them to errors. These four factors are: Complicating the Subject (CS), Wrong Substitution of the subject (WS), Mathematical Error (ME) and Irrational error in solving the question (I).

Briefly, (CS) is a factor when a student complicates a simple subject that then may lead to producing errors. For example, in Item 1, instead of using the simple subject \( y = 3x - 1 \), the student might complicate it by using \( x = \frac{y+1}{3} \) instead. This may cause errors further in their workings.

(WS) is a factor when participant error in substituting their subject into another equation. This can be either literally substituting their subject wrongly or only substituting their subject partially, which this factor will focus on solely. For example, substituting \( y = 3x - 1 \) into the right-hand side of \( y = xy + x^2 - 9 \) only, will not make the elimination of variable \( y \) complete, since there will still be variable \( y \) on the left-side of the equation. Hence, making it impossible to solve the simultaneous equation.

(ME) is self-explanatory where participants made simple mathematical errors such as expanding, rearranging, changing signs or algebraic manipulations, to name a few. It can be due to carelessness of the participant or lack of mathematical skills.

(I) is when participants have no understanding about the question and in solving it. Usually this can be seen when participant tend to give unreasonable solution or answering it as a different topic such as solving both equations from the simultaneous equations
independently or solving it as another topic, for example using $b^2 - 4ac$ from the topic discriminant of intersections.

![Figure 6](image)

**Figure 6.** Factors contributing to participants’ marks on the test

The bar graph from Figure 6 shows that for Item 1, a large number of 12 participants changed their subject from $y = 3x - 1$ into $x = \frac{y+1}{3}$. Out of these 12 participants who committed CS, 9 of them yield 0 mark. This can tell us that indeed complicating the initial subject usually leads to errors and an effort to prevent participant from doing CS might improve their marks. Then 6 participants committed the WS where most of them failed to substitute the subject properly. Only 2 of these participants scored 0 mark while others had a good attempt acquiring 1 or 2 marks. A small number of Irrational errors tell us that most participants have the general idea on solving the question even though the topic has not been covered with them.

For Item 2, there is still a moderate fraction of participants committing the CS, although only 2 out of these 8 participants scored 0 marks indicating a good attempt. This might be due to the simpler nature of the algebraic equation. No WS was recorded since the equation was designed for a one-sided substitution only, unlike Item 1. Most participants were able to score well on this question, but there is an interesting case where a participant who scored full mark on Item 1 and Item 2 committed a CS on Item 2 but not on Item 1. The participant choose variable $x$ as the subject regardless how difficult it can be. This can tell us that although his mathematical skills are very high, a critical thinking might be lacking. One can assume that the reason $x$ is used as the subject on both occasions might be because he is used to it from his previous lower level education, where $x$ is always used as the starting subjects in classroom or exams.

Finally, Item 3 shows that most participants committed the ME, understandably due to the complicated nature of the fraction form. This though can be avoided since the question is designed in such a way that the fraction can be get rid of and changed into a much simpler fraction-less equation by a simple algebraic manipulation, which all of the participants should already possess the skill to by now.

### 3.2. Discussion

Based on the in-depth analysis of the repeating pattern found in the participants’ work, four main factors were detected in affecting participants’ test as mentioned earlier. The three major factors were complicating the subject needed for substitution method (CS),
making error while substituting their subject into the other equation (WS) and simple mathematical error (ME).

The first factor, while mathematically correct, was committed by a total of 20 participants for both Items 1 and 2 of the test, which mostly led participants in making mathematical error for their subsequent workings, scoring them a very low mark in average. The second factor was mostly found in Item 1, where six participants failed to substitute their subject of $y = 3x - 1$ to the both side of $y = xy + x^2 - 9$ rendering their following workings wrong. Both of these factors were perceived by the researchers as misconceptions believed due to the lack of understanding on the meaning of the mathematical process in solving the simultaneous equations, which may have resulted them to be rigidly stuck in their sole method, regardless of how difficult it was for them.

The third factor comprises the highest frequency count out of the other factors (33 total for all items), where participants committed the mathematical errors. Although this result is understandable due to the fact that most participants have the mathematical ability of lower to medium, the mathematical part of the question was designed and allowed to be simpler if certain mathematical skills were to be applied such as simplification, changing fractions to whole numbers or changing negative coefficient of quadratic equations to positive before solving it. Failure in doing it usually leads to participants having to deal with complicated equations or forcing them to commit errors due to carelessness.

These three factors are believed to contribute to the poor results obtained by almost all participants during the test, where 28 out of the 30 them only managed to score marks of below average out of the total 15 marks, where 2 marks being the mode of the result. Consequently, Rohmah and Sutiarso (2018) mentioned that a weak prior knowledge is one of the major problems in solving simultaneous equations. However, the participants of both classes, who are mostly of medium ability participants, managed to score above average marks if not excellent, even though they were not taught on how to solve the simultaneous equations.

4. CONCLUSION

Simultaneous Equations, especially linear equation versus non-linear equation, with its vigorous and heterogeneous nature always intimidate participants in learning it wholeheartedly. They perceive it as a subject that is very difficult to follow rather than something that is yet to be fully understood. This incomplete understanding usually leads them to make misconceptions and common errors along the line that further hinder their study. One of the major factors that resulted from this study was misconception in making only $x$ as the main subject regardless how difficult it can render their subsequent workings, although other variable as subject can offer much simpler workings. Another factor is failing to use their understanding in simplifying equations to achieve simpler mathematical workings in avoiding complications and careless mistakes. It is imperative that we understand the causes of common errors and misconceptions made by participants in their attainment of simultaneous equations, particularly of linear and non-linear equations in two unknowns. This way we may be able to minimise these misconceptions and errors as much as possible and to analyse and reason on every steps taken in calculating that leads to the attainable correct answer.
REFERENCES


