1. Introduction

Given the current conditions of hybrid warfare, civil aviation aircraft perform flights as usual. Air surveillance and control of air objects typically involve radar (RS) with mechanical rotation. Examples of such RSs are P-18 (Ukraine) and their modifications (P-18MA, P-18MU, P-18 Malachite), P-19 (Ukraine), 19Zh6 (Ukraine), 35D6 (Ukraine),
2. Literature review and problem statement

In [1], to increase the accuracy of determining the coordinates of air objects, an increase in the signal/noise ratio by compacting the RS position is proposed. In [2], to improve the accuracy of determining the air objects' coordinates, an increase in the signal/noise ratio was proposed by using RSs of different frequency bands. In [3], to enhance the accuracy of determining the coordinates of air objects, an increase in the signal/noise ratio was proposed through the use of more complex methods of radar information processing. The main disadvantages in [1–3] are the increase in the number of RSs, the cost of radar reconnaissance, the hardware and algorithmic complication of radar information processing methods.

In [4], to improve the accuracy of determining the air objects' coordinates, it is proposed to increase the ratio of signal/noise by combining observation RSs into multi-radar systems. In [5], the methods of coherent signal processing are proposed when uniting two RSs into a multi-radar system. The disadvantages of [4, 5] are the need to increase the number of observation RSs, the need to ensure spatial and time synchronization of the operation of observation RSs.

In [6], to improve the accuracy of determining the coordinates of air objects, spectral estimation methods are proposed. At the same time, a priori information regarding the signal reflected from an air object is additionally used. The disadvantage of [6] is the need to assess the direct or inverse correlation matrix of input signals, a significant amount of calculations, the presence of additive and multiple interference, a decrease in the signal/noise ratio, etc.

In [7], to improve the accuracy of determining the air objects' coordinates, a projection method was proposed. The method is based on the use of the Petrov-Galerkin transformation and the Hallström discerrnment strategy. The disadvantages of the method are the need to have a priori information about the characteristics of air object scattering and significant computational costs.

In [8], to increase the accuracy of determining the coordinates of air objects, RSs additionally use signals of cellular communication stations. This certainly increases the signal/noise ratio and the detection rate of the reflected signal. For example, paper [9] reports calculations on the detection of air objects by a system with two RSs with the additional use of cellular signals. The authors of [9] drew a conclusion about the increased accuracy of determining the coordinates of an air object in the specified system with two RSs. The disadvantages of [8, 9] are the need to ensure the simultaneous operation of RSs and cellular communication stations and the need to optimize the geometric structure of an RS system.

In [10], to improve the accuracy of determining the coordinates of air objects, RSs additionally use signals from space navigation systems. The additional use of energy signals from space navigation systems increases the total signal/noise ratio. The disadvantages of [10] are the need to ensure the simultaneous operation of RSs and space navigation systems and the need to optimize the geometric structure of an RS system.

Increasing the accuracy of determining the air objects' coordinates by RSs by introducing a channel of the distributed reception is proposed in [11]. For the method reported in [11], the authors of [12] calculated parameters of the detection zone of the inspection RS with the additional introduction of the channel of the distributed reception. The disadvantages of [11, 12] are the need to suppress the penetrating signal and the need for a restructuring of RS.

In [13], to increase the accuracy of determining the coordinates of air objects, the possibility of using information from Automatic Dependent Surveillance-Broadcast (ADS-B) transponders is claimed. The disadvantage of [13] is the absence of any calculations and estimates of the accuracy in determining the coordinates of air objects.

In [14], a method of determining the air objects' coordinates by RS with the additional use of ADS-B receivers is proposed. Paper [14] reports the results of calculating the accuracy of determining the coordinates of air objects. It was established that the additional application of ADS-B receivers improves the accuracy of determining the coordinates of an air object. In this case, the value of accuracy depends on the range to the air object. The disadvantage of [14] is the inability to employ the devised method in the absence of ADS-B transponders aboard an air object. In addition, paper [14] does not take into consideration the vulnerability of ADS-B receivers and transponders.

In [15], the Loran-C system (United States of America) is proposed. Underlying the system is a difference-range method for determining navigation parameters. The disadvantage of [15] is the use of the system only in the interests of resolving navigational tasks.

In [16], for air traffic control within an airport, the use of a multi-position radio-monitoring system for determining the location of objects is proposed. In [16], this system was termed multilateration (MLAT). In the MLAT systems, the location of objects is determined based on the assessment of the distances of the object to any number of reference radio-navigation points. These radio-signaling points house signal receivers from objects. The advantages of [16] are the low cost of equipment, increased accuracy of measurement of the coordinates of objects, high reliability, and noise immunity. The disadvantage of [16] is the ability to determine objects' coordinates only within the airport or airfield.

In [17], a global system (Wide area Multilateration (WAM)) was proposed. The WAM system solves the tasks of navigating air objects in the field of space hundreds of kilometers on the Earth's surface. The physical principles of MLAT and WAM operation are the same. The advantages of [17] are the increased accuracy of measuring the
The accuracy of the Bancroft algorithm. The advantage of [22, 23] is the simplicity and insignificance of volume calculations, the use of a linear system of equations. The disadvantages of [22, 23] are the ambiguity of the coordinates obtained, the use of an additional procedure to choose a solution. In addition, the methods reported in [22, 23] can only be applied for an active multilateration system.

In [24], a method for estimating the location of objects for the passive operation of a multilateration system is proposed. The method is based on a combination of algebraic and statistical methods for determining the coordinates of objects using the Bancroft algorithm. The algebraic methods solve a linear system of equations. The statistical methods make it possible to refine the solution obtained from the algebraic methods. The accuracy of estimation of the location of an object using the method from [24] corresponds to the accuracy of the Bancroft algorithm. The disadvantage of [24] is the capacity of the multilateration system operation at small distances (within the airport (airfield)).

In [25], the working zones of multilateration surveillance systems were calculated. The working area parameters were analyzed depending on the system configuration. The main drawback of [25] is that all calculations were performed for the global WAM system without taking into consideration the features of the MLAT system.

Consequently, the use of multilateration technology improves the accuracy of determining the coordinates of air objects. The additional application of multilateration technology in detecting air objects by RSs is one of the possible ways to increase the accuracy of determining the air objects’ coordinates. Therefore, it is advisable to build a method for determining the coordinates of air objects by RSs with the additional use of multilateration technology.

### 3. The aim and objectives of the study

The purpose of this study is to devise a method for determining the coordinates of air objects by RSs with the additional use of multilateration technology. This could make it possible to improve the accuracy of determining the coordinates of air objects by RSs due to the additional use of multilateration technology.

To accomplish the aim, the following tasks have been set:
- to experimentally confirm disruptions in the operation of ADS-B transponders;
- to define the main stages of the method for determining the coordinates of air objects by RSs with the additional use of multilateration technology;
- to assess the accuracy of determining the air objects’ coordinates by RSs with the additional use of multilateration technology.

### 4. The study materials and methods

When experimentally confirming disruptions in the operation of the ADS-B system, the following research methods were used:
1. Systems analysis.
2. Digital signal processing.
3. Statistical theory of detection and measurement of radar signal parameters.
5. Experimental research.
6. Comparative study.

When defining the main stages of the method for determining the air objects’ coordinates by RSs with the additional use of multilateration technology, the following research methods were used:
1. Passive location.
2. Multi-position radar location.
3. Radar theory.
4. Digital signal processing.
5. Theories of probability and mathematical statistics.
7. Systems analysis.
8. Statistical theory of detection and measurement of radar signal parameters.

The following research methods were used to assess the accuracy of determining the coordinates of air objects by RSs with the additional use of multilateration technology:
1. Passive location.
2. Multi-position radar location.
3. Radar theory.
4. Digital signal processing.
5. Theories of probability and mathematical statistics.
7. Statistical theory of detection and measurement of radar signal parameters.
10. Analytical and empirical methods of comparative research.
When validating the proposed solutions, the analytical and empirical methods of comparative research were applied.

The following restrictions and assumptions were accepted during the research:

– we consider two-coordinate survey radars with mechanical rotation (type P-18 (Ukraine) and their modifications (P-18MA, P-18MU, P-18 Malachite), P-19 (Ukraine), 19ZH6 (Ukraine), 35D6 (Ukraine), 79K6 (Ukraine));
– the RS receiver is digital;
– the absence of interference;
– unhindered reception of the signal from an air object is ensured by a multi-positioning system of receivers using multilateration technology;
– the elements of the multi-position receiver system are identical;
– as the elements of the multi-position system of receivers, the receiver FlightAware Piaware (China) is used;
– as signals for a multi-position system of receivers using multilateration technology, we use answer signals from aboard an air object at the request of the secondary radar system;
– the frequency of the request of the secondary radar system is 1,030 MHz;
– the frequency of response signal from aboard an air object is 1,090 MHz.

5. Results of the study on devising a method for determining the coordinates of air objects

5.1. Experimental confirmation of disruptions in the operation of ADS-B transponders

The authors of [14] proposed a method for determining the air objects’ coordinates by RSs with the additional use of ADS-B receivers. It was established that the additional application of ADS-B receivers improves the accuracy of determining the coordinates of an air object. During an experimental study into the operation of the ADS-B system, the authors of [14] observed no disruptions in its work. The RTL-SDR receiver DVB-T+FM+DAB 820T2 & SDR was selected as an ADS-B receiver. An example of monitoring the air object 50816F (the Embraer civilian aircraft, manufactured in Brazil) is shown in Fig. 1 [14]. It was established that the air object 50816F is a civilian aircraft (Kherson-Kyiv flight).

Since ADS-B technology is open, it is exposed to both active and passive attacks [26]. Consequently, the lack of encryption and crypto-resistance makes it possible to send fake radio data and alter information in ADS-B packets. For example, an air object – a violator of the state border – can employ “mimicry” as a real civilian aircraft. It is impossible to detect such a fake based on data from RSs and ADS-B receivers.

An experimental study of disruptions in the operation of the ADS-B system was carried out during our research. The experimental study into the disruptions in the operation of the ADS-B system was performed at the Department of Programmable Electronics, Electrical Engineering, and Telecommunications, the Admiral Makarov National University of Shipbuilding (Mykolaiv, Ukraine). In this case, the FlightAware Piaware receiver was used (Fig. 2) [27].

Fig. 1. Route of the air object 50816F (a civilian aircraft flying from Kherson to Kyiv)

Fig. 2. Receiver FlightAware Piaware [27]

Specifications of the receiver [27]:
– USB SDR ADS-B FlightAware receiver with built-in 1,090 MHz band filter;
– Raspberry Pi 3 B+ single-board PC;
– a GPS receiver to determine the coordinates of the receiver and perform time synchronization;
– a collinear ADS-B antenna with a gain factor of 5.5 dB.

The FlightAware Receiver Piaware makes it possible to receive decoded ADS-B messages with a timestamp in a binary beast format.

As a result of observing the air object CYP487 on August 20, 2021, we registered disruptions in the ADS-B system operation (Fig. 3, 4).

Fig. 3 shows significant distortions in the coordinates of the CYP487 object in the area of the city of Kyiv. Such distortions may be associated with the instability of ADS-B receivers.

Fig. 4 shows less significant failures, which were repeated during the subsequent flight.
Such disruptions may be associated with a significant flow of air objects in the specified area. Thus, our analysis of the results demonstrated in Fig. 3, 4 has experimentally revealed disruptions in the operation of ADS-B receivers. Such disruptions could lead to a decrease in the accuracy of determining the air objects’ coordinates with the joint use of RSs and ADS-B receivers.

According to the Flightradar24 website, it was established that the air object CYP487 is a civilian aircraft flying from Moscow to Larnaca (Cyprus Airways) (Fig. 5).

ADS-B messages are known to use a 24-bit CRC checksum. Therefore, the probability of missing one failed ADS-B message is $1/2^{24} = 6 \cdot 10^{-8}$. The probability of getting multiple failed messages at a short interval is approaching zero. However, our analysis of Fig. 3, 4 indicates that the coordinates of the air object, which did not correspond to the actual coordinates, were indeed transmitted to the air. The most likely cause of the CYP487 air object’s failure coordinates is a GPS module integrated into the ADS-B transponder. The reasons for the failure of the GPS module can be various attacks. The main types of attacks [28] are data-level attacks, attacks on a GPS receiver software, attacks on GPS-dependent systems. The main destabilizing factors in the GPS operation are [28]:

- falsification of ephemerids;
- transmission of incorrect information about the current date;
- timeline synchronization disruptions;
- distortion of the signal – a pseudo-fall code (spoofing).
In addition, the possibility of intentional replacement of ADS-B messages cannot be completely excluded. However, to provide the necessary moving coverage area, it would be necessary to use another air object that would move along approximately the same route. Our analysis of Fig. 3, 4 indicates the absence of such an air object. Thus, the source of the failure coordinates for the CYP487 air object is most likely a GPS module integrated into the ADS-B transponder.

Consequently, the experimentally detected disruptions in the operation of ADS-B transponders could lead to a decrease in the accuracy of determining the air objects’ coordinates with the joint use of RSs and ADS-B receivers.

5.2. Main stages of the method for determining the air objects’ coordinates with the additional use of multilateration technology

Consider the case of the additional use of multilateration technology to determine the coordinates of air objects. A multi-position system of receivers for the implementation of multilateration technology is shown in Fig. 6. The system consists of three receivers as an example.

Assume that an air object with the coordinates \( X_{AO}, Y_{AO} \) is irradiated with signals of trace radars. The signal strength of the secondary radar system is 1,030 MHz. The response signal frequency from aboard an air object is 1,090 MHz. Taking into consideration that the frequency of the response signal from aboard the air object is 1,090 MHz, ADS-B receivers are accepted as receivers of the multi-position system. This can be, for example, the FlightAware Piawar or the RTL-SDR receiver DVB-T+FM+DAB 820T2 & SDR, and a 15-cm antenna (included) [14].

When irradiating an air object with trace radars, the signal is received at three receiving positions (ADS-B receivers). Multilateration technology implements a known difference-range method [29]. When applying the difference-range method [29], the coordinates of an air object are determined as the point of intersection of the three lines of positions from each of the points of reception. At the same time, three ranges to the air object, \( R_1, R_2, R_3 \), and three range differences, \( R_{12}, R_{23}, R_{31} \), are determined.

The range difference between the air object and the receivers is determined from the following expression (1):

\[
\Delta R_j(\alpha, \beta, \beta_j) = c \Delta \tau_j(\alpha, \beta, \beta_j), \tag{1}
\]

where \( \Delta R_j(\alpha, \beta, \beta_j) \) is the difference between the distances from an air object to the \( i \)-th and \( j \)-th reception points; \( \alpha \) is the vector of coordinates of an air object (in this work, for example, the Cartesian coordinate system is considered); \( \beta, \beta_j \) are the vectors of coordinates of reception points with numbers \( i \) or \( j \); the number of points of reception is \( N \); \( \Delta \tau_j(\alpha, \beta, \beta_j) \) is the difference in the moments of signal arrival from an air object to the \( i \)-th and \( j \)-th points of reception, respectively; \( c \) is the speed of light.

We denote the vectors of coordinates of an air object and the points of reception in the form \( \alpha(x, y, z) \), \( \beta_i(x_i, y_i, z_i) \), and \( \beta_j(x_j, y_j, z_j) \). Then expression (1) takes the following form (2):

\[
\begin{align*}
\Delta R_i &= R_i - R_j = \\
&= \sqrt{\left( x - x_i \right)^2 + \left( y - y_i \right)^2 + \left( z - z_i \right)^2 } \\
&+ \Delta \tau_i - \sqrt{\left( x_j - x_i \right)^2 + \left( y_j - y_i \right)^2 + \left( z_j - z_i \right)^2 } \\
&= c \Delta \tau_j.
\end{align*}
\tag{2}
\]

It is known [29] that of the entire set of measured differences in the arrival time of the signal, only \((N-1)\) differences of arrival moments are found to be statistically independent. Therefore, in this work, the differences in arrival moments are calculated in relation to one central point of reception. It is accepted that the index of the reference point of reception is zero; when recording, it is ignored. Therefore, a set of distance differences is \( \Delta R_i, i=1, ..., N \).

According to the essence of the difference-range method [29], the locus of points corresponding to one value of the difference of ranges \( \Delta R_i \) represents the line of position. The intersection of position lines indicates the location of an air object. At three points of reception and a unitary measurement of the difference in the ranges, the position of the air object is found by solving a system of nonlinear equations (2).

System (2) includes four unknown values. These are the three coordinates of the signal source \( \alpha (x, y, z) \) and the residual value \( \Delta R_{\mathrm{res}} = \Delta R_i - \Delta R_j \). The residual value \( \Delta R_{\mathrm{res}} \) appears due to the use of unsynchronized time scales at reception points. Therefore, it is not possible to explicitly solve system (2).

The general structure of the method for determining the air objects’ coordinates with the additional use of multilateration technology is demonstrated in Fig. 7.
Calculate the specified values of coordinates of air object (αS)

Calculate the matrix of partial derivatives AS

Compare the maximum value of vector ζS with the specified threshold level P

 inadequacy of the coordinates of one of reception points; the value of the threshold is taken for the assessment of coordinates. Otherwise, the iteration counter S is increased by one, and the transition to the second stage of the method is carried out.

The main differences of the improved method for determining the coordinates of an air object (Fig. 7) are:

- the secondary location system (frequency of the transmitter, 10.30 MHz) is used as a radiation source;
- the FlightAware Piaware receivers (1090 MHz frequency) are used as receiving points;
- the initial approximations of coordinates of the air object x(0), y(0), z(0) are used;
- the method is only an additional source of information about the coordinates of an air object, which are measured by the main RS.

Thus, we have improved the method for determining the coordinates of an air object by RSs, in which, unlike known ones, multilateration technology is additionally used.

The sequence of stages in the method for determining the coordinates of objects using multilateration technology is as follows:

1. Input of source data: the number of receiving points (ADS-B receivers), the tactical-technical characteristics (TTC) of ADS-B receivers, the coordinates of points of reception x\(_i\), y\(_i\), z\(_i\), the initial approximations of the coordinates of the air object x\(_{T(0)}\), y\(_{T(0)}\), z\(_{T(0)}\) (expression (3));

\[
R_i = \sqrt{(x_i - x_{T(i)})^2 + (y_i - y_{T(i)})^2 + (z_i - z_{T(i)})^2},
\]

where \(x_i, y_i, z_i\) are the coordinates of the \(i\)-th reception point; \(x_{T(0)}, y_{T(0)}, z_{T(0)}\) is the initial approximation of coordinates of the air object.

2. Calculate distances between the \(i\)-th reception point and the air object

3. Calculate the discrepancy vector \(C\) at the \(S\)-th iteration (4):

\[
C_{IS} = R_{IS - 1} - R_{IS - 1} \cdot (T_{i - 1} - T); \quad i = 1, ..., L, \quad (4)
\]

where \(R_{IS - 1}, R_{IS - 1}\) are the distances from the \(i\)-th and 0-th points of reception to the point with coordinates \(x_{T(0)}, y_{T(0)}, z_{T(0)}\), calculated at the \((S-1)\)-th iteration; \((T_{i - 1} - T)\) is the difference between the moment of receiving signals at the \(i\)-th and 0-th point of reception; \(c\) is the speed of light.

4. Calculate the matrix of partial derivatives \(A_S\) taking into consideration the estimates of coordinates \(x_{T(S - 1)}, y_{T(S - 1)}, z_{T(S - 1)}\), which were calculated in the previous step (5):

\[
A_{iso} = \frac{\partial R_i(x_i, y_i, z_i)}{\partial x_s}, \quad A_{iso} = \frac{\partial R_i(x_i, y_i, z_i)}{\partial y_s}, \quad A_{iso} = \frac{\partial R_i(x_i, y_i, z_i)}{\partial z_s}, \quad (5)
\]

5. Calculate the correction \(\zeta_S\) at the \(S\)-th iteration based on the results of calculations at the \((S-1)\)-th iteration (6):

\[
\zeta_S = \left( (A_{3, i})^+ R A_{3, i} \right)^+ (A_{3, i})^+ \text{RC}. \quad (6)
\]

6. Calculate the refined coordinates of an air object (7):

\[
\alpha_S = [x_{T(S)}, y_{T(S)}, z_{T(S)}] - [x_{T(S - 1)}, y_{T(S - 1)}, z_{T(S - 1)}] + \zeta_S, \quad S = 1, 2, ..., K. \quad (7)
\]

7. Compare the maximum value of vector \(\zeta_S\) with the given threshold level \(P\). If the correction is less than the threshold, then one should terminate the cycle of calculation of coordinates, and \(\alpha_S\) is taken for the assessment of coordinates. Otherwise, the iteration counter \(S\) is increased by one, and the transition to the second stage of the method is carried out.
5.3. Estimating the accuracy in determining the air objects’ coordinates by a radar with the additional use of multilateration technology

We shall conduct an experimental study to investigate the method for determining the air objects’ coordinates with the additional use of multilateration technology. Consider the case of using three points of reception (ADS-B receivers). The root mean square error (RMS) in determining the coordinates of an air object by ADS-B receivers is chosen approximate to the GPS coordinate measurement error value \( \sigma_{GPS} = (15–30) \) m [14]. The accuracy in determining the coordinates of an air object is to be estimated by using statistical modeling based on the Monte Carlo method.

It is known [29] that the potential error in determining the azimuth of an air object when using a difference-range method is determined from the following expression (8):

\[
\sigma_\theta = \sigma_{dR} / L \sin(\theta_i),
\]

where \( \sigma \) is the RMS error in determining the azimuth of an air object when using a difference-range method; \( L \) is the distance between the points of reception (base); \( \sigma_{dR} \) is the RMS error of unit measurement of the range of an air object at a receiving point; \( \theta_i \) is the azimuth of an air object relative to the \( i \)-th reception point; \( \sigma_{dR} \) is the RMS error in determining the difference in the range of an air object.

\[
\sigma_{dR} = 2c \sigma_{\tau},
\]

where \( c \) is the speed of light; \( \sigma_\tau \) is the RMS error in determining the delay time of the response signal from the air object at the receiving position.

The results of statistical modeling of the failures in determining the plane coordinates of an air object in the three-position system are shown in Fig. 8. The results are represented in the form of a margin of emergency at the RMS error in determining the range of an air object with a distance between the receiving positions of 50 km, 100 km.

The results of statistical modeling of the angular-range-finding method when determining the plane coordinates of an air object in a single-position autonomous RS are shown in Fig. 9. The RS P-19MA (Ukraine) was used as an RS (the RMS error in the single measuring of the range of an air object \( \sigma_{R}=250 \) m).

The results of statistical modeling (Fig. 8, 9) were obtained using the software package for solving technical calculation tasks Matrix Laboratory (MATLAB), version R2017b.

– the RS P-19 (Ukraine) was considered as a one-position RS;
– the identical FlightAware Piaware (China) receivers (1,090 MHz frequency) were used as elements of the multi-position system of receivers;
– unhindered reception of the signal from an air object by a multi-positioning system of receivers using multilateration technology is provided;
– as signals for a multi-position system of receivers using multilateration technology, response signals from the aircraft object are used at the request from the secondary radar system (1,030 MHz);
– the response signal frequency from aboard an air object was 1,090 MHz.

To assess the quality of additional use of the multilateration system, we shall compare the accuracy of determining the coordinates of an air object with an autonomous RS and an RS with the additional use of multilateration technology. To compare the accuracy of determining the coordinates of an air object by an autonomous RS and an RS with the additional use of multilateration technology, we shall perform statistical
modeling. We shall consider the RS P-19MA as an autonomous RS [30]. We shall assume that the autonomous RS uses an angular-range-finding method for determining the plane coordinates of an air object. The RMS error in determining the range of an air object by an autonomous RS is $\sigma_R = 250$ m.

Table 1 gives errors in determining the coordinates of an air object by an autonomous RS and an RS with the additional use of multilateration technology.

<table>
<thead>
<tr>
<th>Distance to between reception points, km</th>
<th>Autonomous RS P-19MA, $\sigma_{xy}$, m</th>
<th>RS with the additional use of multilateration technology, $\sigma_{xy}$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>154</td>
<td>130</td>
</tr>
<tr>
<td>150</td>
<td>244</td>
<td>130</td>
</tr>
<tr>
<td>200</td>
<td>102</td>
<td></td>
</tr>
</tbody>
</table>

By analyzing Table 1 it was established that the additional use of multilateration technology could ensure a decrease in the error of determining the coordinates of an air object. On average, the accuracy increases by 1.58 to 2.39 times compared to using only an autonomous RS.

The following factors testify to the validity and reliability of our results:
- this work has theoretically justified and practically proven the research methods defined in chapter 4;
- the theoretical calculations, experimental results, and data on the TTC for the RS P-19MA are in good agreement [30];
- the findings do not contradict the basic laws and phenomena of nature; they are clearly physically defined.

6. Discussion of the study results on devising a method for determining the air objects’ coordinates

The experimental study has been performed to confirm disruptions in the operation of ADS-B transponders. The lack of encryption and crypto-resistance in the ADS-B system makes it possible to carry out both active and passive attacks. Our experimental study of disruptions in the operation of ADS-B transponders involved the FlightAware Piaware receiver (Fig. 2). Examples of the disruptions in the operation of ADS-B transponders are shown in Fig. 3, 4. The experimentally detected disruptions in the operation of ADS-B transponders could lead to a decrease in the accuracy of determining the coordinates of air objects with the joint use of RSs and multilateration technology.

The main stages in the method for determining the coordinates of an air object by RS with the additional use of multilateration technology have been defined (Fig. 7). The method for determining the coordinates of an air object by RS using the multilateration technology involves the following (Fig. 7):
- enter the source data;
- calculate distances between the points of reception and an air object;
- calculate a discrepancy vector;
- compute the matrix of partial derivatives, taking into consideration the estimates of the coordinates of an air object at the previous iteration;
- calculate corrections, compute the refined coordinates of an air object.

Thus, we have improved the method for determining the coordinates of an air object by RS, which, unlike known ones, additionally uses multilateration technology.

The accuracy of determining the coordinates of air objects by a radar with the additional use of multilateration technology was estimated. The indicator of accuracy used was an RMS error value (8). It was established that the additional use of multilateration technology could ensure a decrease in the error of determining the coordinates of an air object by, on average, 1.58 to 2.39 times (Table 1).

The following limitations and assumptions are inherent in the present study:
- we have considered two-coordinate survey RSs with mechanical rotation (type P-18 (Ukraine) and their modifications (P-18MA, P-18MU, P-19 Malachite), P-19 (Ukraine), 79K6 (Ukraine), 35D6 (Ukraine), 79K6 (Ukraine));
- the RS receiver is digital;
- the absence of interference;
- nonperturbed reception of the signal from the air object by a multi-positioning system of receivers using multilateration technology is ensured;
- the elements of the multi-position receiver system are identical;
- the FlightAware Piaware receiver is used as an element of the multi-position receiver system;
- as signals for a multi-position system of receivers using multilateration technology, answer signals from aboard an air object are used at the request of the secondary radar system;
- the frequency of the request of the secondary radar system is 1,030 MHz;
- the frequency of response signal from aboard an air object is 1,090 MHz.

Further research should be directed to the study of the accuracy of determining the coordinates of an air object by RS with a different configuration of the receiver system when using multilateration technology.

7. Conclusions

1. An experimental study has been performed to confirm disruptions in the operation of ADS-B transponders. It has been established that the lack of encryption and crypto-resistance in the ADS-B system provides the ability to carry out both active and passive attacks. It was found that the experimentally detected disruptions in the operation of ADS-B transponders could lead to a decrease in the accuracy of determining the coordinates of air objects with the joint use of RSs and multilateration technology.

2. The improved method for determining the coordinates of an air object by RS using the multilateration technology involves the following:
- enter the source data;
- calculate distances between the points of reception and an air object;
- calculate a discrepancy vector;
- compute the matrix of partial derivatives, taking into consideration the estimates of the coordinates of an air object at the previous iteration;
- calculate corrections, compute the refined coordinates of an air object.
Unlike known methods, the improved one additionally uses multilateration technology.

3. The accuracy of determining the coordinates of air objects by a radar with the additional use of multilateration technology was estimated. It was established that the additional use of multilateration technology could ensure a decrease in the error of determining the coordinates of an air object by, on average, 1.58 to 2.39 times compared to using only an autonomous RS.

References
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