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ON ONE METHOD FOR SOLVING DEGENERATING PARABOLIC SYSTEMS BY THE DIRECT LINE METHOD WITH AN APPENDIX IN THE THEORY OF FILRATION

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Abstract:
In this article, we will study the problem arising in the study of the processes of diffusion or filtration of a liquid (gas) in multilayer layers, taking into account convective transfer

Keywords: Degenerate, method of lines, uniformly, estimates, uniform convergence, solution accuracy, linear interpolation, sweep method.

In this article, we will study the problem arising in the study of the processes of diffusion or filtration of a liquid (gas) in multilayer layers, taking into account convective transfer [1,2,3].

In the case of a 3-layer reservoir, the problem is formulated as follows:
Define in

$$\overline{D}=\{0\le x\le 1,\ 0\le z\le 1,\ 0\le t\le T\}\ u\ D_1=\{0\le x\le 1,\ 0\le t\le T\}$$

Correspondingly, continuous functions $u(x,t),\ u_1(x,z,t),\ u_2(x,z,t)$ satisfying the system of equations:

$$\begin{cases}\dfrac{1}{m(x)}\dfrac{\partial}{\partial x}(k(x)\dfrac{\partial u}{\partial x})=A(x,t)\dfrac{\partial u}{\partial t}+B(x,t)K(x)\dfrac{\partial u}{\partial x}+\sum\limits_{i=1}^{2}A_i(x)K_i(z)\left.\dfrac{\partial u_i}{\partial z}\right|_{z=1}+f(x,t,u)+\int\limits_0^t R(t,s)u(x,s)ds\\ \dfrac{1}{m_i(z)}\dfrac{\partial}{\partial z}(K_i(z)\dfrac{\partial u_i}{\partial z})=A_i(z,t)\dfrac{\partial u_i}{\partial z}+f(x,z,t,u_i)\end{cases}\quad(1)$$

initial conditions $u(x,0)=\varphi(x),\ u_i(x,z,0)=\varphi_i(x,z)$ (2) and boundary conditions

$$\begin{cases}\left.(K(x)\dfrac{\partial u}{\partial x}-a_1(t)u)\right|_{x=0}=0\\ (K(x)\dfrac{\partial u}{\partial x}+a_2(t)u)|_{x=1}=0\\ (K_2(z)\dfrac{\partial u_2}{\partial z}-a_{12}(t)u_i)|_{z=0}=0,\\ u(x,t)=u_i(x,1,t),\qquad i=1,2\end{cases}\qquad \text{если}\ \int\limits_0^1\dfrac{dz}{K_i(z)}<+\infty\qquad(3)$$

Here
$F(x,t,u),\ F_i(x,z,t,u_i),\varphi(x),\varphi_i(x,z),K(x),m(x),K_i(x),A(x,t),A_i(z,t),B(x,t),A_2(x),a_k(t),\ (k=1,2),R(t,s)\ a_{12}(t)$
given functions, and
$K(x),m(x)>0,\ A(x,t)\ge A>0,\ A_i(z,t)\ge A_{i0}>0,\ A_i(x)>0,K_i(0)=0,\ K_i(z)\ u\ m_i(z)$ - positive for $z\to 0$.
We will assume that the solutions themselves and all known functions in the equations are smooth

If a $\int_0^1 \frac{dz}{K_i(z)} = +\infty, \quad \int_0^1 \frac{\int_0^z m_i(\xi)d\xi}{K_2(z)} < +\infty$, then the condition $(K_i(z)\frac{\partial u_1}{\partial z} - a_{12}(t)u_i)\big|_{z=0} = 0$ is replaced by the condition $\left|u_i(x,z,t)\big|_{z=0}\right| < +\infty, \ i=1,2$.

The peculiarity of these problems is that the desired function enters the equations of the problem in such a way that each of the equations has a "Main" unknown function, while the rest are either not contained or are represented by their own boundary conditions.

Introducing division $t_j = j\tau, \ \ j=0,1; \ N = \left[\frac{T}{\tau}\right]$ we will look for an approximate solution $\{u_j(x) \ u \ u_{ij}(x,z)\}$ using Rote schemes

$$\begin{cases} Lu_j = A(x,t_j)\delta_{\bar{\tau}}u_j + B(x,t_j)K(x)\dfrac{du_t}{dx} + \sum\limits_{i=1}^{2} A_i(x)K_i(z)\dfrac{\partial u_{ij}}{\partial z}\bigg|_{z=1} + f(x,t_{j,}u_{j-1}) + \tau\sum\limits_{i=0}^{j-1} R_{j,i}u_i \\ L_z u_{ij} = A_i(z,t_j)\delta_{\bar{\tau}}u_{ij} + f_i(x,z,t_j,u_{ij-1}) \end{cases} \quad (3)$$

$$u_0(x) = \varphi(x), \ \ u_{i0}(x,z) = \varphi_i(x,z), \ i=1,2$$

$$\left|\bar{K}(x)\frac{dv_j}{dx} - a_1(t_j)u_j)\right|_{x=0} = 0, \ \ (K(x)\frac{du_j}{dx} + a_2(t_j)u_j)\big|_{x=1} = 0$$

$$(K_i(z)\frac{\partial u_{ij}}{\partial z} - a_{12}(t_j)u_{ij})\bigg|_{z=0} = 0, \ \ u_{ij}(x,z) = u_j(x) \quad (4)$$

where $\delta_{\bar{t}}U_j = \dfrac{U_j - \mathrm{U}_{j-1}}{\tau}, \ \ Lu_j = \dfrac{d}{m(x)}\dfrac{d}{dx}(K(x)\dfrac{dU_j}{dx}), \ \ l_z u_{ij} = \dfrac{1}{m_i(z)}\dfrac{\partial}{\partial z}(k(z)\dfrac{\partial u_{ij}}{\partial z})$

task for j everyone (3) (4)- linearly relative $\{u_j(x), \ u_{ij}(x,z)\}$ and has the only solution[1,2,3]. Introducing the norm $\|u\|_j \le \max\limits_{1\le k\le i}|u_k| \ \ u \ \|\cdot\| = \max|\cdot|$ according to the maximum principle, the evaluation [2]

$$\{|u_j|,|u_{ij}|\} \le \max\left\{\left\|\frac{\dfrac{A(x,t_j)}{\tau}u_{j-1} + f(x,t_j,u_{j-1}) + \tau\sum\limits_{i=1}^{j-1}R_{ji}u_i}{\dfrac{A(x,t_j)}{\tau}}\right\|_c, \max\left\|\frac{-\dfrac{A(z,t_j)}{\tau}u_{ij-1} + f(x,z,t_j,u_{j-1})}{\dfrac{A_i(x,t_j)}{\tau}}\right\|\right\} \text{ from here}$$

$$\{\|u_j\|;\|u_{ij}\|\} \le (1+c_2T\tau)\max\{\|u\|_j;\|u_i\|_j\} + c_1\tau.$$

By induction

$$\{\|u_j\|;\|u_{ij}\|\} \le \max\{\|\varphi(x)\|;\|\varphi_i(x,z)\|\}e^{c_2T^2} + \frac{c_1}{Tc_2}(e^{c_2T^2}-1)$$

$c_1, c_2 -$ some constants depend on input functions.

Functions $\Phi_j = \delta_{\bar{\tau}}u_j, \ \Phi_{ij}(x,t) = \delta_{\bar{\tau}}u_{ij}$ satisfy the system of equations of type a (3), (4).

For solutions $\Phi_j(x) \ u \ \Phi_{ij}(x,t)$ by induction we uniformly estimate

$$\{\|\Phi_j\|,\|\Phi_{ij}\|\} \le c_3\exp(c_4T) + c_3\frac{\exp(c_4T)-1}{c_4} = M_1$$

In the future, through M_k we will denote constants depending on the input data of the problems. Let us establish the uniform boundedness of the families of quantities.

$$\{\|u_j\|,\|u_{ij}\|\}, \{\|\Phi_j\|, \|\Phi_{ij}\|\}, \{\|L\Phi_j\|;\|L_z\Phi_{ij}\|\}, \{\|Lu_j\|,\|L_zu_{ij}\|\}, \{\|\delta_{\bar{\tau}}\delta_{\bar{\tau}}u_{j-12}\|;\|\delta_{\bar{\tau}}\delta_{\bar{\tau}}u_{i,j-1}\|\}$$

From the established estimates, we have

$$\left|\frac{du_j}{dx}\right| \le M_4\psi(x), \quad \left|\frac{d\Phi}{dx}\right| \le M_5\psi(x),$$

$$\begin{cases} \left|\dfrac{du_{ij}}{dz}\right| \le M_6\psi_i(z), \\ \left|\dfrac{d\Phi_{ij}}{dz}\right| \le M_7\psi_i(z) \end{cases}, \qquad z>0 \qquad (6)$$

Where $\psi(x)=\dfrac{\int_0^x m(\xi)d\xi}{K(x)}$ - uniformly bounded function. And for the function $\psi_i(x)=\dfrac{\int_0^z m_i(s)ds}{K_i(z)}$ consider the following cases:

Case 1. Limit $\overline{\lim_{z\to 0}}\psi(z)$ finite. Then the right-hand side of (6) is bounded by a constant that does not depend on the partitioning method. In the limit $\tau\to 0$ linear interpolations $u^\tau(x,t)$ u $u_i^\tau(x,z,t)$ accordingly in D and D_1 coinciding with $u(x, j\tau)$ u $u_i(x,z, j\tau)$ at $t=j\tau$ and linearly depending on t inside the layers $j\tau\le t\le (j+1)\tau$ give a solution $u(x,t)$ u $u_i(x,z,t)$, $i=1,2$ in area D u D_1 respectively

Case 2. If $\lim_{z\to+\infty}\psi_i$ $t($ is infinite or does not exist, then we cannot use estimate (6) to prove the equicontinuity of families. However, using the boundary conditions at z=0, one can be convinced of the equicontinuity of the families in this case as well.

Note 1

For the numerical solution, a modified version of the differential sweep is used [2] and is implemented by the Maple software system

Note 2

The approximate solution constructed by the method of lines converges to the exact one with the speed $o(\tau)$, $\tau-$ time step

LITERATURE

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