Backstepping Nonlinear Control by using Sliding Mode Observer for controlling Blood Sugar

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Abstract. Accurate control and stability of vital variables in patients whose natural control system has been undermined for some reasons are essential. One of these fundamental variables is the blood glucose level of a diabetic person. However similar to most artificial control systems associated with the human body, many complexities and uncertainties force us to use advanced control methods. In this paper, by choosing a successfully applied model, nonlinear controller and observer are designed to control blood glucose level. This mathematical model, which is called Bergman minimal model, approximates the dynamic reaction of a diabetic patient's blood glucose concentration to the insulin injection. These equations are non-normal nonlinear with some uncertainties. The backstepping method is applied to control the system duo to its non-normal equations. In addition, to estimate state variables and uncertainty, a nonlinear observer-based sliding mode theory has been designed. The simulation results prove the proper performance of this method.

Keywords: Glucose Control, Nonlinear Control, Backstepping Theory, Nonlinear Observer, Sliding Mode

1-Introduction

IntroductionDiabetes mellitus is a well-known and very common disease. It is one of the metabolic disorder consisting of high blood glucose levels.[1] In recent years, applying new control methods in various processes has become widespread. Among them, novel control techniques in medical equipment have been used to improve treatment methods. For example, they can be used to control disease such as diabetes. Diabetes is a complex, chronic disease characterized by the level of blood sugar or glucose. High blood glucose in a healthy body is controlled by insulin. Insulin is a hormone produced by the pancreas, controls the glucose level by regulating the production and storage of glucose [2]. Partial or total defects in the insulin secretion system, in glucose regulation, or both lead to diabetes. In fact, in diabetes, the effect of insulin on glucose uptake is reduced or totally eliminated.

Due to lack of Insulin, the blood glucose of the patient remains dangerously high for long periods of time. Untreated hyperglycemia can result in long-term diseases such as cardiovascular diseases, damaging nerves, damaging kidneys, damaging the retina leading to blindness, bone and joints diseases, skin diseases, etc. Similarly, very low blood glucose levels can also result in many diseases like a failure of consciousness i.e. coma which can be fatal[3].In a type of diabetes, called type I, insulin secretion stops. In this case, glucose remains high in the blood so that cells can't absorb it. Therefore, the cells are exposed to glucose

deficiency. And the solution is to inject insulin into the bloodstream. Surely this injection should be completely controlled, otherwise, too much insulin in the blood can cause death. In order to approach the condition of a diabetic person to normal, it is necessary to measure blood glucose regularly and inject insulin as needed. Therefore, the following items are required:

- 1. Glucose sensor
- 2. Insulin pump
- 3. Insulin control unit

The glucose sensor can measure the electrical current proportional to glucose. Due to frequentglucose changes in diabetics, it is necessary to use a closed-loop control system to achieve the right amount of it. Many efforts have been made to model diabetes mathematically [4-9]. In some references, a linear model has been obtained, but in general, the model of the human body system is completely nonlinear and has some uncertainties. Uncertainty in modeling can have adverse effects on nonlinear systems. Therefore, in any practical design, it must be explicitly considered.

To cope with this problem in diabetes mellitus patients, different controllers have been proposed by researchers. Controllers based on linear models include proportional-integral-derivatives,PID [10], pole placement technique [11], model predictive control strategy [12] and H robust control scheme [13]-[14].In addition [15], the prediction of blood glucose fluctuations in diabetic patients has been done by neural networks. For this purpose, real data sampled from three patients with type I has been used and a model extracted to predict glucose fluctuations. In [16-18], blood sugar control in the body was performed using model-based controllers. In [19], using neural network and fuzzy algorithm, an intelligent insulin injection system has been designed in diabetic patients so that neural networks were used to identify the nonlinear model, and fuzzy control was applied to control blood sugar. In [20], in addition to identifying and estimating the model, adaptive control was used to control glucose. Finally, in the [21], a fuzzy proportional-integral controller was utilized.

In most mentioned references, the model of the human body is obtained and then linearized. Therefore, the designed controller is valid around the linearization point and may not work well in other areas. It should be mentioned that the uncertainties are not considered in most models. Due to the complexities of the human body and the possibility of changing insulin sensitivity during the day and also the effect of various disturbances on glucose such as food intake, exercise, stress, disease, etc., it would be better to apply the control algorithms that can resist to these disturbances and adapt to the current disturbances instantly.

There are two basic and complementary methods challenging uncertainty of model, robust control and adaptive control. A robust control method and the main topic of this paper, which is a powerful approach to control nonlinear and uncertain systems, is sliding mode control. In [22,23], a nonlinear model with uncertainty is considered inwhich, high-order sliding mode control has been used to eliminate chattering, to ensures asymptotic stability. But to improve the performance of the controller, it is better to use limited time control methods. Another problem can be the use of sliding mode control as standard. In other words, normalization of dynamic equations has been done. However, due to the existence of nonadoptive uncertainty in the model, in case of normalization, uncertain derivatives will appear which cannot be measured. In result, the designed controller is not reliable or should be used by approximating and estimating the upper bound on uncertainty derivatives. In addition, controlling blood sugar does not require very high precision and in the range of 70-90 is sufficient. So the reduction in accuracy due to the continuous approximation method is acceptable.

In this paper, we use the back stepping method to control blood sugar in diabetics. Therefore there is no need to normalize the equations, which leads to ease of design. For this purpose, the body of a diabetic human has been modeled. Insulin has been used as input and glucose has been considered as output of this model. A nonlinear observer has been also used to estimate state variables and uncertainty.

The paper is organized as the following: In section 2 we demonstrate a concise overview of mathematical model named Bergman body model, whereas the control design procedure is illustrated in section3. Then a sliding mode observer is designed in section 4. To proof the advantages of introduced method, numerical simulation is shown in section 5. Finally, the conclusions are represented in section 6.

2-Mathematical Model

As mentioned in the previous section, several modeling's are presented to model diabetes. Dr. Richard Bergman is one of the pioneers that have made many efforts to model diabetes and developed the so-called 'Bergman's three-state minimal patient model' (BEM). There are many advantages of this model including a minimum number of parameters and describing the interaction between main components such as insulin and glucose concentrations without biological complexity [24-28]. Therefore in this paper, the Bergman nonlinear model is considered.

$$G(t) = -p_1[G(t) - G_b] - X(t)G(t) + D(t)$$

$$\dot{X}(t) = -p_2X(t) + p_3[I(t) - I_b]$$

$$\dot{I}(t) = -n[I(t) - I_b] + \gamma[G(t) - H]^+ t + u(t)$$
(1)

Where t is the time glucose enters the blood, G(t) is the loud glucose concentration in mg / dL, X(t) is the effect of active insulin in 1/ min, I (t) is the plasma insulin concentration in μ U/ml, G_b is the basal pre-injection level of glucose (mg/dl), I_b is the basal pre-injection level of insulin (μ U/ml), p_1 is the insulin independent rate constant of glucose uptake in muscles and liver (1/min), p_2 is the rate for the decrease in tissue glucose uptake ability (1/ min), p_3 is the insulin-dependent increase in glucose uptake ability in tissue per unit of insulin concentration above the basal level ((μ U/ml)/min^(-2)), n is the first-order decay rate for insulin in the blood (1/min), h is the threshold value of glucose above which the pancreatic β cells release insulin (mg/dl), γ is the rate of the pancreatic β cells release of insulin rate in (mU/min). Also, D(t) is a disturbance in blood sugar is caused by the patient eating food or other factors.

As shown in Eq.(1), this system is non-normal nonlinear with three state variables. In addition, uncertainty is non-adaptive related to the input.

The control system output is blood glucose Concentration (G(t)) and controls input is the injected insulin rate. To normalize and make the input appear in the new equations, the output should be derived [15]. Therefore, the derivatives of uncertainties will also appear in the new equations. To avoid it, back stepping theory is used to design the controller in the next section.

3-Back stepping nonlinear controller design

backstepping technique designs the controller by breaking the complex nonlinear systems into smaller subsystems and designing the Lyapunov control functions and virtual controls for these subsystems. An important feature of this method is to prevent eliminating useful nonlinearities.

As we mentioned previously, due to the nonlinearities and abnormality of the system model presented in Eq.(1), the backstepping theory will be used. The design will be done by considering three separate dynamic parts for the system.

It should be noted that using the observer to estimate noise D (t), it is assumed that the noise is known. The first dynamic part considered as:

$$\dot{G}(t) = -p_1[G(t) - G_b] - X(t)G(t) + D(t)$$
(2)

where X(t) is considered as a pseudo control input which should control G(t) for tracking desired G_b in presence of uncertainties D(t).

Thus according to Sliding Mode Control (SMC) theory, the Lyapunov function introduced as:

$$V_{1}(t) = \frac{\left(G(t) - G_{b}\right)^{2}}{2}$$
(3)

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To investigate the system stability and find a suitable controller, it is necessary to peruse the derivative of the Lyapunov function. To stability, the derivative of Lyapunov function must be negative. Using the design algorithm and deriving from Eq.(3), $X_d(t)$ will be obtained:

$$\dot{V}_{1} = (G(t) - G_{b})\dot{G}(t) =$$

$$G(t)(-p_{1}[G(t) - G_{b}] - X_{d}(t)G(t) + D(t)) < 0 \Longrightarrow$$

$$X_{d}(t) = \frac{1}{G(t)} \left[-p_{1}[G(t) - G_{b}] + D(t) + (G(t) - G_{b}) \right]$$
(4)

The next step will start by considering the second Lyapunov function in form (5):

$$V_{2}(t) = (X(t) - X_{d}(t))^{2}$$
(5)

Where $X_d(t)$ is considered as a constant. By continuing and deriving from relation (5) we have:

$$\dot{V}_{2} = (X(t) - X_{d}(t))(\dot{X}(t) - \dot{X}_{d}(t)) < 0$$
(6)

Therefore, the following Equation will be obtained:

$$I_{d}(t) = \frac{1}{p_{3}} \left(p_{2}X(t) + p_{3}I_{b} + \dot{X}_{d}(t) - \left(X(t) - X_{d}(t)\right) \right)$$
(7)

In the last step, by considering the third relation of Eq.(1) and Lyapunov function as follows, the control input will be obtained:

$$V_{3} = \frac{\left(I(t) - I_{d}(t)\right)^{2}}{2} < 0$$
(8)

$$\dot{V}_{3} = (I(t) - I_{d}(t))(\dot{I}(t) - \dot{I}_{d}(t)) < 0$$
⁽⁹⁾

$$u(t) = n[I(t) - I_b] - \gamma[G(t) - H]^+ t + \dot{I}_d(t) - (I(t) - I_d(t))$$
(10)

4-Sliding mode nonlinear observer design

Designing the controller depends on availability of system state variables, but in all cases the variables are not measurable. Also, the disturbances in real systems are inevitable. One way to eliminate these disturbances and estimate the unmeasurable state variables is applying an observer.

An observer is a system that its state variables are an estimate of the main system state variables. So that its estimation error, that is the difference between the actual state of the system and its observer, is zero.

As it was mentioned, since there are some uncertainties in the system, using a sliding mode nonlinear observer is a suitable method, which will be discussed in the following.

Consider the three variables glucose Bergman's model as described by Eq.(1). In this system, the design of the observer is desired by, assuming that the second derivative D(t) equals zero.

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The following structure is considered as sliding mode nonlinear observer:

$$\begin{aligned} \dot{\hat{G}}(t) &= -p_1[\hat{G}(t) - G_b] - \hat{X}(t)\hat{G}(t) + \hat{D}(t) + k_1 S \operatorname{gn}(y - \hat{y}) \\ \dot{\hat{X}}(t) &= -p_2 \hat{X}(t) + p_3[\hat{I}(t) - I_b] + k_2 S \operatorname{gn}(y - \hat{y}) \\ \dot{\hat{I}}(t) &= -n[\hat{I}(t) - I_b] + \gamma [\hat{G}(t) - H]^+ t + u(t) + k_3 S \operatorname{gn}(y - \hat{y}) \\ \dot{\hat{D}}(t) &= k_4 S \operatorname{gn}(y - \hat{y}) \\ \dot{\hat{y}}(t) &= \hat{G}(t) \end{aligned}$$

$$(11)$$

Error dynamics are written as follows:

$$\begin{cases} e_1 = y - \hat{y} = G - \hat{G}_1 \\ e_2 = X - \hat{X} \\ e_3 = I - \hat{I} \end{cases}, \begin{cases} \dot{e}_1 = \Delta f_1 - k_1 S \operatorname{gn}(e_1) \\ \dot{e}_2 = \Delta f_2 - k_2 S \operatorname{gn}(e_2) \\ \dot{e}_3 = \Delta f_3 - k_3 S \operatorname{gn}(e_3) \end{cases}$$
(12)

Where $\Delta f_i = \hat{f}_i - f_i$.

Now, to prove the observer's stability, we begin with the first relation of Eq.(12). The Lyapanov function is selected for this as follows:

$$V = \frac{1}{2}e_1^2$$
(13)

The derivative of this Lyapunov function will be as follows:

$$\dot{V} = e_1 \dot{e}_1 = e_1 \left(\Delta f_1 - k_1 Sgn(e_1) \right)$$
(14)

To ensure the limited time convergence, the sliding condition must be maintained:

$$\dot{V} \le -\eta \left| e_1 \right| \tag{15}$$

(16)

So:

$$\dot{V} = e_1 \left(\Delta f_1 - k_1 Sgn(e_1) \right) \le -\eta \left| e_1 \right|$$

$$\dot{V} = \frac{e_1}{\left| e_1 \right|} \left(\Delta f_1 - k_1 Sgn(e_1) \right) + \eta \le 0$$

To establish this condition, $\mathbf{k_1}$ must be selected as follows:

$$k_1 = \alpha_{e_2} + \eta \tag{17}$$

Where α_{e_2} is maximum error value of variable Δf_1 second state. by selecting this value for k_1 , Converging e_1 to zero in limited time is assured. Then the linear system matrix is obtained as follows:



Technium BioChemMed Vol. 2, Issue 1 pp.119-130 (2021) ISSN: 2734-7990 www.techniumscience.com

$$A = \begin{bmatrix} \frac{-k_2}{k_1} & 1 & 0\\ \frac{-k_3}{k_1} & 0 & 1\\ \frac{-k_4}{k_1} & 0 & 0 \end{bmatrix}$$
(18)

To stabe estimated variables ranging from second to nth the real part of the eigenvalues of the above matrix must be negative. In addition, the following condition must be maintained.

 $k_n \ge |\Delta f|$

Where $\Delta \mathbf{f}$ is the maximum error of estimation of function f.

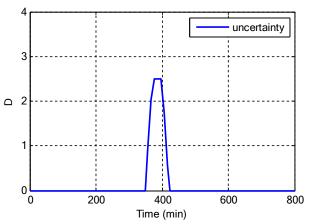
5-Numerical simulation

In this section, the performance of the proposed algorithm based on sliding mode nonlinear observer is evaluated by simulation. First, the open-loop response of healthy human and sick body models without using of a controller is examined.

The parameters considered for both patient and healthy person are given in Table 1.

Table 1.		
Parameter	Healthy	Patient
p_1	0.03	170
p_2	0.0123	0.0123
<i>p</i> ₃	8-10×8.2	8-10×8.2
γ	5-10×6.5	0
п	0.2659	0.2659
h	79.0353	0
G_b	70	70
I _b	7	7
G_0	140	200
Ι ₀	20	20

Uncertainty, which is caused by meal, is entered into the system as shown in Fig. (1).



The uncertainty effect on glucose levels in the openloop response of a healthy person and a patient is shown in Fig.(2). As seen in this figure, the body of a healthy human is in a stable state and the blood sugar follows the desired value of 70.

In addition, the system behaved well in response to the disturbance at t=350min and could compensate the effect of disturbance Whichis caused by eating or drinking that leads to an increase in glucose level up to 140 mg/dl. As shown in this figure, the response of the sick human body to a disturbance equivalent to 40% of the disturbance inflicted on the healthy body is undesired and leads to an increase in glucose up to dangerous level of 250 mg/dl.

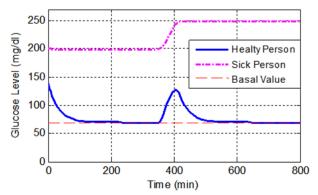


Fig. 2: Open-loop response of glucose in healthy and patient body

Now, the performance of observer in estimating the state variables and uncertainty is examined. Fig.(3) shows the changing curve of the first variable G(t) comparing with its estimated value. Grandiosity and error diagram are also illustrated in this figure where X_1 represents blood glucose concentration or the first state variable of system and Xh_1 represents its estimated value. As shown in diagram, the observer is able to estimate the state variable in less than 1 second. It should be noted that the first state is determined so that it can estimate others along with control input.

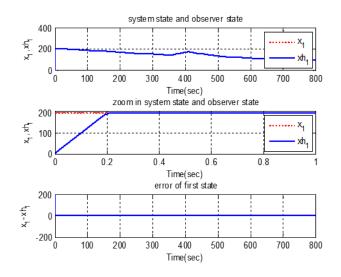


Fig. 3: Blood glucose concentration, grandiosity and error.

Fig.(4) shows the changing curve of the second state variable or X (t) compared to its estimated value, magnification and error. In the diagrams, the effect of active insulin and its estimated amount is marked by X_2 and Xh_2 , respectively. As shown in this figure, the observer has been able to estimate the state variable in a short time. It is also clear that the error value is less than 0.0001 which is appropriate.

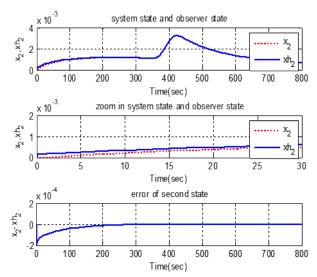


Fig. 4: Effect of active insulin, grandiosity and error

The simulation is repeated for the third state variable of the system or plasma insulin concentration. Fig. (5) indicates the proper performance of the observer in estimating I(t).

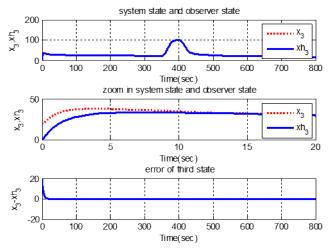


Fig. 5: Plasma insulin concentration, grandiosity and error

The performance of the designed sliding mode observer in estimating disturbance is shown in Fig.(6). In this figure, D represents disturbance and Dh shows the estimated parameter. As can be seen, the estimate was made at the right time and error was reduced to zero.

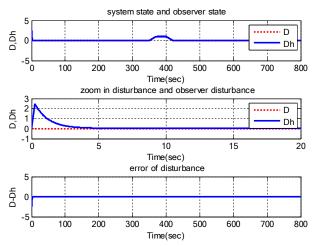


Fig. 6: Estimated distribution, grandiosity and error

As shown in the figures, all estimates are made at the right time so that the error has tended to zero. Therefore, during the next step, estimated amounts can be used instead of the second and third state variables as well as the error to designe controller.

In the following, the performance of glucose controller is examined. Fig. (7) and (8) show the changes in the control input (injected insulin rate) and the variable G(t), which is the output of the system. As it can be seen, the general trend of changes is in the direction of lowering glucose so that the disturbance in t=340s is well controlled and in t=700s, the blood glucose level drops below 100.

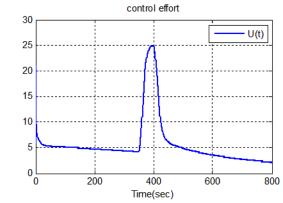
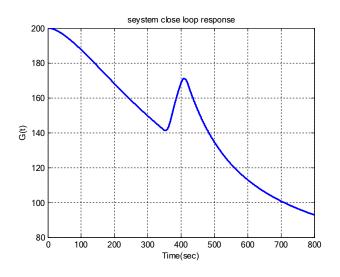
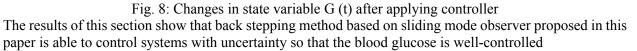


Fig. 7: u(t) and signal generated by the third virtual control input





6-Conclusion

In the present work, the backstepping method has been innovatively applied to control glucose in diabetics by sliding mode. For this purpose, the controller was designed using back stepping method in three steps. To estimate the state variables and uncertainty a nonlinear sliding mode observer was used. The simulation results show the proper performance of the observer in estimating the state variables and uncertainty and also the great performance of the controller.

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