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Investigation on Self-Similar Analysis of the Problem Biological Population Kolmogorov-Fisher Type System

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Abstract: In this work we considered a parabolic system of two quasilinear reaction-diffusion equations for a biological population problem of the Kolmogorov-Fisher type describes the process of a biological population in a nonlinear two-component medium. We studied the qualitative properties of the solution to Cauchy problem based on self-similar analysis and its numerical solutions using the methods of modern computer technologies, to study the methods of linearization to the convergence of the iterative process with further visualization.

Keywords: Cauchy problem, quasilinear, reaction-diffusion, biological population, numerical solutions.

We can consider a parabolic system of two quasilinear reaction-diffusion equations for a biological population problem of the Kolmogorov-Fisher type in the following domain

$Q=\{(t,x)\colon 0< t < \infty, x\in R^2\}$

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_1^{\sigma_1}\dfrac{\partial u_1}{\partial x}\right) + k_1(t)u_1\cdot\left(1-u_2^{\beta_1}\right) \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_2^{\sigma_2}\dfrac{\partial u_2}{\partial x}\right) - k_2(t)u_2\cdot\left(1-u_1^{\beta_2}\right) \end{cases} \quad (1)$$

$$u_1\big|_{t=0} = u_{10}(x), \quad u_2\big|_{t=0} = u_{20}(x), \quad (2)$$

It describes the process of a biological population in a nonlinear two-component medium, the diffusion coefficient of which is equal to $D_1 u_1^{\sigma_1}$ and $D_2 u_2^{\sigma_2}$, $\sigma_1,\sigma_2,\beta_1,\beta_2$ - positive real numbers, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - sought solutions.

The Cauchy problem and boundary value problems for system (1) in the one-dimensional and multidimensional cases have been studied by many authors.[1]

The purpose of this work is to study the qualitative properties of the solution to problem (1), (2) based on self-similar analysis and its numerical solutions using the methods of modern computer technologies. Also the article has a purpose to study the methods of linearization to the convergence of the iterative process with further visualization. The main estimations of the solutions and the resulting free boundary have been found, which makes it possible to choose appropriate initial approximations [...] Each of them has their own counting systems.

Now we will start constructing a self-similar system of equations for (1) - (2). It is a simpler system of equations for research.

We construct a self-similar system of equations by the method of nonlinear splitting.

Instead of (1)

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$u_1(t,x)=e^{k_1 t}v_1(t,x),$
$u_2(t,x)=e^{k_2 t}v_2(t,x)$

This will lead (1) to the form:

$$\begin{cases}\dfrac{\partial v_1}{\partial \tau}=\dfrac{\partial}{\partial x}\left(D_1 v_1^{\sigma_1}\dfrac{\partial v_1}{\partial x}\right)+k_1 e^{((\beta_1+1)k_1-(\delta_1+1)k_2)t}v_1 v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial \tau}=\dfrac{\partial}{\partial x}\left(D_2 v_2^{\sigma_2}\dfrac{\partial v_2}{\partial x}\right)+k_2(t)e^{((\beta_2+1)k_2-(\delta_2+1)k_1)t}v_1^{\beta_2}v_2,\end{cases}\qquad(3)$$

$v_1|_{t=0}=v_{10}(x)$, $v_2|_{t=0}=v_{20}(x)$,

Where $a_1=(\delta_1 k_1)^{b_1}$, $a_2=(\delta_2 k_2)^{b_2}$

$$b_1=[(\beta_1+1)k_1-(\sigma_1+1)]k_2/\sigma_1 k_1,\quad b_2=(\beta_2+1)k_2-(\sigma_2+1)k_1/\sigma_2 k_2$$

After choosing $\sigma_1 k_1=\sigma_2 k_2$, we can achieve the following forms of equation systems.

$$\begin{cases}\dfrac{\partial v_1}{\partial \tau}=\dfrac{\partial}{\partial x}\left(D_1 v_1^{\sigma_1}\dfrac{\partial v_1}{\partial x}\right)-a_1\tau^{b_1}v_1 v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial \tau}=\dfrac{\partial}{\partial x}\left(D_2 v_2^{\sigma_2}\dfrac{\partial v_2}{\partial x}\right)-a_2\tau^{b_2}v_1^{\beta_2}v_2,\end{cases}\qquad(4)$$

In the following, we can write one of the ways of auto-model systems for equation systems (4). It is done like in the following:

$$\begin{cases}\dfrac{d\bar{v}_1}{d\tau}=-a_1\tau^{b_1}v_1 v_2^{\beta_1},\\ \dfrac{d\bar{v}_2}{d\tau}=-a_2\tau^{b_2}v_1^{\beta_2}v_2,\end{cases}$$

It has a solution in the following:

$\bar{v}_1(t)=c_1(\tau+T)^{\gamma_1}$, $\bar{v}_2(t)=c_2(\tau+T)^{\gamma_2}$, $T>0$,

And then systems of solution are sought in the following steps. (3)-(4)

$v_1(t,x)=\bar{v}_1(t)w_1(\tau,x)$,

$v_2(t,x)=\bar{v}_2(t)w_2(\tau,x)$,

here $\tau=\tau(t)$ is chosen in the following way:

$$\tau(t)=\int\bar{v}_1^{-\sigma_1}(t)dt=\frac{1}{\gamma_1\sigma_1+1}(T+t)^{\sigma_1\gamma_1+1},\qquad \gamma_1\sigma_1+\ \neq$$

$$u\,\tau(t)=\ln(T+t),\qquad \gamma_1\sigma_1+\ =$$

Here we can choose equation system for $w_i(\tau,x),\ i=1,2$

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$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial x}(D_1 w_1^{\sigma_1} \dfrac{\partial w_1}{\partial x}) - \theta_1 (w_1 w_2^{\beta_1} - w_1) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial x}(D_2 w_2^{\sigma_2} \dfrac{\partial w_2}{\partial x}) + \theta_2 (w_2 w_1^{\beta_1} - w_2) \end{cases},$$

$$\eta_1 = b_1 + 1 + \beta_1 (b_2 + 1) \neq 0$$

$$\eta_2 = -\beta_2 (b_1 + 1) + (b_2 + 1) \neq 0$$

It is as $\gamma_1 \sigma_1 > 0$, $\gamma_1 \sigma_1 = \gamma_2 \sigma_2$, $d_i > 0$. In this case we can rely on e $w_i(\tau(t), x) = y_i(\xi), \quad \xi = |x| / \tau_1^{1/2}$, $i = 1, 2$,

We can choose the following equation system considering the fact that equation for $w_i(\tau, x)$ without little members is always auto-model.

$$\begin{cases} \xi^{1-N} \dfrac{d}{d\xi}(\xi^{N-1} y_1^{\sigma_1} \dfrac{dy_1}{d\xi}) + \dfrac{\xi}{2\theta_1} \dfrac{dy_1}{d\xi} - \mu_1 (y_1 - y_1 \, y_2^{\beta_1}) = 0 \\ \xi^{1-N} \dfrac{d}{d\xi}(\xi^{N-1} y_2^{\sigma_2} \dfrac{dy_2}{d\xi}) + \dfrac{\xi}{2\theta_2} \dfrac{dy_2}{d\xi} + \mu_2 (y_2 - y_2 \, y_1^{\beta_2}) = 0 \end{cases} \quad (6.105)$$

Here $\mu_i = \dfrac{1}{\theta_i \sigma_i} \quad \theta_i = \begin{cases} 1, & i = \\ \gamma_1^{-\sigma_1} \gamma_2^{\sigma_2}, & i = \end{cases}$

The study of the qualitative properties of the system (1) - (2) made it possible to perform a numerical experiment depending on the values included in the system of numerical parameters. For this purpose, the constructed asymptotic solutions were used as an initial approximation. In the numerical solution of the problem for the linearization of system (1) - (2), linearizations by the Newton and Picard methods were used. The method of nonlinear splitting is proposed to solve the problem of a biological population.

$\omega_h = \{x_i = ih, \; h > 0, \; i = 0, 1, \ldots, n, \; hn = l\}$,

Temporary grid $\omega_{h_1} = \{t_j = jh_1, \; h_1 > 0, \; j = 0, 1, \ldots, n, \; \tau m = T\}$.

The main problem in nonlinear problems is the appropriate choice of the initial approximation and the way to linearize equation (3).

We replace problem (3) - (4) with an implicit difference scheme and obtain a difference problem with an error $O(h^2 + h_1)$.

$$\psi_1(t) = \bar{v}_1(t), \quad v_{10}(t, x) = \psi_1(t) \cdot \left(a - \frac{\sigma_1}{4} \xi^2 \right)_+^{1/\sigma_1},$$

$$\psi_2(t) = \bar{v}_2(t), \quad v_{20}(t,x) = \psi_2(t) \cdot \left(a - \frac{\sigma_2}{4}\xi^2\right)_+^{1/\sigma_2},$$

$$\xi = \frac{x}{[\tau(t)]^{1/2}}, \quad \tau(t) = \int_0^t [\psi(y)]^{\sigma} dy.$$

$(a)_+$ means $(a)_+ = \max(0, a)$.

As a conclusion, he results of numerical experiments have shown the effectiveness of the proposed approach. Asymptotes of various solutions of the system of type (1) - (2) made it possible to simulate the processes of mutual reaction-diffusion in the form of visualization with animation.

All in all, we can emphasize the importance of a joint study of migration and demographic processes. To analyze the population dynamics of interacting populations, it is important to jointly study the processes of fertility, mortality, trophic interactions, and various migrations. The introduction of nonlinearity into migration flows is the first step towards an adequate description of spatio-temporal population dynamics.

REFERENCES

1. A.V.Martynenko, A.F.Tedeev, The Cauchy problem for a quasilinear parabolic equation with a source and no homogeneous density, Comput. Math. Math. Phys. 47, 2007, no. 2, pp. 238–248.
2. M.Aripov, Sh.Sadullaeva, To properties of the equation of reaction diffusion with double nonlinearity and distributed parameters, Journal of Siberian Federal University. Mathematics & Physics, 2013, Vol. 6, Issue 2, pp 157–167.
3. Sh.Sadullaeva, Numerical Investigation of Solutions to a Reaction-diffusion System with Variable Density, Journal of Siberian Federal University. Mathematics & Physics, 9(1), 2016, pp. 90-101.
4. S.A.Sadullaeva, A.T.Khaydarov, F.A.Kabilianova, Modeling of Multidimensional Problems in Nonlinear Heat Conductivity in Non-Divergence Case, 3rd International Symposium on Multidisciplinary Studies and Innovative Technologies, ISMSIT 2019 – Proceedings, 8932954.
5. S.A.Sadullaeva, M.B.Khojimurodova, Properties of solutions of the cauchy problem for degenerate nonlinear cross systems with convective transfer and absorption, Springer Proceedings in Mathematics and Statistics, 264, 2018, pp. 183-190.
6. M.Aripov, S.Sadullaeva, An asymptotic analysis of a self-similar solution for the double nonlinear reaction-diffusion system, J. Nanosyst. Phys. Chem. Math., 2015, 6 (6), pp. 793-802.
7. M.Aripov, S. Sadullaeva, Qualitative properties of solutions of a doubly nonlinear reaction-diffusion system with a source, J. Appl. Math. Phys., 2015, 3, pp. 1090-1099.
8. D.K.Muhamediyeva, The property of the problem of reaction diffusion with double nonlinearity at the given initial conditions, International Journal of Mechanical and Production Engineering Research and Development, Volume 9, Issue 3, 2019, IJMPERDJUN2019117, Pages 1095-1106.
9. A.Mersaid, The fujita and secondary type critical exponents in nonlinear parabolic equations and systems, Springer Proceedings

https://journals.researchparks.org/index.php/IJOT e-ISSN: 2615-8140 | p-ISSN: 2615-7071

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in Mathematics and Statistics, 2018, 268, pp. 9-23.

10. D.K.Muhamediyeva, Properties of self similar solutions of reaction-diffusion systems of quasilinear equations, International Journal of Mechanical and Production Engineering Research and Development, 8 (2), 2018, pp. 555-566.
11. S.A. Sadullaeva, G. Pardaeva, Numerical Investigation one System Reaction-Diffusion with Double Nonlinearity, Journal of Mathematics, Mechanics and Computer Science, 2018, 86 (3), 58-62.
12. M.Aripov, O.Djabbarov, Sh.Sadullaeva, Mathematic modeling of processes describing by double nonlinear parabolic equation with convective transfer and damping, AIP Conference Proceedings 2365, 060008 (2021); https://doi.org/10.1063/5.0057492
13. M.Aripov, S.Sadullaeva, N.Iskhakova, Numerical Modeling Wave Type Structures in Nonlinear Diffusion Medium with Dumping, 4th International Symposium on Multidisciplinary Studies and Innovative Technologies, ISMSIT 2020 - Proceedingsthis link is disabled, 2020, 9254988
14. S.Sadullaeva, Z.Fayzullaeva, D.Nazirova, Numerical Analysis of Doubly Nonlinear Reaction-Diffusion System with Distributed Parameters, 4th International Symposium on Multidisciplinary Studies and Innovative Technologies, ISMSIT 2020 - Proceedingsthis link is disabled, 2020, 9255106
15. A.Nematov, E.S.Nazirova, R.T.Sadikov, On numerical method for modeling oil filtration problems in piecewise-inhomogeneous porous medium, IOP Conference Series: Materials Science and Engineeringthis link is disabled, 2021, 1032(1), 012018
16. E.Nazirova, A.Nematov, R.Sadikov, I.Nabiyev, One-Dimensional Mathematical Model and a Numerical Solution Accounting Sedimentation of Clay Particles in Process of Oil Filtering in Porous Medium, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)this link is disabled, 2021, 12615 LNCS, стр. 353–360
17. N.Ravshanov, E.S.Nazirova, V.M.Pitolin, Numerical modelling of the liquid filtering process in a porous environment including the mobile boundary of the oil-water section, Journal of Physics: Conference Seriesthis link is disabled, 2019, 1399(2), 022021
18. S.Anarova, S.Ismoilov, Nonlinear mathematical model of stress-deformed state of spatially loaded rods with account for temperature, AIP Conference Proceedingsthis link is disabled, 2021, 2365, 070019
19. S.A.Anarova, S.M.Ismoilov, O.S.Abdirozikov, Software of Linear and Geometrically Non-linear Problems Solution Under Spatial Loading of Rods of Complex Configuration, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)this link is disabled, 2021, 12615 LNCS, стр. 380–389