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Quantitative methods in the field of economic sciences

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Abstract

The usage of quantitative tools for creating the mathematical models of functioning different economic facilities abounds the opportunity for better understanding and acquaintance of the surrounding reality. A lot of thinkers identify even universality of the particular branch of knowledge with the extent of its 'mathematization'. Applying mathematical methods so called quantitative provide great and not to overestimate services not only in the science research of technique, physics, astronomy, biology and medicine, but also – within the qualitative methods- in the field of social science in the sphere of the control of the quality of production or in the process of service management or decision making. Complex nature of the social and economic phenomena requires making the usage of the most modern means and the ubiquitous computerization significantly confirms the usefulness of these methods. Progressing 'mathematization' and computerization of the science forces creating and applying quantitative (mathematical) models including economic science. The model of operating of studied system was considered in two variants. I. when the process of the product delivery to the store represents inclusively the subsystem of production and the subsystem of the transportation – it could be then said that the level of filling the store up is controlled by the aggregated process of the delivery of the product.

II. when the process of the product delivery to the store takes into account explicate both the production process and also the operating of transportation subsystem, so it is then the structural process of the product delivery. Both in the aggregated and structural version, the analyses of the functioning of the system was made in three variants of the store filling: intermediate state of the store filling; zero state of the store filling that is lower barrier; the state of full storage of the store, that is the upper barrier. The result of my analyses are two proprietary probabilistic models of system operation which are presented through the system of differential equations both in the aggregated and structural variant. Probabilistic models of functioning of the system in both variants presented throughout the probabilistic model also enable determining sizing prognosis which are characteristic for the functioning of this system. These prognoses are transferred to the unit of the management system and they provide the premises to the streamline of its functioning.

These tools create the basics of theoretical and methodological constructed computer programmes of the informative systems of decision-making support.

Keywords: methods – quantitative (mathematical) models, system – subsystem, service enterprise, probabilistic model, forecast, system parameters.

Introduction

More than two centuries ago, Polish mathematician and philosopher Jan Śniadecki stated, that (I quote): "... the human mind in its thoughts, views and reasoning would be unequally happier and more confident, if it could, in any case, know how close or far it is from surety what to expect from calculation of chance adapted to other sciences, as it is used

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today in physics and astronomy. It would then be a reliable and effective *criterium veritatis*, which logicians and metaphysics look for in, not very useful, talk. Although the great difficulty of the calculation seems to be the major obstacle to it, the time may come, when from general literary formulas, tables will be formed for every science to help us be closer to the truth, just as today's bankers can easily know how much interest they are paid for each loan. It seems to me, that this reality will be present in every science in the future and everyone will know, that counting means reasoning with certainty, and the mathematics, which has done so much to the society, the science and arts, will become the leader of the human mind in all cognitions". (Śniadecki, 1954, p. 287)

The use of quantitative tools in building mathematical models of the functioning of various businesses creates the opportunity for a better understanding and control of the surrounding reality.

The need to quantify business activity was already mentioned in the Old Testament, in which we read (I quote): "Whatever stores you issue, do it by number and weight, spendings and takings, put everything in writing". (Peter, Walkiewicz, 1998, p. 222). Moreover, many thinkers equate the universality of a given field of knowledge with the degree of its "mathematization" according to the saying of Immanuel Kant that "every science contains as much knowledge as much mathematics it contains". (Żwirbla, 2007).

The use of mathematical methods known as quantitative ones gives today great and invaluable benefits not only in natural research in the field of technology, physics, astronomy, biology or medicine, but also, along with qualitative methods, in the fields of social sciences, e.g. in the control of production quality or in the service management or decision making processes.

The complex nature of social and economic phenomena requires the use of the most modern means, and ubiquitous computerization significantly confirms the usefulness of these methods.

Results and discussion

Selected quantitative models in service management

Progressive mathematization and computerization of science forces the building and use of quantitative (mathematical) models also in social sciences. The methodology of the modeling process distinguishes several types of quantitative (mathematical) models, as follows:

- deterministic, in which parameters are known and constant,
- stochastic, in which parameters are random variables with a known probability distribution,
- operational, with a short time horizon,
- strategic, with far-reaching consequences,
- single- or multi-criteria optimization,
- linear, as part of the scope of computational techniques and linear programming.

In assessing the reliability of the service system (Nowakowski, 2011, p. 128) it is the time factor that is usually of primary importance. For that reason, the probability P of completing a specific task within a set time limit t_0 is assumed to be a measure of this reliability R(t), i.e.

$$R(t) = P(T < t_0)$$

where: T – random time of a specific task (e.g. delivery), t_0 – assumed (determined) time limit for completing the task.

If we, however, take into account the requirement that a specific task should be done "on time", means with a certain time tolerance Δt (not too early, but also without unacceptable delays), then the reliability of the logistics system can be defined as probability:

$$R(t) = P\left(t_0 - \frac{\Delta t}{2} \leq T < t_0 + \frac{\Delta t}{2}\right)$$

where: Δt – acceptable time interval for a specific service.

Technical reliability of the service system, defined as the probability of failure-free operation of the material flow subsystem, depends not only on the reliability of its individual components, but also on the arrangement and mutual relations between those individual elements forming the structure of this subsystem.

In this context, systems with serial, parallel, serial – parallel, parallel – serial and threshold structures are analyzed.

The subsystem with a serial structure can be represented in the figure below:

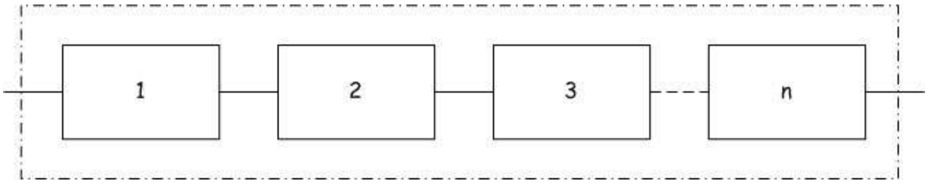

Fig. 1 – Subsystem with serial functional structure.

Reliability R_{sz} of a subsystem with a serial structure containing n – elements, is expressed by the result of multiplication of reliability of individual elements:

$$R_s = R_1 \cdot R_2 \cdot \ldots \cdot R_n$$

In the subsystem with serial structure:

- the fewer the components, the greater the reliability;
- reliability cannot be greater than the reliability of the most failing element (the weakest link effect);
- in the case of a large number of components even the high reliability of a single element does not affect the reliability of the entire subsystem;
- inability to function properly of at least one element causes the entire subsystem to fail.

A subsystem with a parallel structure can be represented in the figure:

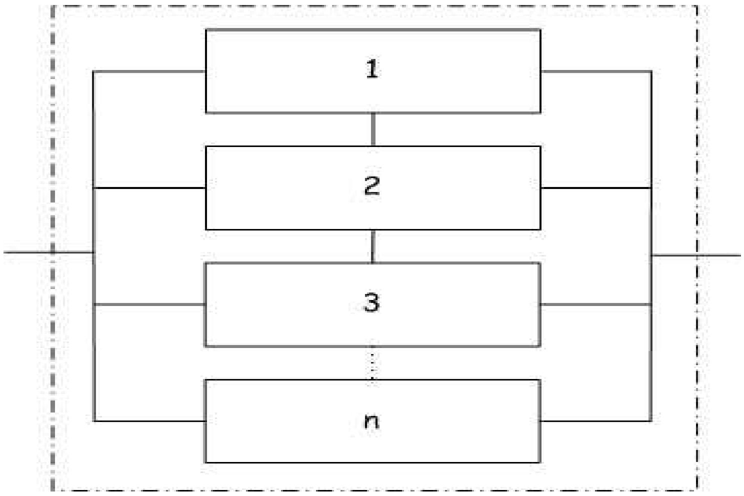

Fig. 2 –Subsystem with a parallel functional structure.

One of the most characteristic feature of a subsystem with a parallel structure is the fact that it can function correctly if at least one of its components is correct.

Failure (probability of damage) F_{fail} of this subsystem containing *n – elements,* is expressed by the result of multiplication of the failures of individual elements $F_{fail(i)}$.

According to the above, the reliability (probability of correct operation) of such a subsystem is equal to:

$$R_r = 1 - (1 - R_1) \cdot (1 - R_2) \cdot \ldots \cdot (1 - R_n)$$

It implicates that the reliability of a subsystem with a parallel structure is the greater, the more elements this structure contains. For this reason – in order to increase the reliability of systems with a parallel structure – a number of so-called redundant (reserve) elements is added.

The following figure presents a subsystem with a serial – parallel structure.

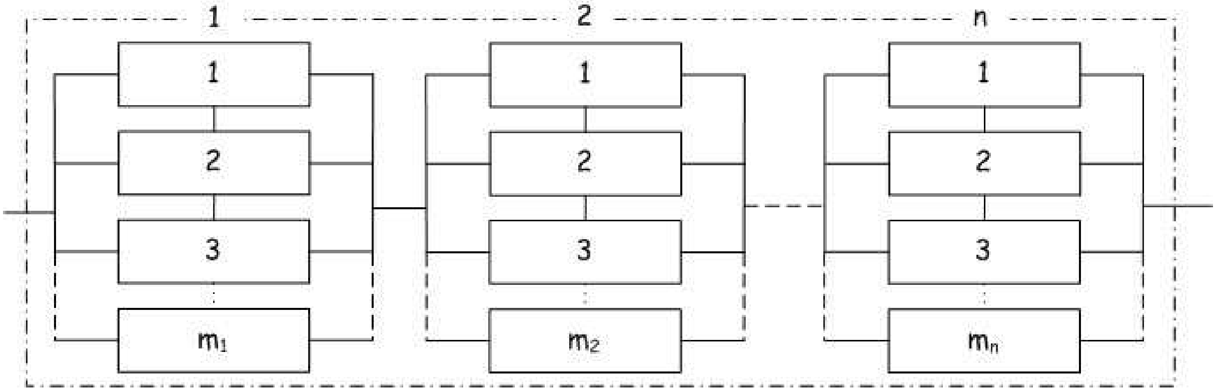

Fig. 3 – Subsystem with serial – parallel functional structure.

The subsystem with a serial – parallel structure is a serial connection of *n* – units, each of which containing m_i elements connected in parallel.

The subsystem can function properly only when each of its *n* – units works without a failure.

Reliability of the subsystem with serial – parallel functional structure, containing *n* – units, each of which having m_i parallel connected elements, is equal to:

$$R_{sr} = \prod_{j=1}^{n}\left[1 - \prod_{i=1}^{m}(1 - R_{ij})\right]$$

where: R_{ij} – reliability of the *i* – element installed in the *j* – unit.

In a special case, when the subsystem with a serial – parallel structure is homogeneous and regular - which means it has the same number of equal components in each unit – reliability R_{sr} can be defined as:

$$R_{sr} = [1 - (1 - R^m)]^n$$

A subsystem with a parallel – serial structure is presented below.

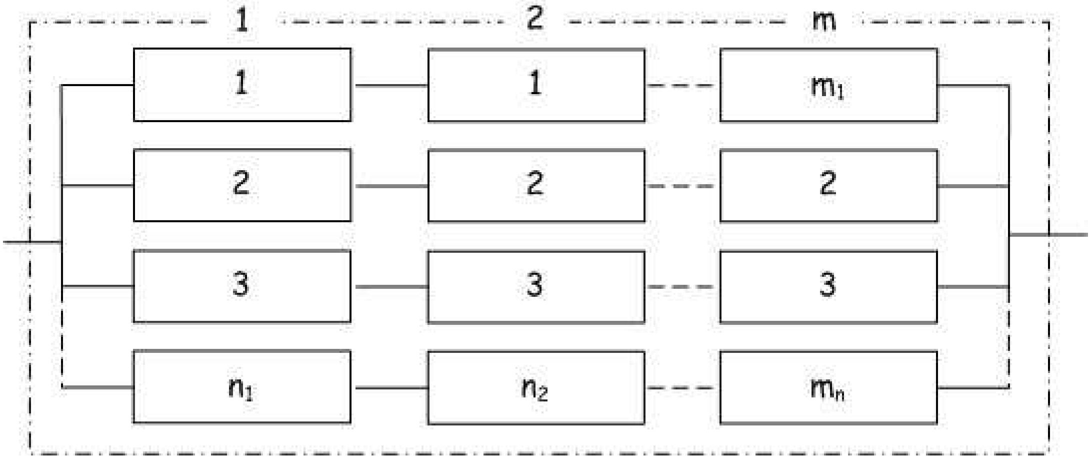

Fig. 4 – Subsystem with parallel – serial functional structure.

The subsystem with a parallel – serial functional structure is a parallel connection of *n* – units, each of which containing m_i – elements connected serially.

Such subsystem functions correctly when at least one of its *n* – units works properly, and its reliability R_{sr} is equal to:

$$R_{rs} = 1 - \prod_{j=1}^{n}\left(1 - \prod_{i=1}^{m}(1 - R_{ij})\right)$$

where: R_{ij} reliability of the *i* – element installed in the *j* – unit.

Similarly, in a situation when the subsystem is homogeneous and regular - which means it has the same number of equal components in each unit – reliability R_{sr} can be defined as:

$$R_{\mathrm{rs}} = 1 - (1 - R^m)^n$$

A special case of logistics subsystems of material flow are the subsystems of the so-called threshold structure. Threshold subsystems (also called subsystems of the "*k out of n*" type) are in the mode of proper functioning, when *k* out of *n* components (when 1≤ *k* ≤ *n*) work properly.

In the "*k* out of *n*" type of threshold subsystems, it is allowed for some predetermined number of elements (n - k) to be faulty, and the object can still be considered as fit for action. For each "k out of n" type of threshold object, the so-called *p* parameter is defined, as threshold of object:

$$0 \leq p = \frac{k}{n} \leq 1$$

On this basis, threshold subsystems (objects) of "*k out of n*" type are classified as:

- minority objects, where *p =k/n* < 0,5;
- equality objects, where *p =k/n* =0,5;
- majority objects, where 0,5 < *p =k/n* < 1;0.

Simple subsystems with serial structure or parallel structure are special cases of threshold objects, namely:

- serial subsystem is a threshold object of "*n out of n*" type,
- parallel subsystem is a threshold object of "*1 out of n*" type.

The evaluation of the reliability of operation of such systems, taking the probability of meeting particular partial tasks into account, is defined as follows (Nowakowski, 2011, p. 135):

$$\left.\begin{array}{c} P_{cpr} = \dfrac{N_{cpr}}{N}\ ;\ P_{cpl} = \dfrac{N_{cpl}}{N}\ ;\ P_{ccu} = \dfrac{N_{ccu}}{N}\ ; P_{cti} = \dfrac{N_{cti}}{N}\ ; \\ P_{cql} = \dfrac{N_{cql}}{N}\ ;\ P_{cqt} = \dfrac{N_{cqt}}{N}\ ;\ P_{cdo} = \dfrac{N_{cdo}}{N}\ ; \end{array}\right\}$$

where: P_i – the probability of ensuring the operation of *i* – feature of the whole logistic system, *N* – number of all completed tasks (deliveries), N_{cpr} – number of correct products, N_{cpl} –- the number of products delivered to the right place, N_{ccu} – number of products delivered to the correct recipient (customer), N_{cti} – the number of products delivered within the correct timeframe, N_{cql} – the number of the right quality products delivered, N_{cqt} – the number of products delivered in the right quantity, N_{cqt} – the number of products delivered at the correct price.

If the individual features of the service system are statistically independent random variables and the meaning of these features for the recipient is the same (the same weights), then for the system defined in this way, we are able to denominate the so-called "perfect order completion" index as:

$$R_d = P_{cpr} \cdot P_{cpl} \cdot P_{ccu} \cdot P_{cti} \cdot P_{cql} \cdot P_{cqt} \cdot P_{cdo}$$

However, if the individual features (requirements) of the system have different meanings for the recipient, this fact should be taken into account – introducing the vector of partial weights:

$$w = [w_{cpr}, \mathrm{w}_{cpl}, \mathrm{w}_{ccu}, \mathrm{w}_{cti}, \mathrm{w}_{cql}, \mathrm{w}_{cqt}, \mathrm{w}_{cdo}]$$

wherein the components of this vector are in the range <0,1> and also fulfill the condition:

$$w_{cpr} + w_{cpl} + w_{ccu} + w_{cti} + w_{cql} + w_{cqt} + w_{cdo} = 1$$

Accordingly, for a system whose features have different meanings for the end user, the "perfect order completion" index is expressed as follows:

$$R_d = P_{cpr} \cdot w_{cpr} + P_{cpl} \cdot w_{cpl} + P_{ccu} \cdot w_{ccu} + P_{cti} \cdot w_{cti} + P_{cql} \cdot w_{cql} + P_{cqt} \cdot w_{cqt} + P_{cdo} w_{cdo}$$

In assessing the reliability of logistic systems, it is possible to use the so-called *OTIF* (*On-time, In-full, Error-free*) index, defined as:

$$\mathrm{OTIF} = P_{o-t} P_{i-f} P_{e-f}$$

where: P_{o-t} – the probability of completing the order on time, P_{i-f} – the probability of fully completing the order, P_{e-f} – the probability of completing the defect-free order.

According to the OTIF index, a perfectly completed order is when:

- the product has been delivered without any delay,
- all line items have been delivered,

- no part of the delivery has been damaged in the logistic process.

An example of such an approach to the issue is the reliability of the internal transport subsystem in the warehouse system, which is assessed in binary categories, i.e. the state of suitability and the state of unsuitability.

Another example of a quantitative (mathematical) model is the Economic Order Quantity (EOQ) method.

It involves searching for a compromise between the costs of maintaining the stock of goods and the costs of placing orders. The basic assumptions of the simple EOQ model for replenishing goods are as follows (Coyle, Bardi, Langley, 2010, p. 49):

- continuous, permanent and known demand,
- permanent and known stock replenishment cycle (delivery time),
- full satisfaction of demand,
- fixed purchase and transport costs, independent of the order size and time,
- the lack of goods in transit,
- homogeneity of the product (one assortment),
- infinite planning horizon,
- unlimited amount of available financial resources (capital).

Economic Order Quantity determines the point at which the total costs of placing orders and maintaining the goods are the lowest. The value of the EOQ is calculated from the following formula:

$$Q = \sqrt{\frac{2RA}{VW}}$$

where: R – annual demand for a given assortment, A – the cost of ordering an assortment unit, V – value of the inventory item, W – percentage share of the cost of a given product inventory in total costs.

Economic Order Quantity calculated on the basis of the formula, may need to be adjusted for reasons, such as changes in transport costs or price discounts. In general, unit transport prices depend on the weight or size (dimensions) of the transported cargo. When transporting small loads, the price of transporting a unit of mass (e.g. 1 ton) is generally higher than in the case of large loads. In addition, with larger orders, suppliers usually use higher price discounts. This affects the change of the initial assumptions of the simple EOQ model and should be taken into account when calculating Economic Order Quantity.

Quantitative (mathematical) models, in the form of specific formulas, are present when examining the performance of service systems.

Among the factors characterizing the material flow subsystems in terms of technology, the following are of particular importance.

Theoretical efficiency of the transport subsystem components Q (also called a bandwidth), resulting from the values of technical parameters of individual technical measures forming the subsystem structure. The bandwidth meter is the maximum number/mass of loads per time unit that can theoretically flow through the individual elements of the subsystem.

The bandwidth Q of the continuously operating components depends on the maximum permissible speed v of the load movement and the minimum allowable distance a between the loads - according to the formula:

$$Q = 3600\frac{v}{a}$$

The bandwidth Q of the cyclically operating components, on the other hand, depends on the speed v, the transport distance L and the number of the technical measures used M - according to the following formula:

$$Q = 3600\frac{M}{t_0 + \frac{2L}{v}}$$

where: t_0 is the total time of loading / unloading, acceleration and braking of the means of transport.

Actual efficiency of the subsystem Q_{rz} is its ability to transfer uninterrupted load streams through all components forming the structure of the transport subsystem. Actual efficiency includes, among others, a possibility to form pile-ups and queuing in the material flow subsystem. In other words, the actual efficiency expresses the maximum intensity of the material stream at which the time of stacking of loads and the length of the generated queues do not exceed the permissible values.

A necessary condition for undisturbed flow of cargo through certain component of the transport subsystem is that the instantaneous intensity of stream of materials (charges) λ_i was not greater than the theoretical efficiency (bandwidth) Q_i of this component, that is – that the so-called utilization of transport potential index ρ_i was not greater than 1:

$$\rho_i = \frac{\lambda_i}{Q_i} \leq 1$$

The postulate above is a sufficient condition for the efficient operation of the transport subsystem only in simple deterministic cases, i.e. when the deliveries (loads) arrive at regular intervals and the time of their handling (acceptance into the system) is constant and shorter than the time of new arrivals.

In real logistic systems, however, material flows usually take place in a random manner, i.e.:

- loads arrive at irregular intervals,
- the service time of individual loads varies depending on the type and size of loads.

Under these conditions, the analysis of the operation of the transport subsystem requires consideration of random aspects of the material flow process – in particular:

- probability distribution of incoming loads to the system,
- probability distribution of times of handling particular loads in the system.

For simple, single-channel service systems – operating in random conditions in which it can be assumed that:

- deliveries are subject to Poisson distribution with a fixed parameter λ;
- delivery handling time has an exponential distribution with a fixed parameter μ;

from the theory of mass service the following dependences arise on:

- probability density function of the queue for n deliveries waiting for service, i.e.:

$$f_N = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

- cumulative distribution function of the queue for n deliveries waiting for service, i.e. the probability:

$$F_N = P(N \leq n) = 1 - \left(\frac{\lambda}{\mu}\right)^{1+N}$$

- the expected value of the queue length, that is:

$$H = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1 - \rho}$$

- density function of time spent in the queue for n deliveries, i.e. the probability:

$$f_T(t) = \lambda \left(1 - \frac{\lambda}{\mu}\right) e^{-(\mu - \lambda)t}$$

- cumulative distribution function of time spent in the queue for n deliveries, i.e. the probability:

$$F_T(t) = P(T \leq t) = 1 - \frac{\lambda}{\mu} e^{-(\mu - \lambda)t}$$

- the expected value of the waiting time in the queue, that is:

$$\bar{t} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\rho}{1 - \rho}$$

where: $\rho = \lambda/\mu$ is a system usage index.

There are also other quantitative (mathematical) models used in certain formulas that calculate i.e.:

- transport infrastructure assessment indices;
- costs of the flow of the cargo stream (Jacyna, 2012, p. 277-281);
- storage and transport costs;
- conditions for the costs minimization.

Certain mathematical formulas are also expressed in certain values, allowing for:

- calculating the so-called regions of gravity of the modal point service areas of the logistics network;
- expected demand in a given place (the HUFF model of mutual spatial interaction);
- coordinating transport in the city (the gravity model). (Gołembska, 2009, p. 144-146)

Quantitative (mathematical) models are also present in optimization issues, such as the calculation of the optimal batch of production, the optimization of the purchase part, or the optimization of transport links (Skowronek, Sariusz-Wolski, 2008, p. 150-169) as well as the optimal location of logistic facilities in the logistics network (Jacyna, 2012, p. 335-338).

Quantitative models in the form of formulas or indices can also be found in the analysis of purchasing processes both in the functional and decision-making sphere in relation to the size of the dynamics and structure of purchase and the impact of prices on the operating costs of the logistics system (Skowronek, Sariusz-Wolski, 2008, p. 312-315).

The essence of service system operation processes is the control of material flows over time. In this respect, currently used methods of forecasting are methods of time series analysis called methods of development tendency - in particular, adaptive models of short-term forecasting. Such forecasting uses econometric causal models to plan logistics processes on a strategic scale.

These models are limited to regression equations of one or more variables, e.g.:

- single-parameter Brown model adequate to stationary time series can be presented as follows:

$$\bar{y}_t = xy_t + (1-\alpha)\bar{y}_{t-1}$$

with the forecast for the period $t+T$ has a form

$$\hat{y}_{t+T} = \bar{y}_t$$

where: $\bar{y}_t$ $\bar{y}_{t-1}$ – average values after periods, respectively t i $t-1$, y_t – last implementation of the forecasted variable, α – exponential smoothing parameter taking values from the interval $(0;1)$, $\hat{y}_{t+T}$ – the forecast of variable y for the period $t+T$,

- two-parameter adaptive C.C. Holt model appropriate for use in the case of significant changes in the development trend of the forecasted variable and consisting of three equations:

data smoothing:

$$a_t = \alpha y_t + (1-\alpha)(a_{t-1} + b_{t-1})$$

trend smoothing:

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1}$$

forecast for a period:

$$t+T: \hat{y}_{t+T} = a_t + b_t T$$

where: a_t – average demand for the period t t inclusive, b_t – the difference between the averages being an assessment of trend growth, α, β – smoothing parameters from the interval $(0;1)$.

There are also other models based on exponential data smoothing, e.g. model P.R. Wintera also taking into account seasonal fluctuations or the Z. Pawłowski's unrestricted exponential – autoregressive smoothing model (Skowronek, Sariusz-Wolski, 2008, p.217-232).

The dependencies between individual elements of the logistics system are usually very complex and multidirectional. The econometric models that have been presented above include only the essential elements and omit the less important ones – they are therefore too general.

3. Probabilistic models of the system's operation and their application to build the forecasts of service enterprise effective functioning

The object of my interest was the following inventory management system, constituting a production and supply system supporting a service company. The system is schematically illustrated in the figure below.

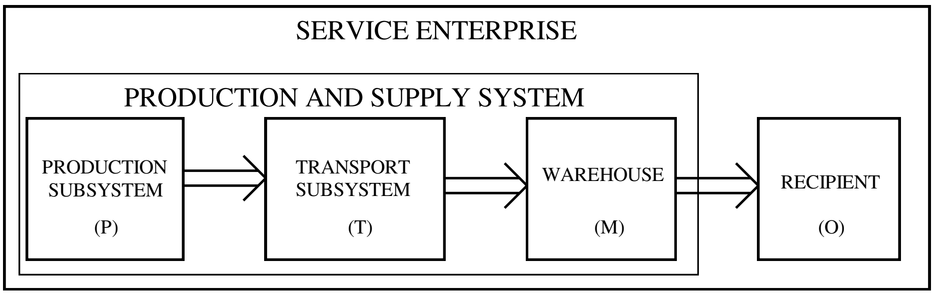

Fig. 5 – Service company and its subsystems

Source: Own elaboration.

The aim of the study was to present new, precursory tools – quantitative methods supporting the service management process. These tools are a new methodology for determining forecasts of characteristics (processes) describing the functioning of the service company and its subsystems. Such forecasts, so that they could support the process of managing the service enterprise, should depend on the parameters describing the operation of the production subsystem, transport subsystem and the level of filling the warehouse. Then, these forecasts give the opportunity to predict the process of managing a service enterprise, and hence its functioning and support through the use of the parameters of these subsystems. If the forecasts prove to be unfavorable for the functioning of a service company, then it is possible to adjust the values of these parameters in order to obtain forecasts beneficial for the operation of a service company, and thus improve the quality of its management and its efficiency.

The functioning of the production and supply system in a service enterprise is illustrated in the figure below.

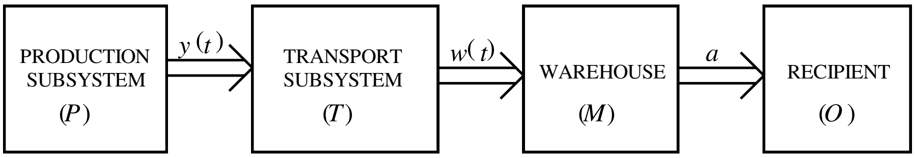

Fig. 6 – The general scheme of the functioning of the production and supply system in a service enterprise.

Source: Own elaboration.

The operating model of the tested system has been considered in two variants:

I. when the process of product delivery to the warehouse is represented jointly by the production subsystem and the transport subsystem – then it can be said that the level of filling the warehouse is controlled by the aggregated process of product delivery, i.e.: aggregated variant: $w(t) = y(t)$; the process $w(t)$ represents the production subsystem *P* and the transport subsystem *T* together, and

II. when the product delivery process to the warehouse takes into account *explicite* both the production process and the operation of the transport subsystem, i.e. it is a structural product delivery process, i.e.: a structural variant: $w(t) = y(t)v(t)$; the process $w(t)$ takes into account *explicite* both the subsystem *P* (product stream $y(t)$), as well as the subsystem *T*, the process $v(t)$) describes the operation of the transport subsystem.

Both in the aggregated version and in the structural version, the analysis of the system operation was carried out in three variants of filling the warehouse:

- intermediate state (not extreme) of filling the warehouse;
- zero state of filling the warehouse, i.e. the bottom barrier;
- the state of full storage of the warehouse, i.e. the upper barrier.

Let the time function $z(t)$such that $z(t) = z(t,\omega)$ equals to the current warehouse storage *M* of finite capacity *V*, where ω is a random parameter. The function *h* will be defined as follows:

$$h(z)=\begin{cases}0, & \text{when} \quad z\le 0\\ z, & \text{when} \quad 0<z<V\\ V, & \text{when} \quad z\ge V, \quad V=const, \quad V>0.\end{cases}$$

Let us note that the stochastic process $z(t)$ in the range $[\alpha_1, \alpha_2)$of constancy of the process: $w(t) = y(t)$ meets the following condition for $\alpha_1 \le t_1 < t < \alpha_2$:

$$z(t,\omega) = h\,[z(t_1) + (y(t_1) - a)\,(t - t_1)\,].$$

The stochastic process is a size dependent on random factors (ω) and time (t).

In order to solve many problems related to improving the efficiency of the service enterprise, including obtaining appropriate forecasts, it is enough to determine the following probabilities for the aggregated variant:

- partial filling of the warehouse:

$$P(0 < z(t) < \alpha, \quad x(t) = x_k) = \int_0^\alpha f_k(z,t)dz,$$

- the bottom barrier:

$$P(z(t) = 0, \quad x(t) = x_k),$$

- the upper barrier:

$$P(z(t) = V, \quad x(t) = x_k),$$

where: $0 < \alpha < V$, $f_k(z,t)$ equals the probability density, and x_k is the k – th state of the auxiliary process introduced in the form of $x(t) = y(t) - a$ $(x_k = y_k - a, k = 1,2, \dots, n)$.

System operation in the variant: $0 < z(t) < V$ is characterized by the probabilities of the following form:

$$Q_k^{(l)}(\alpha, t) = P(0 < z(t) < \alpha, \quad x(t) = x_k) = \int_0^\alpha f_k^{(l)}(z,t)dz, \quad 0 < \alpha < V, \quad t \in T_l.$$

Analysis of system operation in the period T_l $(l = 1,2, \dots, m)$ in case when the product stock level meets the condition: $z(t) = 0$, is characterized by the probabilities of the form:

$$Q_k^{(l)}(\{0\}, t) = P(z(t) = 0, \quad x(t) = x_k), \quad t \in T_l.$$

In turn, the system's operation in the period T_l if the stock level of the product meets the condition: $z(t) = V$, is characterized by the probabilities below:

$$Q_k^{(l)}(\{V\}, t) = P(z(t) = V, \quad x(t) = x_k), \quad t \in T_l.$$

The result of my analysis are two author's probabilistic models of system operation, which are presented by system of differential equations in the aggregated variant:

$$\begin{cases} \frac{\partial f_k^{(l)}(z,t)}{\partial t} = -\pi_k^{(l)} f_k^{(l)}(z,t) - x_k \frac{\partial f_k^{(l)}(z,t)}{\partial z} + \sum\limits_{i \neq k} f_k^{(l)}(z,t)\pi_{ik}^{(l)}, \quad \text{dla } 0 < z < V, \quad t \in T_l, \; k = 1,2,\dots,n \\ \frac{\partial Q_k^{(l)}(\{0\},t)}{\partial t} = -\pi_k^{(l)} Q_k^{(l)}(\{0\},t) - x_k f_k^{(l)}(0,t) + \sum\limits_{\substack{i \neq k \\ x_i \leq 0}} Q_k^{(l)}(\{0\},t)\pi_{ik}^{(l)}, \quad \text{for } x_k \leq 0, \quad t \in T_l \\ Q_k^{(l)}(\{0\},t) = 0, \quad \text{for } x_k > 0, \quad t \in T_l \\ \frac{\partial Q_k^{(l)}(\{V\},t)}{\partial t} = -\pi_k^{(l)} Q_k^{(l)}(\{V\},t) + x_k f_k^{(l)}(V,t) + \sum\limits_{\substack{i \neq k \\ x_i \geq 0}} Q_i^{(l)}(\{V\},t)\pi_{ik}^{(l)}, \quad \text{for } x_k \geq 0, \quad t \in T_l \\ Q_k^{(l)}(\{V\},t) = 0, \quad \text{for } x_k < 0, \quad t \in T_l \end{cases}$$

Such a probabilistic model of functioning of the tested production and supply system, supporting a service company, enables the construction of magnitudes that can be used both in the process of effective management of this system and the design phase of this system.

Let us consider the following example.

The index $w_1 = P(W_1) = \sum_{y_k < a} Q_k^{(m)}(\{0\})$ is the probability of a product supply shortage *a* for the recipient *O*, i.e. loss for the recipient.

The index $w_2 = P(W_2) = \sum_{y_k > a} Q_k^{(m)}(\{V\})$shows the assessment of the loss for the supplier.

Generally, the indices: $w_s = r_s\left(\pi_{ik}^{(m)}, y_l, a, V\right)$ depend on the following parameters: intensity $\pi_{ik}^{(m)}$, controlling process y_t the product level *a* and the capacity *V*.

Therefore, it is possible to optimize the values of given indices by appropriate changes in the size of these parameters. Improving the organizational elements of the system's operation that affect the values of intensity $\pi_{ik}^{(m)}$ controlling process y_t, we can influence the values of indices w_s, and thus increase the efficiency of the production and supply system supporting the service company.

The probabilistic models of the system functioning expressed in the probabilistic model also allows determining the forecasts of the values characterizing the operation of this system. These forecasts are passed to the system management body and provide premises to increase the efficiency of its functioning.

The examples of the forecasts are presented below.

Let $t \in T_m \cap \tilde{T}_i$, and $\tilde{T}_i$ means the forecast horizon of the *i*-th forecasted value. The forecast, e.g. $\widehat{w}_1(t)$ of the probability of the product supply shortage for the customer *O* at time *t* is calculated using the following formula:

$$\widehat{w}_1(t) = \sum_{y_k < a} Q_k^{(m)}(\{0\}, t).$$

The forecast $\widehat{w}_2(t)$ of the probability of loss for the supplier (of the subsystem *P*) due to the filling of the warehouse *M* at the time *t* is determined using the following formula:

$$\widehat{w}_2(t) = \sum_{y_k > a} Q_k^{(m)}(\{V\}, t).$$

The forecast $\widehat{w}_3(t)$ of the assessment index of usage of the subsystem *M* in the tested system at time *t* is calculated on the basis of the following formula:

$$\widehat{w}_3(t) = \sum_{k} Q_k^{(m)}(\{V\}, t).$$

In the structural variant, the level of inventories $z(t)$ of the subsystem *M* is controlled by the process: $w(t) = y(t)v(t)$, where $y(t)$ describes the production volume of the subsystem *P* and

$$v(t) = \begin{cases} 1, & \text{when the subsystem } T \text{ is in the operation mode,} \\ 0, & \text{when the subsystem } T \text{ is not operating (is in a failure mode).} \end{cases}$$

The functioning of the tested system is now characterized by a three-dimensional stochastic process: $\big(y(t), v(t), z(t)\big)$.

The operation of the system will be analyzed in three variants (non-extreme state of subsystem inventory level *M*: $0 < z(t) < V$, bottom limit state of subsystem inventory level *M*: $z(t) = 0$, upper limit state of subsystem inventory level *M*: $z(t) = V$) because they have different operating conditions.

For further purposes, it is sufficient to determine, analogically to the aggregate variant, probabilities (corresponding to three variants of the analysis of the considered system) in the following form:

- for non-extreme inventory filling:
$$P(0 < z(t) < \alpha,\ x(t) = x_k,\ v(t) = u) = \int_0^{\alpha} f_k^{(u)}(z,t)dz,$$
$$0 < \alpha < V,$$
- for the bottom barrier:
$$P(z(t) = 0,\ x(t) = x_k,\ v(t) = u),$$
- for the upper barrier:
$$P(z(t) = V,\ x(t) = x_k,\ v(t) = u),$$

where: $f_k^{(u)}(z,t)$ indicates the density function of the probability distribution, and x_k is the *k*-th state of the process $x(t) = y(t) - a$ $(x_k = y_k - a,\ k = 1,2,\dots,n)$; $u = 1$ (operation) or $u = 0$ (failure).

Functioning of the system in the case when the level of product inventory in subsystem *M* meets the condition: $0 < z(t) < V$, is characterized by the probabilities of the form:

$$Q_k^{ul}(\alpha, t) = P(0 < z(t) < \alpha,\ x(t) = x_k,\ v(t) = u) = \int_0^{\alpha} f_k^{ul}(z,t)dz,$$
$$0 < \alpha < V,\ t \in T_l.$$

Operation of the tested system in the case when the level of product inventory in subsystem *M* fulfills the condition: $z(t) = 0$, that is, it reaches the bottom limit state is described by the probabilities of the form:

$$Q_k^{ul}(\{0\}, t) = P(z(t) = 0,\ x(t) = x_k,\ v(t) = u), \text{ for } t \in T_l,\ u = 1 \text{ or } u = 0.$$

Functioning of the tested system in the case when the level of product inventory in subsystem M meets the condition: $z(t) = V$, that is, it reaches the upper limit state is, accordingly, described by the probabilities of the following form:

$$Q_k^{ul}(\{V\}, t) = P(z(t) = V, x(t) = x_k, v(t) = u),$$
$$\text{for } t \in T_l,\ u = 1 \text{ or } u = 0.$$

The result of the analysis of the production and supply system in the structural variant is the following author's probabilistic model of the system operation:

$$\begin{cases} \dfrac{\partial f_k^{1l}(z,t)}{\partial t} = -x_k \dfrac{\partial f_k^{1l}(z,t)}{\partial z} - \left(\pi_k^{(l)} + \pi_1^{*l}\right) f_k^{1l}(z,t) + \sum_{i \neq k} \pi_{ik}^{(l)} f_i^{1l}(z,t) + \pi_0^{*l} f_k^{0l}(z,t), \text{for } 0 < z < V, t \in T_l, k = 1,2,\dots,n \\ \dfrac{\partial f_k^{0l}(z,t)}{\partial t} = a \dfrac{\partial f_k^{0l}(z,t)}{\partial z} - \left(\pi_k^{(l)} + \pi_0^{*l}\right) f_k^{0l}(z,t) + \sum_{i \neq k} \pi_{ik}^{(l)} f_i^{0l}(z,t) + \pi_1^{*l} f_k^{1l}(z,t), \text{for } 0 < z < V, t \in T_l, k = 1,2,\dots,n \\ \dfrac{\partial Q_k^{1l}(\{0\},t)}{\partial t} = \pi_0^{*l} Q_k^{0l}(\{0\},t) - \left(\pi_k^{(l)} + \pi_1^{*l}\right) Q_k^{1l}(\{0\},t) - x_k f_k^{1l}(0,t) + \sum_{\substack{i \neq k \\ x_i \leq 0}} Q_i^{1l}(\{0\},t) \pi_{ik}^{(l)}, \text{for } x_k \leq 0, t \in T_l \\ Q_k^{1l}(\{0\},t) = 0, \text{for } x_k > 0, t \in T_l \\ \dfrac{\partial Q_k^{0l}(\{0\},t)}{\partial t} = \pi_1^{*l} Q_k^{1l}(\{0\},t) - \left(\pi_k^{(l)} + \pi_0^{*l}\right) Q_k^{0l}(\{0\},t) + a f_k^{0l}(0,t) + \sum_{i \neq k} Q_i^{0l}(\{0\},t) \pi_{ik}^{\{l\}}, \text{for } k = 1,2,\dots,n, t \in T_l \\ \dfrac{\partial Q_k^{1l}(\{V\},t)}{\partial t} = x_k f_k^{1l}(V,t) - \left(\pi_k^{(l)} + \pi_1^{*l}\right) Q_k^{1l}(\{V\},t) + \sum_{\substack{i \neq k \\ x_i \geq 0}} Q_i^{1l}(\{V\},t) \pi_{ik}^{(l)}, \text{for } x_k \geq 0, t \in T_l \\ Q_k^{1l}(\{V\},t) = 0, \text{for } x_k < 0, t \in T \\ Q_k^{0l}(\{V\},t) = 0, \text{for } k = 1,2,\dots,n \end{cases}$$

The above model allows to determine the forecast of some unfavorable events.

Examples are as follows.

The forecast $\widehat{w}_1(t)$ of the probability of the product supply shortage for the customer *O* at time *t* is calculated using the formula:

$$\widehat{w}_1(t) = \sum_{x_k<0} Q_k^{1m}(\{0\},t) + \sum_k Q_k^{0m}(\{0\},t).$$

The forecast $\widehat{w}_2(t)$ of the probability of loss for the supplier (of subsystem *P*) at time *t* is determined by the following formula:

$$\widehat{w}_2(t) = \sum_{x_k>0} Q_k^{1m}(\{V\},t) + \sum_k Q_k^{0m}(\{0\},t).$$

The forecast $\widehat{w}_3(t)$ of the assessment index of usage of the subsystem *M* in the tested system at a future time *t* shall be calculated from the formula:

$$\widehat{w}_3(t) = \sum_k Q_k^{1m}(V,t) + \sum_k Q_k^{0m}(V,t).$$

Two-dimensional process $\big(z(t), v(t)\big)$ characterizes the functioning of the subsystem *M* and the transport subsystem *T*. Thus, it is possible to predict the probability of occurrence at the time *t* of the following random events:

- the stock level of the warehouse *M* is not extreme, i.e. $0 < z(t) < V$ and the transport subsystem *T* is in the state *u*, then the forecast of this event is:

$$\widehat{w}_4(t) = \sum_k \int_{c_1}^{c_2} f_k^{um}(z,t)dz, 0 \leq c_1 < c_2 \leq V, u = 0 \text{ or } u = 1,$$

- the stock level of the subsystem *M* reaches the bottom barrier $z(t) = 0$ and the transport subsystem *T* is in the state *u*, then the forecast of this event is:

$$\widehat{w}_5(t) = \sum_k Q_k^{um}(\{0\},t), u = 0 \text{ or } u = 1,$$

- the stock level of the subsystem *M* reaches the upper barrier, i.e. $z(t) = V$ and the transport subsystem *T* is in the state *u*, then the forecast of this event is:

$$\widehat{w}_6(t) = \sum_k Q_k^{um}(\{V\},t), u = 0 \text{ or } u = 1.$$

The process $\big(y(t), v(t)\big)$ describes the supply of the product $y(t)$ of the subsystem *P* and the operation $y(t)$ of the transport subsystem *T*. The forecast of the probability of a random event occurring at a future time where: the product supply of the subsystem *P* reaches the state y_k, and the transport subsystem *T* is in the state *u* ($u=1$ – operation, $u=0$ – failure), is calculated from the following formula:

$$\widehat{w}_7(t) = Q_k^{um}(\{0\},t) + Q_k^{um}(\{V\},t) + \int_0^V f_k^{um}(z,t)dz.$$

The process $\big(z(t), y(t)\big)$ in turn, characterizes the functioning $z(t)$ of the warehouse *M* and the product supply $y(t)$ of the subsystem *P*. It is also possible to determine the probability forecast of the following random events at a future time:

- the stock level of the warehouse *M* is not extreme, i.e. $0 < z(t) < V$, and the product supply $y(t)$ of the subsystem *P* reaches the state Y_k:

$$\widehat{w}_8(t) = \int_{c_1}^{c_2} f_k^{1m}(z,t)dz + \int_{c_1}^{c_2} f_k^{0m}(z,t)dz,$$

- the stock level of the subsystem *M* reaches the bottom barrier $z(t) = 0$ and the product supply $y(t)$ of the subsystem *P* reaches the state Y_k:

$$\widehat{w}_9(t) = Q_k^{1m}(\{0\},t) + Q_k^{0m}(\{0\},t),$$

- the stock level of the subsystem *M* reaches the upper barrier, i.e. $z(t) = V$ and the product supply $y(t)$ of the subsystem *P* reaches the state Y_k:

$$\widehat{w}_{10}(t) = Q_k^{1m}(\{V\},t) + Q_k^{0m}(\{V\},t).$$

The above forecasts $\widehat{w}_s(t)$ of the characteristics describing the functioning of the tested system in the case of a structural product delivery process are calculated on the basis of a probabilistic model.

This allows tracking changes in the trajectory of forecast values depending on changes in the value of these parameters, and thus the anticipation of changes in the functioning of the tested production and supply system supporting the service company.

The obtained stochastic descriptions of the functioning of the production and supply system supporting the service company enable determination of the defined quantitative characteristics of the system, e.g. the product supply for the customer *O* deficit index, the assessment index of the production loss for the supplier *P*, the assessment of the subsystem *M* use in the tested system, the occurrence of the barriers in the subsystem *M* indexes, or the index describing the correctness of designing the system under consideration.

In addition, the presented two models of the production and supply system also give the possibility to determine forecasts of specific system operation characteristics. These types of tools form the theoretical and methodological basis for the computer programs and information systems created to support the decision-making process.

Conclusions

In the research work, various techniques are generally used. In parallel with the use of quantitative methods, researchers also use qualitative and mixed methods.

Qualitative procedures are a slightly different approach to scientific work than quantitative methods. In qualitative research, other research strategies are adopted, other methods for collecting the analysis and interpretation of the collected data are used. However, qualitative methods may, in a sense, be considered or act as the quantitative ones (Creswell, 2013, p. 189) due to used processes of schematization, table organization, ordering, and manual or computer data coding.

Validation of both procedures: qualitative and quantitative research in the field of social sciences does not diminish the role and popularity of the use of mixed methods (Creswell, 2013, s. 219). In particular, in the social sciences and health and human sciences, a quantitative or only qualitative study turns out to be insufficient. An interdisciplinary combination of both research methods provides a broader, in-depth and multilateral understanding of research problems.

Due to the growing independence of business entities operating in conditions of growing competition, it is necessary to introduce significant changes in the management procedures. A modern decision-making process requires the development of objective solutions that are derived from the results of the analysis of the reliability of information.

These solutions are often obtained through the use of mathematical modeling and appropriate quantitative methods, called mathematical methods, that allow rationalization of management in many areas of activity.

The complexity of the management process entails the indispensability of their wider use; the intuition and experience of decision-makers cease to be enough. The development of modern, innovative and effective management and planning is currently impossible without the use of statistical and mathematical methods supported by modern information technology.

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