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A linear programming DEA model for selecting a single efficient unit

Reza Akhlaghi[1] , Mohsen Rostamy-Malkhalifeh[2]

[1] *Department of Applied Mathematics, Islamic Azad University, Rasht, Iran*

[2] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran*

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Abstract

In recent years, several mixed integer linear programming (MILP) models have been proposed for finding the most efficient decision making unit (DMU) in data envelopment analysis. This paper introduces a new linear programming (LP) model to determine the most BCC-efficient decision making unit. Unlike previous models, which are not convex, the new model is linear programming and so that it can be solved efficiently to discover the most efficient DMU. Moreover, it is mathematically proved that the new model identifies only a single BCC-efficient DMU by a common set of optimal weights. To show applicability of proposed model, a numerical example which contains a real data set of nineteen facility layout designs (FLDs) is used.

1. Introduction

Data envelopment analysis (DEA) is a well-known operations research (OR) technique for evaluating the relative efficiency of a set of homogeneous decision making units (DMU). In DEA, DMUs can be assessed on the basis of multiple inputs and outputs, even if the production function is unknown. In this assessment, the efficiency measure of each DMU is a scalar ranging between zero and one. This nonparametric approach solves an LP formulation per DMU [5]. It assigns efficiency scores of less than 1 to inefficient DMUs and scores of strictly 1 to efficient
DMUs, thus, all of the efficient DMUs have the same efficiency score. This lack of discriminatory power has motivated numerous researchers to develop different ranking methods for use with DEA [3, 6]. Sometimes, DMU ranking is not a main concern. For example, in DEA applications such as robot selection [4, 9], flexible manufacturing system (FMS) selection [11], The results indicated that customer satisfaction and the amount customer expectations are met have maximum weight or priority in achieving efficiency among recognized factors [10] and computer numerical control (CNC) machine selection [12], what the decision maker (DM) is concerned about is the selection of the most efficient DMU, rather than DMU ranking. So, in these situations, there is no need to measure the performance of every DMU and a very practical way is to develop a model to find the most efficient DMU directly without evaluating the performances of the other DMUs. To provide more methodological and model options for the decision maker, we propose in this paper a new LP model for finding the most efficient DMU. In comparison with

*Corresponding author email address: mathe8877@gmail.com

the existing DEA models for finding the most efficient DMU, the proposed alternative LP model is, more practical and more reliable and contains only essential constraints and decision variables. The rest of the paper is organized as follows: Section 2 briefly reviews existing models for finding the most efficient DMU and points out their drawbacks. Section 3 proposes the new LP model. Section 4 compares the models mentioned and the capability of these models is illustrated using data from a real life problem. Conclusions are presented in the final section.

2. Existing models for finding the most efficient DMU

Suppose that there are n DMUs to be evaluated in terms of m inputs and s outputs. Denote by $x_{ii}(i = 1, \dots, m)$ and $y_{ri}(r = 1, \dots, s)$the input and output values of DMU$_j$ $(j = 1, \dots, n)$.To find the most efficient DMU, Amin and Toloo proposed the following integrated DEA model [2]

$$
\begin{aligned}
&\min M \\
&M - d_j \geq 0, \quad j = 1, \dots, n, \qquad\qquad (1)\\
&\sum_{i=1}^{m} w_i x_{ij} \leq 1, \quad j = 1, \dots, n, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1, \dots, n, \\
&\sum_{i=1}^{m} d_j = n - 1, \\
&0 \leq \beta_j \leq 1, \quad j = 1, \dots, n, \\
&d_j \in \{0,1\}, \quad j = 1, \dots, n, \\
&w_i \geq \varepsilon^*, \quad i = 1, \dots, m, \\
&u_r \geq \varepsilon^*, \quad r = 1, \dots, s,
\end{aligned}
$$

Where, ε^* is the optimal non-Archimedean epsilon. But Amin has shown that model (1) may sometimes obtain more than one efficient DMU [1]. To eliminate this drawback and to find a single most efficient DMU, he proposed an improved integrated DEA model, as shown below:

$$
\begin{aligned}
&\min M \\
&M - d_j \geq 0, \quad j = 1, \dots, n, \qquad\qquad (2)\\
&\sum_{i=1}^{m} w_i x_{ij} \leq 1, \quad j = 1, \dots, n, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j = 0, \quad j = 1, \dots, n, \\
&\sum_{i=1}^{m} \theta_j = n - 1, \\
&\theta_j - d_j \beta_j = 0, \quad j = 1, \dots, n, \\
&d_j \geq 0, \beta_j \geq 1, \theta_j \in \{0,1\}, \quad j = 1, \dots, n, \\
&w_i \geq \varepsilon^*, \quad i = 1, \dots, m, \\
&u_r \geq \varepsilon^*, \quad r = 1, \dots, s,
\end{aligned}
$$

It is claimed that the following nonlinear constraints ensure that only one of the deviation variables can take a zero value.

$$\begin{cases} \theta_j - d_j\beta_j = 0, & j = 1, \dots, n, \\ \sum_{i=1}^{m} \theta_j = n - 1. \end{cases}$$

But Foroughi has shown that model (2) may sometimes be infeasible [8]. To overcome the infeasibility of model (2), he introduced a new mixed integer linear model for selecting the best DMU, which is provided as follows:

$$\begin{aligned} & \max \ d \\ & \sum_{i=1}^{m} w_i x_{ij} \le 1, \quad j = 1, \dots, n, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + t_j + d \le 0, \quad j = 1, \dots, n, \\ & -\sum_{r=1}^{s} u_r y_{rj} + \sum_{i=1}^{m} w_i x_{ij} + t_j \le 1, \quad j = 1, \dots, n, \\ & \sum_{i=1}^{n} t_j = 1, \\ & t_j \in \{0,1\}, \quad j = 1, \dots, n, \\ & \{u_r\} \in U, \\ & \{w_i\} \in W, \end{aligned} \tag{3}$$

Where U and W are the sets of all acceptable weights, specified by the decision maker (DM). Model (3) produces the efficiency score of one for the most efficient DMU and efficiency scores of less than one for the other DMUs. Toloo and Nalchigar [13] proposed the following integrated MIP–DEA model to achieve the best BCC-efficient DMU:

$$\begin{aligned} & \min M \\ & M - d_j \ge 0, \quad j = 1, \dots, n, \\ & \sum_{i=1}^{m} w_i x_{ij} \le 1, \quad j = 1, \dots, n, \\ & \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1, \dots, n, \\ & \sum_{i=1}^{m} d_j = n - 1, \\ & 0 \le \beta_j \le 1, \quad j = 1, \dots, n, \\ & d_j \in \{0,1\}, \quad j = 1, \dots, n, \\ & w_i \ge \varepsilon^*, \quad i = 1, \dots, m, \\ & u_r \ge \varepsilon^*, \quad r = 1, \dots, s, \\ & u_0 is\ free. \end{aligned} \tag{4}$$

Wang and Jiang [14] proposed MILP model under constant returns to scale (CRS) is formulated as:

$$
\begin{aligned}
&\min \ \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} x_{ij} \right) - \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj} \right) \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq I_j, \qquad j = 1, \dots, n, \\
&\sum_{i=1}^{n} I_j = 1, \\
&I_j \in \{0,1\}, \qquad j = 1, \dots, n, \\
&u_r \geq \frac{1}{(m+s)\, max_j \{y_{rj}\}}, \qquad r = 1, \dots, s, \\
&v_i \geq \frac{1}{(m+s)\, max_j \{x_{ij}\}}, \qquad i = 1, \dots, m.
\end{aligned} \tag{5}
$$

Model (5) can never take a zero weight and can therefore make the best use of input and output information without the need of specifying any assurance regions for input weights and output weights. They also introduced the following input-oriented mixed integer linear model for selecting the most BCC-efficient DMU under variable returns to scale (VRS), which is provided as follows:

$$
\begin{aligned}
&\min \ \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} x_{ij} \right) - \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj} \right) - n u_0 \\
&\sum_{r=1}^{s} u_r y_{rj} + u_0 - \sum_{i=1}^{m} v_i x_{ij} \leq I_j, \qquad j = 1, \dots, n, \\
&\sum_{r=1}^{s} u_r y_{rj} + u_0 \geq 0, \qquad j = 1, \dots, n, \\
&\sum_{i=1}^{n} I_j = 1, \\
&I_j \in \{0,1\}, \qquad j = 1, \dots, n, \\
&u_r \geq \frac{1}{(m+s)\, max_j \{y_{rj}\}}, \qquad r = 1, \dots, s, \\
&v_i \geq \frac{1}{(m+s)\, max_j \{x_{ij}\}}, \qquad i = 1, \dots, m. \\
&u_0 \ is \ free.
\end{aligned} \tag{6}
$$

They also have shown that the use of MILP model (6) for finding the most BCC-efficient DMU under VRS is always feasible and can always find an optimal DMU.

3. Proposed model

In this section, we propose the following linear programming model to determine the most efficient unit:

$$
\begin{aligned}
&\min M \\
&M - d_j \geq 0, \qquad j = 1, \dots, n, \qquad (7) \\
&\sum_{i=1}^{m} w_i x_{ij} \leq 1, \qquad j = 1, \dots, n,
\end{aligned}
$$

$$\sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j = 0, \qquad j = 1, \dots, n,$$
$$\sum_{i=1}^{n} \theta_j = n - 1,$$
$$0 \le d_j \le \theta_j \le 1, \qquad j = 1, \dots, n,$$
$$\theta_j \le N d_j, \qquad j = 1, \dots, n,$$
$$w_i \ge \varepsilon^*, \qquad i = 1, \dots, m,$$
$$u_r \ge \varepsilon^*, \qquad r = 1, \dots, s,$$

Where ε^* is the optimal non-Archimedean epsilon, N is large enough number, d_j represents the deviation variable of DMU_j and M represents maximum inefficiency which should be minimized. First constraint implies that M is equal to maximum inefficiency. Second constraint shows input-oriented nature of the Model (7).

In the next theorem, we show that model (7) can be applied to obtain a single efficient DMU.

Theorem 1. Solving model (7) provides only a single efficient DMU.

Proof. Let $(M^*, d^*, w^*, u^*, \theta^*, u_0^*)$ be an optimal solution of model (7). If $d_k^* = 0$, then according to

$$\sum_{r=1}^{s} u_r^* y_{rk} - u_0^* - \sum_{i=1}^{m} w_i^* x_{ik} + d_k^* = 0,$$

DMU_k is efficient. On the contrary suppose that there exist no k such that$d_k^* = 0$. Let

$$e = min\{d_i^* | i = 1, \dots, n\}, dd_i^* = d_i^* - e, i = 1, \dots, n,$$

Therefore $(M^* - e, w^*, u^*, u_0^* - e, dd^*, \theta^*)$ is also feasible for (7), which is contradiction with the optimality of $(M^*, d^*, w^*, u^*, \theta^*, u_0^*)$ and so there exists at least one BCC-efficient unit. Also due to the constraints of model (7)

$$\begin{cases} \theta_j \le N d_j, & j = 1, \dots, n, \\ \sum_{i=1}^{n} \theta_j = n - 1, \\ 0 \le \theta_j \le 1, & j = 1, \dots, n, \end{cases}$$

In any optimal solution exactly one variable d_k^* can be equal to zero.

4. Application

In this section we indicate a numerical illustration to show the applicability of the proposed model. The numerical example contains a real data set of nineteen facility layout designs (FLDs) originally provided by Ertay et al. [7]and used in Amin and Toloo [2], Amin [1], and Toloo and Nalchigar [13]. Two inputs: cost and adjacency score, is used by each FLD to produce four outputs: shape ration, flexibility, quality, and hand-carry utility. This data are presented in Table 1. To find the most efficient FLD(s) given in Table 1 we apply the proposed model (3) with $\varepsilon = 0.000026$. Using this value and solving Model (3), DMU_{14} is identified as most BCC-efficient DMU. The comparison of results from Model (3) and other models is presented in Table 2.

Table 1. Inputs and outputs of DEA

DMU	DEA Inputs		DEA outputs			
	Cost	Adjacency score	Shape ratio	Flexibility	Quality	Hand-carry utility
1	20309.56	6405	0.4697	0.0113	0.041	30.89
2	20411.22	5393	0.438	0.0337	0.0484	31.34
3	20280.28	5294	0.4392	0.0308	0.0653	30.26
4	20053.2	4450	0.3776	0.0245	0.0638	28.03
5	19998.75	4370	0.3526	0.0856	0.0484	25.43
6	20193.68	4393	0.3674	0.0717	0.0361	29.11
7	19779.73	2862	0.2854	0.0245	0.0846	25.29
8	19831	5473	0.4398	0.0113	0.0125	24.8
9	19608.43	5161	0.2868	0.0674	0.0724	24.45
10	20038.1	6078	0.6624	0.0856	0.0653	26.45
11	20330.68	4516	0.3437	0.0856	0.0638	29.46
12	20155.09	3702	0.3526	0.0856	0.0846	28.07
13	19641.86	5726	0.269	0.0337	0.0361	24.58
14	20575.67	4639	0.3441	0.0856	0.0638	32.2
15	20687.5	5646	0.4326	0.0337	0.0452	33.21
16	20779.75	5507	0.3312	0.0856	0.0653	33.6
17	19853.38	3912	0.2847	0.0245	0.0638	31.29
18	19853.38	5974	0.4398	0.0337	0.0179	25.12
19	20355	17402	0.4697	0.0113	0.041	30.89

Table 2. Comparison results of proposed model with different models

variable	Model(7) New Model	Toloo and Nalchigar's Model [4]	Amin and Toloo's Model [2]	Amin's Model [1]	Foroughi's Mode [3]
d_j^*	$d_{14}^* = 0,$ $d_{j\neq 14}^* \neq 0$	$d_{14}^* = 0,$ $d_{j\neq 14}^* \neq 0$	$d_{16}^* = 0,$ $d_{j\neq 16}^* \neq 0$	$d_{16}^* = 0,$ $d_{j\neq 16}^* \neq 0$	$d_{10}^* = 0,$ $d_{j\neq 10}^* \neq 0$
w_1^*	0.000026	0.000025	0.000026	0.000026	0.00004485
w_2^*	0.000026	0.000028	0.328456	0.00002	0.000005
u_1^*	0.247648	0.556423	0.000026	0.146	1.17219727
u_2^*	2.969236	0.860412	0.020036	0.151	1.45114046
u_3^*	0.000026	0.000025	0.000027	0.000026	0.43580088
u_4^*	0.010426	0.026153	0.000026	0.0176	0
u_0^*	0.007299	0.458908	------	------	------

According to the table 2, the single efficient unit of our linear model and the non-linear modified Toloo model of the two units (DMU) number 14 is the same with regard to the weights almost the same results. It has been not noted that in other models ranking is also done, but our new model gets one-time solving

5. Conclusion

The identification of the most efficient DMU is sometimes the main concern of decision makers. In this paper, we have proposed a new LP model for finding the most BCC-efficient DMU. Using the proposed model, decision maker is able to find most BCC-efficient DMU by solving only one LP. To illustrate the model capability it has been applied to a real data set consisting of 19 FLPs and has compared with earlier models to determine the most efficient DMUs in DEA.

Compared to the nonlinear models compared in this paper, our continuous linear model determines the efficiency of a single-unit load model. Our continuous linear model has the capability to declare only one unit, but in non-linear models of the other Effective units should be ranked. The number of constraints in our model is less than the nonlinear models mentioned in the paper.

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