International journal of industrial engineering and operational research (IJIEOR)
Volume 1, ISSUE 1
www.ijieor.bgsarman.ir ISSN:

Optimization of Minimum Spanning Tree and Traveling Salesman Problems Arising in a University Campus Network

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Article info:

Received: 2018/12/28
Revised: 2018/12/31
Accept: 2018/12/31

Keywords:
Binary integer Programming, Campus Network, Minimum Spanning Tree, Traveling Salesman Problem.

Abstract

The network problem of a university campus is investigated in this study. The facilities such as departments, administration buildings, dormitory and etc. on the network are considered as the nodes. The aims of this study are twofold: finding the shortest distance which visits all nodes and finding the minimum path which connects all nodes. To achieve the aforementioned goals, the problem is modeled as a traveling salesman problem (TSP) and minimum spanning tree (MST), respectively. Two different binary integer programming models are developed and applied to the case study of Gaziantep University campus network with 72 nodes. The results of the models provide the feasible route to visit all nodes and the feasible path which interconnects every node pair of.

1. Introduction

Graph theory is an important branch of operations research and its aim is to study how to plan and control network systems effectively [1]. The minimum distanced route which visits all nodes and the minimum path which interconnects all nodes in a network are one of the most popular issues need to be solved. The former situation should be modeled as TSP, whereas the latter should be modeled as MST problem [7]. Although the aforementioned two problems seem very similar, they are quietly different at one point. While the MST problem seeks to build a tree that connects all nodes and has minimum total weight, the TSP searches to find a trip that visits all nodes with minimum total weight (and coming back to the starting point) [10]. Due to the NP-hard nature of TSP and MST, the real case applications of these two problems are uncommon.

One of the real case applications of TSP is studied by Murray and Chu [14]. They solved a case of drone-assisted parcel delivery using TSP formulation. Then, Groba et al. [6] applied TSP for the recovery of fish aggregating devices by tuna vessels. In health services, Anderson et al. [2] minimized the route for the kidney paired donation using TSP. Later, real-sized in-port routing and scheduling problems in chemical shipping are solved as a TSP by Arnesen et al. [3] and finally, Yapicioglu [19] implemented the TSP to an exam booklet distribution problem with 110 nodes.

In addition to the real cases of TSP above, there have been some studies which applied MST algorithm to the real case problems. As one of the oldest researches, Hill [8] showed how a comparison of price levels across a group of countries can be made by chaining bilateral price indexes across a spanning tree. It is argued that they should use the MST algorithm whose resulting multilateral price indexes are least sensitive to the choice of bilateral formula. Later on, AssunÇão et al. [4] presented an efficient method for regionalization. The cost of each edge in the graph is inversely proportional to the similarity between the regions it joins. They applied MST to summarize the neighborhood structure. Afterwards, Mahdavi et al. [12] constructed an optimal natural gas distribution network among stations and consumers using MST technique. Their aim is to determine both locations and types of stations minimizing location-allocation costs in the network. As a manufacturing application, Vasseur et al. [17] proposed a MST methodology to qualify the spatial intensity distribution of a laser beam on a rough surface. In the renewable energy area, Hou et al. [9] developed a model based on MST algorithm to optimize the cable connection layout of large-scale offshore wind farms. The objective is to minimize the production cost of an offshore wind farm by optimizing the cable connection configuration. Finally, the theory of MST was used by Xu [18] to develop for the optimal layout of canals and ditches.

Differently from the aforementioned studies, a campus network which includes 72 nodes (dormitory, departments, administration, sports center and etc.) are modeled as TSP and MST. The aim of TSP application is finding the minimum route which visits all facilities within the campus. This information should be used for student/staff service planning or security audit. The aim of MST approach is calculating the minimum path which interconnects all nodes each other. The result of MST should be used for the electricity, heating or internet infrastructure of the campus.

In the next section, the mathematical formulations of TSP and MST models are given. Then, the case of Gaziantep University campus is introduced and results are discussed in the third section. The paper is concluded with future directions in the last section.

2. Mathematical Models

In this section, the mathematical formulations of two well-known combinatorial optimization problems are described.

2.1. Traveling salesman problem

TSP is defined on a complete graph as TSP $G=(N, A)$, where $N=\{n_1, n_2, \ldots .n_n\}$ is a set of n vertices (nodes) and $A= \{(n_i, n_j)\ l\ n_i, n_j \in N, i\neq j\}$ is a set of arcs, together with a non-negative distance matrix $D=(d_{ij})$ associated with A. The problem is considered to be symmetric if $d_{ij}=d_{ji}$ for all $(n_i, n_j) \in A$, and asymmetric otherwise [17]. In this study symmetric TSP structure is considered. In the TSP, every city must be visited and all vehicles depart from a city and return to the starting city. The mathematical model of TSP is given below [5]:

$$minimize \sum_{i,j \in N} d_{ij} x_{ij} \quad (1)$$

$subject\ to$

$$\sum_{j \in N} x_{ij} = 1 \qquad \forall_{i;i \neq j} \quad (2)$$

$$\sum_{i \in N} x_{ij} = 1 \qquad \forall_{j;i \neq j} \quad (3)$$

$$U_i - U_j + N x_{ij} \leq N - 1 \qquad \forall_{i,j=2,3,4 \ldots N; i \neq j} \quad (4)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in A \quad (5)$$

where d_{ij} is the distance between i and j, x_{ij} is 1 if there is a visit from i to j. The objective function (1) minimizes the total distance of route which visits all nodes. Eqs. (2) and (3) ensure that each city must be departed and visited exactly once. Eq. (4) is used to prevent sub-tours, which are degenerate tours that are formed between intermediate cities and not connected to the origin. Finally, Eq. (5) enforces binary restrictions on the decision variables.

2.2. Minimum spanning tree problem

The MST problem is a well-known problem in network optimization, with a wide range of applications in communication, transportation, and computer networks. The standard MST problem is stated as follows: given a weighted graph whose vertices might represent cities and whose edges serve as possible communication links with edge weights representing the cost of building a link or the length of the link, the aim is to select a set of communication links that would connect all the vertices such that the tree has the minimum total weight [11]. In our problem, the cities (nodes) are considered as buildings in the campus. The mathematical model of MST problem is given below [15]:

$$minimize \sum_{i,j \in N} d_{ij} x_{ij} \quad (6)$$

$subject\ to$

$$\sum_{i,j \in N} x_{ij} = n - 1 \quad (7)$$

$$\sum_{i,j \in N: i \in S, j \in S} x_{ij} \leq |S| - 1 \qquad \forall S \subseteq V \quad (8)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in N \quad (9)$$

where d_{ij} is the distance between i and j, x_{ij} is 1 if edge i and j is in the tree T. The objective function (6) minimizes the total distance of the tree which reaches all nodes. Eq. (7) ensures that there must be $n - 1$ edges in the tree. Eq. (8) is the sub-tour elimination constraint. Any subset of n vertices must have at most $n - 1$ edges contained in that subset. Finally, Eq. (9) is the binary restriction constraint.

3. The Case of Gaziantep University Campus

The two mathematical models which are given in the previous section are applied to the campus network of Gaziantep University. University campus is located in southwestern Gaziantep and shown in Figure 1. The road network of campus is shown with red in Figure 1. University has almost 55,482 population including students, administrative and academic staff on an area with 3,113,084 m^2 [13].

Figure 1. Map of Gaziantep University campus

The considered nodes (department buildings, administration offices, dormitories and etc.) are given in Table 1 and illustrated in Figure 2. All facilities on the campus are taken into consideration.

Table 1. 72 nodes in Gaziantep University campus

#	Name	Abbrv.	#	Name	Abbrv.
1	Department of Electric & Electronics Engineering (B Block)	A	**37**	Girls' Dormitory (B Block)	L1
2	Department of Industrial Engineering	B	**38**	Mosque	M1
3	Department of Mechanical Engineering (B Block)	C	**39**	Congress & Art Center	N1
4	Central Classroom (A Block)	D	**40**	Football Field Carpet	O1
5	Department of Electrical & Electronic Engineering (A Block)	E	**41**	Amphitheater	P1
6	Department of Mechanical Engineering (A Block)	F	**42**	Technopark	R1
7	Data Processing Center	G	**43**	Gaziantep Provincial Directorate	S1
8	Engineering Department Headquarter	H	**44**	Restaurant	U1
9	Department of Civil Engineering	I	**45**	Bank	V1
10	Faculty of Arts & Sciences	J	**46**	Hospital	W1
11	Department of Conservatory	K	**47**	Faculty of Architecture	X1
12	Cultural Center	L	**48**	Faculty of Communication	Y1
13	Directorate of Construction & Technical Works	M	**49**	Faculty of Dentistry	Z1
14	Central Classroom (B Block)	N	**50**	Dean of Medicine Faculty	A2
15	Vocational School of Technical Sciences (A Block)	O	**51**	School of Physical Education & Sports	B2
16	Vocational School of Technical Sciences (B Block)	P	**52**	Academic Housing	C2
17	Office of Student Affairs	R	**53**	Faculty of Economic & Administrative Sciences	D2
18	Rectorate Building	S	**54**	Department of Biology	E2
19	Ataturk Monument	T	**55**	Turkish Employment Agency	F2
20	Department of Physic, Optics & Acoustic Engineering	U	**56**	Energy Management Center	G2
21	Department of Textile Engineering	V	**57**	Bazaar	H2
22	Central Classroom (C Block)	W	**58**	Culture Center	I2
23	Faculty of Education	X	**59**	Department of Health Culture	J2
24	Faculty of Theology	Y	**60**	School of Physical Education & Sports Hall	K2
25	Faculty of Law	Z	**61**	Civil Engineering Laboratory	L2
26	Faculty of Fine Arts	A1	**62**	Central Business Center	M2
27	English Prep School (A Block)	B1	**63**	Plant Management Directorate	N2
28	English Prep School (B Block)	C1	**64**	Directorate of Vehicle Management	O2
29	Department of Food Engineering	D1	**65**	Multidisciplinary Research Center	P2
30	Food Engineering Laboratory	E1	**66**	Vocational School of Social Sciences	R2
31	Library	F1	**67**	Vocational School Workshops	S2
32	Dining Hall	G1	**68**	Cafeteria	T2
33	Staff Dining Hall	H1	**69**	Vocational School of Health Services	U2
34	Faculty of Aeronautics & Astronautics	I1	**70**	Medical School Annex Building	V2
35	Sports Center	J1	**71**	Institute of Social Sciences	W2
36	Girls' Dormitory (A Block)	K1	**72**	Mental Health Center	X2

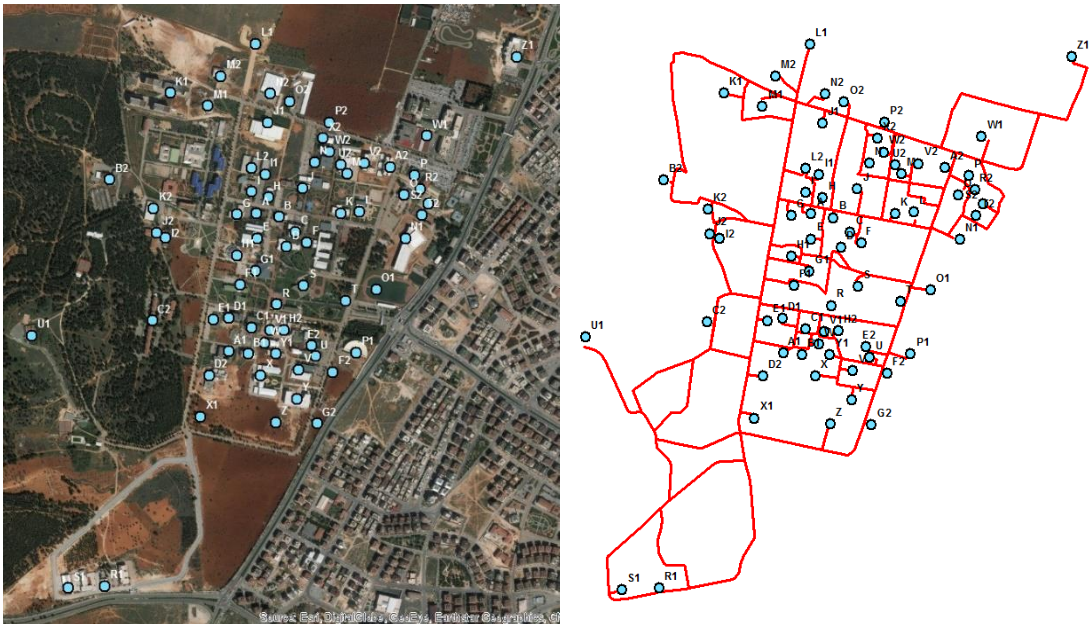

Figure 2. Satellite image of campus (left) and road network (right) of the campus

In the study area, road network of the campus is vectored as line features via geographic information system (GIS) with ESRI ArcGIS 10.2 software. Two kinds of GIS data which are facilities (nodes) as a point layer and roads as a line layer are used. Road data is also used as network data set in GIS environment. For this reason, at first, university road map is collected as line data. Then, line-shape road layer is used to generate network between all nodes. The distances (d_{ij}) between all nodes (72x72 matrix) obtained by GIS are available at http://ibs.gantep.edu.tr/duyuru/files/articles/72x72-data-web12669.xlsx.

3.1. Application of location-allocation models

In this section, the models of TSP and MST are applied to the network of Gaziantep University campus. It must be noted that all runs were completed on a server with 1.8 GHz Intel Core processor and 4 GB of RAM. LINGO 18.0 optimization package program is used to solve the models. Figure 3 indicates the network design (with 72 nodes) which is used in the models. While the circles in Figure 3 indicate the nodes, the numbers on the arcs represent the distances (meters) between nodes.

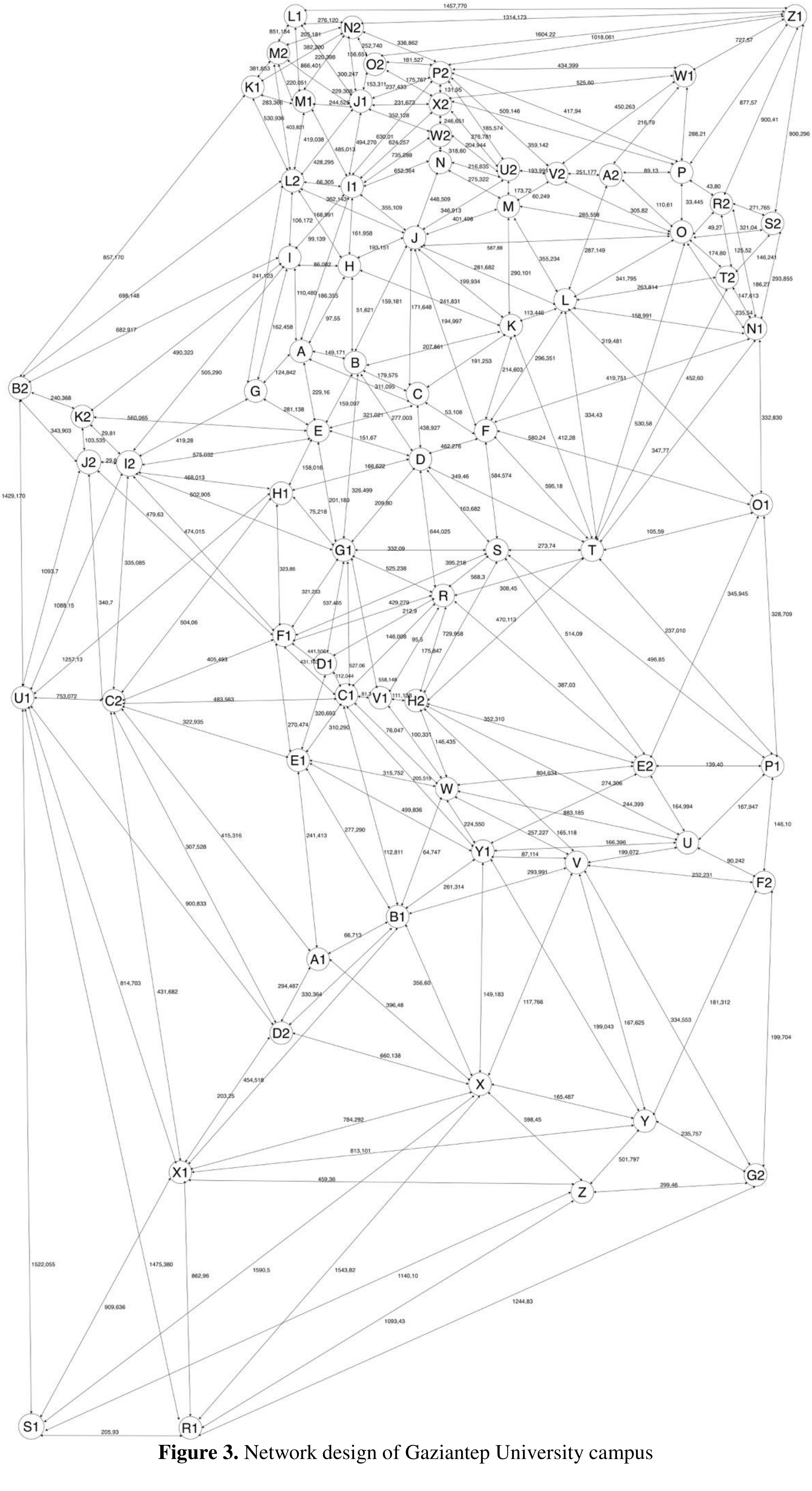

Figure 3. Network design of Gaziantep University campus

3.1.1. Results of traveling salesman problem

The mathematical formulation of TSP (Eqs. 1 to 5) is run using the data described in the previous section. The TSP model includes 5184 binary variables and 5328 constraints. Due NP-hard nature of the problem, the computation time is limited with 3 hours. Obtained feasible solution includes an upper bound with 16,327.70 meters and a lower bound with 15,749.60 meters. Therefore, the gap is only 3.54% between the bounds. The route is: A→G→B2→K2→J2→I2→C2→F1→E1→D2→X1→R1→S1→U1→Z→G2→Y→X→V →Y1→W→B1→A2→D1→C1→V1→R→H2→U→F2→E2→P1→T→O1→S→D→H1→ G1→E→H→B→J→F→C→K→L→N1→T2→S2→W1→Z1→A2→R2→O→P→V2→M →U2→W2→P2→N→X2→O2→J1→N2→L1→M2→M1→K1→L2→I1→I→A. The feasible route which visits all nodes is illustrated in Figure 4. The total distance in Figure 4 is the upper bound which is 16,327.70 meters.

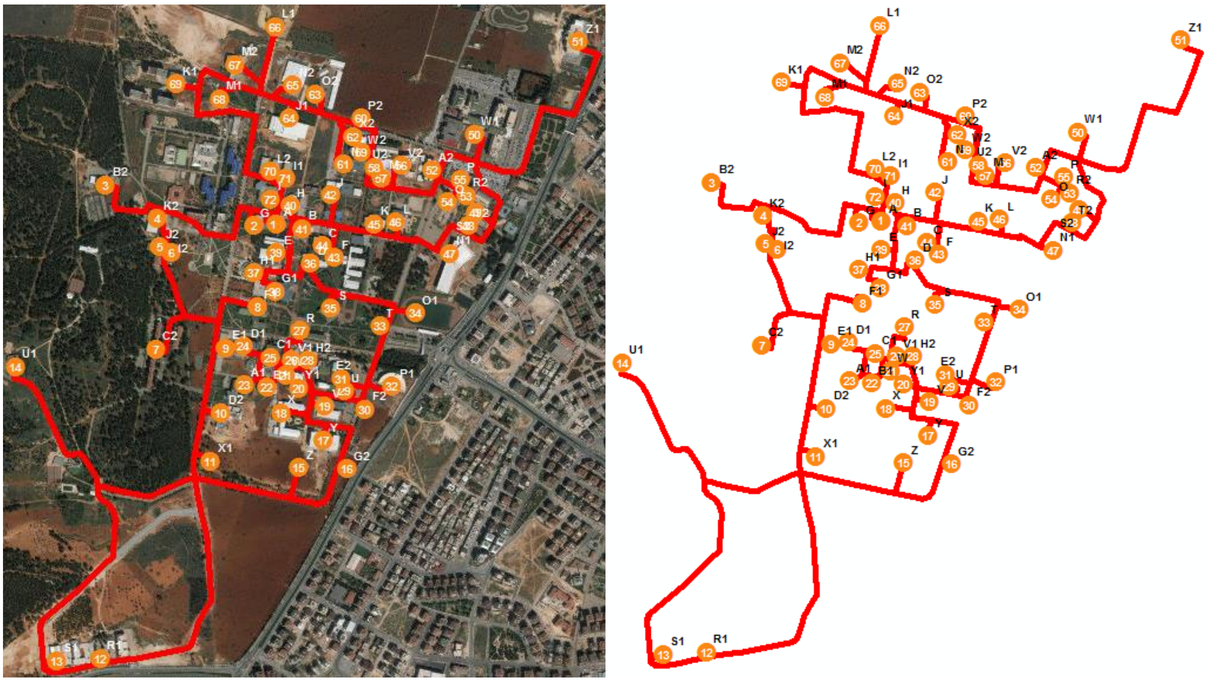

Figure 4. The feasible solution of TSP

The feasible route of TSP is also shown as a network in Figure 5. According to TSP solution, the route is started at Department of *Electric & Electronics Engineering – B Block* (A) then visits *Data Processing Center* (G), *School of Physical Education & Sports* (B2) and ends at *Electric & Electronics Engineering – B Block* (A).

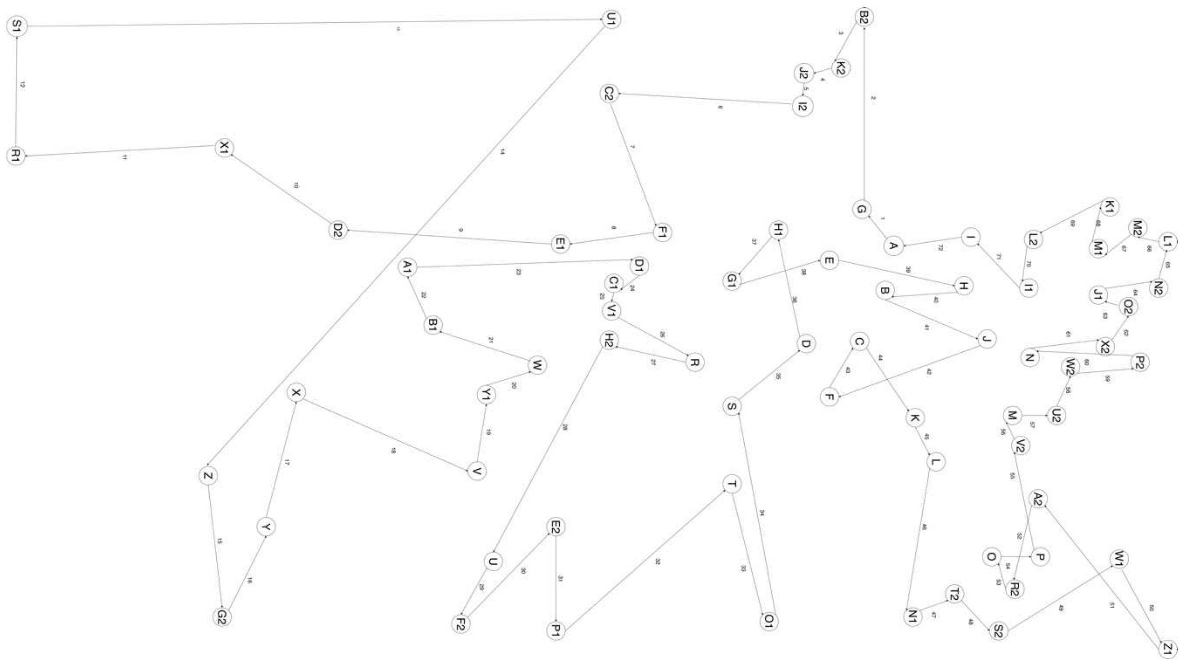

Figure 5. The feasible route of TSP on campus network

3.1.2. Results of minimum spanning tree

After showing the total distance which visits all nodes in the campus, we implemented MST model to find the total line distance which connects all nodes together. Therefore, the mathematical formulation of MST is applied to the case study. The MST model includes 5256 binary variables and 5114 constraints. As the TSP model, the computation time is limited with 3 hours. The minimum length spanning tree which connects all nodes together is found as 12,637.31 meters. The feasible spanning tree is represented in Figure 6.

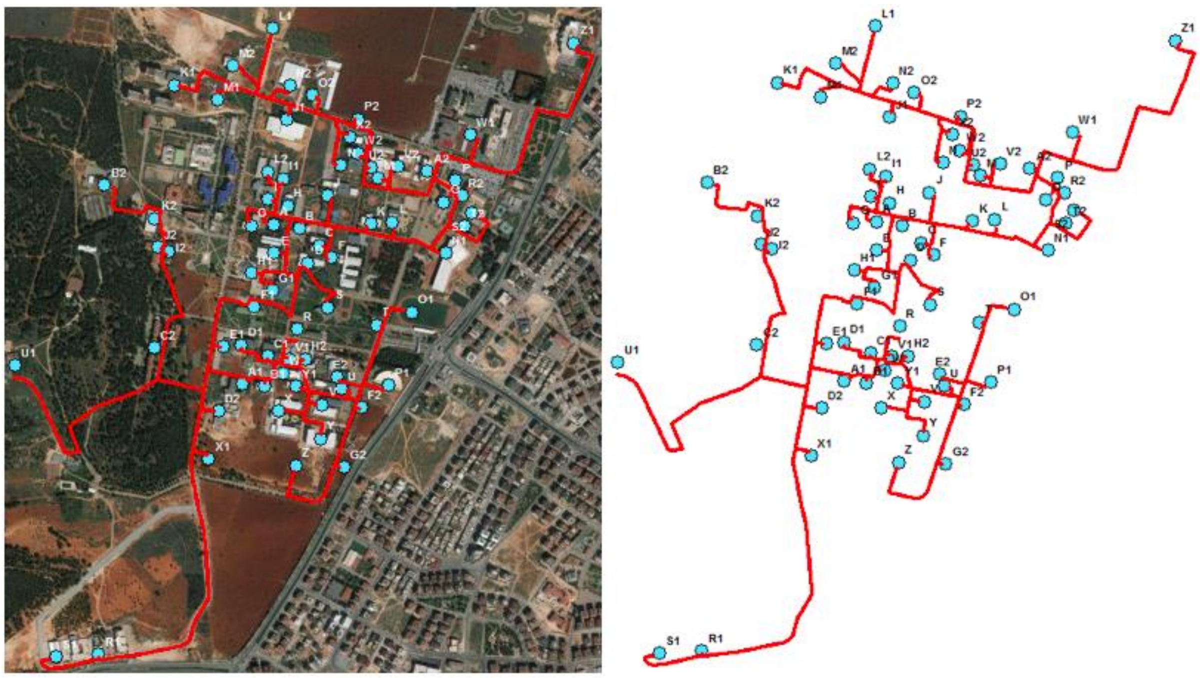

Figure 6. The feasible solution of MST problem

The feasible solution of MST as a network is also illustrated in Figure 7. As can be seen Figure 7, all nodes are connected each other and there is no loop.

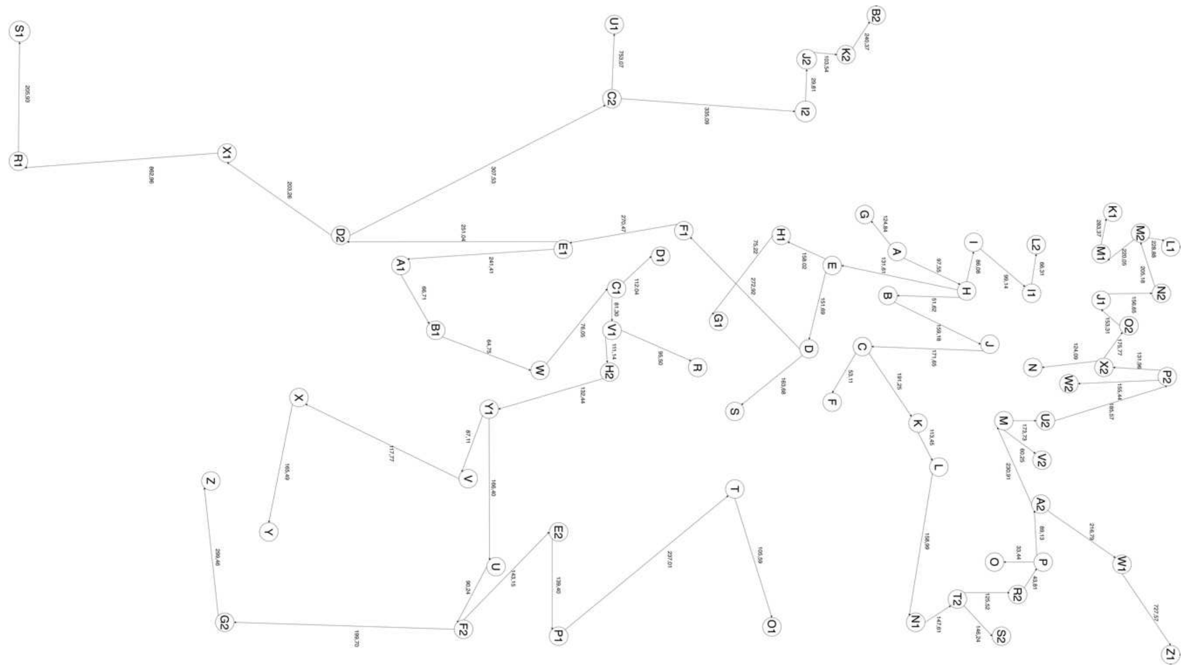

Figure 7. The feasible spanning tree of campus network

4. Conclusion

In this paper, the problems of traveling salesman and MST are applied to the campus network of Gaziantep University. To the best knowledge of the authors, it is one of the first studies which apply an arc based network design model to a university campus. With the applied two models, both the total distance which visits all nodes and the minimum length spanning tree which interconnects the nodes together are obtained. These solutions can shed light in transportation between facilities and also in communication infrastructures if the need arises.

The other well-known combinatorial models such as shortest path, maximum flow or vehicle routing problems should be tested on the problem introduced in this paper. As mentioned in the application section, the solution time of two models is limited with 3 hours due to NP-hardness of the problems. Thus, heuristic and meta-heuristic approaches should be applied to get acceptable solutions in a reasonable time.

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