Розглянуто особливості процесу визначення координат звукових аномалій за даними звукових рядів. Показано, що звукові аномалії є джерелом інформації про події, явища, що вже відбуваються, чи є їх передвісниками. Означено, що системи прослуховування є доповненням до тепловізорів, та при комплексному використанні з урахуванням переваг, які досягаються при використанні безпілотних літальних апаратів, забезпечують економію фінансових та людських ресурсів. Викладено методи, що дозволяють вирішувати задачу спостереження та прогнозу шляхом знаходження координат звукових аномалій. Запропоновані непрямі методи розв'язку задач пошуку координат звукової аномалії для трьох мікрофонів за лінійною схемою наближення та лінійною і квадратичною апроксимацією. Розв'язки доведено до аналітичних завершених виразів, які дозволяють проводити розрахунок координат за вхідними умовами для трьох або чотирьох мікрофонів. Також поставлено та розв'язано прямими методами задачу пошуку координат звукової аномалії для трьох та чотирьох мікрофонів. Представлено розв'язки як вирази, що дозволяють обчислювати координати звукової аномалії. Проведено чисельні експерименти, в ході яких обчислювались координати звукових аномалій, абсолютна похибка їх визначення на кожній ітерації та загальний час, що витратився на розрахунок. Продемонстровано, що найбільшу похибку мають системи, у яких координати мікрофонів і джерел звуку або практично однакові, або співпадають. За цих умов для прямих методів значення коефіцієнтів рівнянь зменьшуються практично до нуля або обертаються у нуль, а різниця значень шуканих координат між ітераціями різко зростає, що гальмує процес збіжності розв'язків. Показано, що застосування до пошуку координат наближених методів, шляхом розв'язку задач мінімізації із залученням методу рекурентної апроксимації, дозволяє будувати прості алгоритми. Їх реалізація для розв'язку задач чисельних експериментів дає швидкі та практично точні значення координат. Встановлено, що застосування до побудови алгоритмів методів логічного аналізу та правил логічного висновку зменшує кількість ітерацій та загальний час розрахунку

Ключові слова: звукова аномалія, функціонал, рекурентна апроксимація, аналітичні розв'язки, числовий експеримент

UDC 519.6: 629.7-519
DOI: 10.15587/1729-4061.2020.201103

FORMING A METHOD FOR DETERMINING THE COORDINATES OF SOUND ANOMALIES BASED ON DATA FROM A COMPUTERIZED MICROPHONE SYSTEM

A. Trunov
Doctor of Technical Sciences, Professor,
Head of Department*
E-mail: trunovalexandr@gmail.com

Zh. Byelozyorov
Postgraduate Student*
E-mail: zhan.byelozyorov@gmail.com

*Department of Automation and Computer-Integrated Technologies
Petro Mohyla Black Sea National University
68 Desantnykiv str., 10,
Mykolaiv, Ukraine, 54003

Received date 18.02.2020
Accepted date 20.04.2020
Published date 30.04.2020

1. Introduction

Advantages of integrated video and audio surveillance and the new technical capabilities of sound anomaly recording open up new prospects for widespread monitoring including emergency automated management systems (AMS) [1].

Sound anomalies found in surveying from drones are a source of additional meaningful information about events or phenomena [2] that are occurring or are precursors of subsequent events [3]. In the city, steppe, mountainous or forested areas, sound anomalies are sometimes the only source of information [4]. In this regard, recording, storage, and analysis of audio information [5] are of interest for a systemic collection, processing by various institutions, including government institutions and dual-purpose services [6]. Lack of financial and human resources is one of the reasons explaining the growth of scientific and applied interest in designing computerized systems for the collection and processing of audio information [7, 8]. However, further design and development of applied systems for recording, recognizing, documenting and mapping sound anomalies and constructing information flow AMS [9, 10] should be preceded by a process of theoretical analysis and a search for new theoretical bases and patterns of calibration and training [11, 12]. The methods of collecting, rapid processing, and recognition of potentially dangerous events or preceding events evidenced by sound anomaly have become a topical subject of prospective scientific studies [13, 14].

However, innovative development of water economy, problems connected with gamekeeper control, unlicensed lumbering and hunting, road accidents, fires, disturbance of silence, and efficiency of corresponding services depend directly on accuracy and speed of position determination [15]. The solution of the latter and construction of models capable of providing valid and accurate information on sound anomaly coordinates for AMS operation has become a subject of recent studies [16]. In this regard, the development of an effective method of coordinate determination which will extend the functionality of such services and improve the efficiency of their work is an urgent task. Undoubtedly, the latter can be achieved through the technical improvement of tools including the improvement of methods, models, and algorithms of their work [15–17].

At present, the construction of models and development of methods for determining coordinates required for the sound anomaly recording systems of the cannon sound reconnaissance requiring immediate development and improvement in view of the real actions and threats from the aggressor is the problem of particular urgency [18].

2. Literature review and problem statement

Recently, data about new possibilities of technical means of recording acoustic waves propagating in the air are increasingly reported in the literature [1]. Sensitivity and resolution of such systems suggest that terrestrial audio interception systems, such as Trembita M, will replace or informationally supplement thermal imagers and night vision devices [1]. Also, according to [5], innovative approaches to building systems of switching sensors for sensory processing which have previously been tested in adaptive robotic means of flexibly restructuring enterprises [7] are further developed today. The proposed approach based on programmable logic controllers (PLC) [19] is well adaptable for the case when multiple sensors of the same type are switched to the processing module [20]. However, the advantage of the new proposed systems [19] implies the possibility of further expansion in accordance with requirements to the quality of sensors and realization of tasks in *VHDL* language which is implemented in various PLC procedures. Thus, the technical preconditions permit further innovative improvement of both fixed and mobile systems operating on the basis of a mobile device in cooperation with unmanned aerial vehicles (UAV) [15]. As shown in [21], an increase in the number of sound receivers through the use of UAV swarms can expand the technology capabilities. At the same time, it was also proved that such a solution further increases the number of problems to be solved [21]. The method presented in [22] makes it possible to choose UAV operation zones for effective solution of surveillance and forecasting problems. However, the influence of the features of the present-day formation and terrain topology complicates their solution [22]. Application of the multiagent event isolation method [23] based on multicharacter sets and use of machine learning algorithms, obviously, can be further implemented after additional improvements to solve problems of surveillance and forecast of sound anomalies [24]. Accuracy of recording the sound anomaly, separating it from superimposed signals, and recording the start and end time of a single event remains the task of improving means, algorithms and sensors of UAV [15, 20].

The idea of propagation of a spherical acoustic wave regardless of the sound origin, be it sound from a sniper position or explosion of a cannon projectile from a closed position was used in [25]. There are studies in which bursts of self-propelled self-homing missiles are considered as sources of spherical acoustic waves propagating at a constant speed [26] in conditions of imposing sounds of other events. The latter is interference in describing processes and construction of models and algorithms of determining coordinates of the source of sound anomaly [27]. The task of determining coordinates in the context of excessive information shows that, regardless of the binaural effect, the problem of functional minimization is solved by direct or numerical methods. However, the use of the fastest descent method provided convergence regardless of the choice of initial conditions but the Newton method did not do this. Such conclusions can be explained by the presence of irrationality and not simple roots in which denominator of the recurrent formula tends to zero and the initial functional oscillates. As shown in [15], the necessity of recording the sound anomaly in such a model of the phenomenon requires an increase in the number and location of several spatially separated microphones [28]. By its essence, this technical solution makes the functional uniquely positive for all microphones. However, the location of microphones on quadrocopters generates an error in coordinates because of the error in the microphone coordinates. Influence on the error of measuring the target angle was studied in [28] but the algorithm given there did not present a method of correcting the methodical error that arises. The methodological error determination algorithm proposed in [29] makes it possible to estimate it under the conditions of a simplified model and accurate solutions available. Thus, for the tasks in which initial approximation is selected from the conditions of the expected events that requires for its use equipment that uses a binaural effect, it will also require refinement of both the model with a functional and a solution method. Search for and implementation of design solutions that realize the binaural effect [29] requires prediction of the event direction which cannot be predicted for civilian tasks. Under these conditions, the problem of determining the coordinate of sound anomaly is usually reduced to a problem of either minimizing the functional [30] or solving the system of nonlinear algebraic equations [31].

Theoretical generalizations of the studies devoted to the use of direct and indirect methods of solving the problem of determining coordinates of existing models are presented in [32]. Analysis of the studies [30–32] shows that the operation of square-rooting both for indirect gradient methods [30] and the methods of quadratic programming with constraints [31] causes ambiguity. A comparison of the results for direct and indirect methods presented in the applied mathematics study [32] just confirmed the assumption of non-singularity. The latter is a major cause of oscillations between solution iterations and, as a consequence, an increase in time, and discrepancies for some conditions.

Thus, the presence of a square root as a source of non-singularity is a major not solved problem and therefore the first reason for searching for and studying other forms of representation of initial mathematical models.

Further development of systems for management of emergencies, traffic on highways, etc., requires joint application of geoinformation systems and mapping tools which was substantiated and presented in [16] as a field of the current study. The contradiction between mapping systems that work with a continuous scale and the numerical methods of finding coordinates [30–32] using finite increments is about averaging of the first and second derivatives. The continuity requirement creates the need for interpolation and as a consequence generates an error.

Thus, the requirement of continuity of functions and their derivatives and discreteness inherent in numerical methods contradict. The latter is the second major reason for finding effective applied mathematical methods for determining coordinates of sound anomalies. However, the practical implementation of deterministic models for solving the mapping problem is also a problem of completeness and solution of the quadratic equation system. As substantiated and demonstrated in [33], its simplification implies a recurrent approximation of operators which will be effective for continuous vector functions or algebraic nonlinear items of differential models [34]. Search for the ways that can provide fast calculations or represent values of physical quantities by means of analytic membership functions is demonstrated in [35]. The advantages of adding capabilities of the apparatus of mathematical analysis demonstrated for management problems are convincing in its reality [36] in the presence of analytical solutions. In addition, its application requires additional numerical and physical experiments [37] and a special assessment of the possibilities of providing the specified accuracy and magnitude of errors. Another way to simplify a mathematical problem by applying indirect methods consists in reducing it to a recurrent sequence that allows rapid calculations [38]. However, for its practical application, this methodology requires a concerted action: the reformation of the model, proposal of new analytical methods and algorithms, and study of their convergence in the course of numerical experiments which is an unrelated task as well.

3. The aim and objectives of the study

The study objective is to develop a method that makes it possible to determine coordinates of a sound anomaly with a given accuracy.

To achieve this objective, the following tasks were set:

– to construct a mathematical model that relates coordinates of sound anomaly according to the data of sound series to such parameters as the speed of sound propagation in air, the distance between microphones and their number and location;

– to construct an approximate analytic solution of the problem of finding coordinates of a sound anomaly for three microphones using a scheme of linear approach and a linear and quadratic approximation;

– to substantiate the algorithms of search by methods of indirect and direct solution of the problem of calculating the sound anomaly coordinate for the model form with a selection of three and four microphones;

– to conduct a numerical experiment to determine coordinates of a sound anomaly with a specified accuracy.

4. Model construction and statement and solution of the problem of determining coordinates of sound anomalies

4. 1. Description of the computerized microphone system and a model of determining coordinates of sound anomalies based on sound series

Let us consider a system consisting of three or four microphones and controllers. The system is capable of receiving and recording in time audio signals previously converted into electrical signals. Suppose that the system is provided with channels for receiving and transmitting data from a controller of each microphone to the processor. The latter generates frame data for a protocol storage, prior to forming a series from all microphones. According to a command formed according to the algorithm, it inquires, processes and saves the processed data in the time of recording the sound anomaly by individual microphones. Denote the source of sound anomaly and the sound receiver by letters c and r, respectively, in the subindex. Also, denote the speed of the sound wave propagation via v_n. Under these conditions, denote the point and coordinates of the signal receiver $M_j(x_{rj}, y_{rj}, z_{rj})$ in the Cartesian coordinate system. Suppose that coordinates of the microphone on the j-th vehicle, that is, UAV or at a stationary surveillance station are defined at this point. Denote by t_{ci} and t_{rj}, respectively, the time of occurrence and recording of the sound anomaly arising at the point C_i with coordinates of the location (x_{ci}, y_{ci}, z_{ci}) and recorded by the microphone at point $M_j(x_{rj}, y_{rj}, z_{rj})$ in the same time recording system. Suppose that the sound wave is spherical and its front is recorded as an instantaneous impulse. Under these assumptions and notations, reduce the original mathematical model to a system of equations which essentially are conditions of equality of distance between the source of sound anomaly and the microphone which are determined by their coordinates and the path passed by the spherical wave to that microphone. For example, write for the case of use of three microphones:

$$\begin{cases}(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\ +(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2=0;\\ (x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\ +(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2=0;\\ (x_{r3}-x_{ci})^2+(y_{r3}-y_{ci})^2+\\ +(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2=0.\end{cases} \quad (1)$$

Such a system contains three unknown coordinates and a fourth unknown, that is, the time of sound anomaly appearance. In essence, it can be considered as a problem with a parameter or requiring the search for an additional fourth equation.

4. 2. Approximate analytical solution of the problem of finding coordinates of the sound anomaly for three microphones

The formed system (1) is nonlinear, search for its solution is not obvious and unique. Let us explore possible ways of the search for its solution and analyze and compare them. To construct the approximate method, form a functional $F(x_{ci}, y_{ci}, z_{ci}, t_{ci}, t_{rj})$, from the system (1) equations as a sum of deviation squares:

$$F(x_{ci}, y_{ci}, z_{ci}, t_{ci}, t_{rj})=\sum_{j=1}^{m}\left\{\begin{array}{l}(x_{rj}-x_{ci})^2+(y_{rj}-y_{ci})^2+\\ +(z_{rj}-x_{ci})^2-\left[v_n(t_{rj}-t_{ci})\right]^2\end{array}\right\}^2. \quad (2)$$

The items under the sum sign in the functional expression are always positive because they are squares of real numbers, that is, the values of coordinates. In addition, the error in determining coordinates is a bounded quantity determined by the problem content. Denote it by $[\Delta l]$. Thus, formulate the original problem (1) as a problem with a minimization parameter with constraints:

$$\min_{x_{ci}, y_{ci}, z_{ci}} F(x_{ci}, y_{ci}, z_{ci}, t_{ci}, t_{rj}, m);$$

$$(\Delta x_{ci})^2 \le [\Delta l]^2;\ (\Delta y_{ci})^2 \le [\Delta l]^2;\ (\Delta z_{ci})^2 \le [\Delta l]^2. \quad (3)$$

Record the required condition of minimum for all sought coordinates of a sound anomaly:

$$\begin{cases} \dfrac{\partial F}{\partial x_{ci}} = 0; \\ \dfrac{\partial F}{\partial y_{ci}} = 0; \\ \dfrac{\partial F}{\partial z_{ci}} = 0. \end{cases} \tag{4}$$

after direct differentiation, it can be presented as follows:

$$\frac{\partial F}{\partial x_{ci}} = -4(x_{r1}-x_{ci})\begin{bmatrix}(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\+(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2\end{bmatrix} - -4(x_{r2}-x_{ci})\begin{bmatrix}(x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\+(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2\end{bmatrix} - -4(x_{r3}-x_{ci})\begin{bmatrix}(x_{r3}-x_{ci})^2-(y_{r3}-y_{ci})^2-\\-(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix} = 0;$$

$$\frac{\partial F}{\partial y_{ci}} = -4(y_{r1}-y_{ci})\begin{bmatrix}(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\+(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2\end{bmatrix} - -4(y_{r2}-y_{ci})\begin{bmatrix}(x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\+(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2\end{bmatrix} - -4(y_{r3}-y_{ci})\begin{bmatrix}(x_{r3}-x_{ci})^2-(y_{r3}-y_{ci})^2-\\-(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix} = 0;$$

$$\frac{\partial F}{\partial z_{ci}} = -4(z_{r1}-z_{ci})\begin{bmatrix}(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\+(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2\end{bmatrix} - -4(z_{r2}-z_{ci})\begin{bmatrix}(x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\+(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2\end{bmatrix} - -4(z_{r3}-z_{ci})\begin{bmatrix}(x_{r3}-x_{ci})^2-(y_{r3}-y_{ci})^2-\\-(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix} = 0.$$

The formulated problem can be solved (including the method of recurrent approximation) in two ways.

4. 2. 1. The solution of the problem of finding the sound anomaly coordinates by a linear scheme of approximants and a linear approximation

Expand the system (4) and reduce the problem (3) to a linear system of algebraic equations using the method of recurrent approximation [33, 35] and limiting ourself by a linear scheme of approximants and a linear approximation:

$$\begin{cases} \dfrac{\partial F}{\partial x_{ci}} + \dfrac{\partial^2 F}{\partial x_{ci}^2}\Delta x_{ci} + \dfrac{\partial^2 F}{\partial y_{ci}\partial x_{ci}}\Delta y_{ci} + \dfrac{\partial^2 F}{\partial z_{ci}\partial x_{ci}}\Delta z_{ci} = 0; \\ \dfrac{\partial F}{\partial y_{ci}} + \dfrac{\partial^2 F}{\partial x_{ci}\partial y_{ci}}\Delta x_{ci} + \dfrac{\partial^2 F}{\partial y_{ci}^2}\Delta y_{ci} + \dfrac{\partial^2 F}{\partial z_{ci}\partial y_{ci}}\Delta z_{ci} = 0; \\ \dfrac{\partial F}{\partial z_{ci}} + \dfrac{\partial^2 F}{\partial x_{ci}\partial z_{ci}}\Delta x_{ci} + \dfrac{\partial^2 F}{\partial y_{ci}\partial z_{ci}}\Delta y_{ci} + \dfrac{\partial^2 F}{\partial z_{ci}^2}\Delta z_{ci} = 0. \end{cases} \tag{5}$$

Introduce notation of respective coefficients for increments of the sought unknowns and derive their expressions by direct differentiation:

$$a_{11} = \frac{\partial^2 F}{\partial x_{ci}^2} = 4\begin{bmatrix}3(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\+(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2+\\+3(x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\+(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2+\\+3(x_{r3}-x_{ci})^2+(y_{r3}-y_{ci})^2+\\+(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix};$$

$$a_{12} = \frac{\partial^2 F}{\partial y_{ci}\partial x_{ci}} = 8\begin{bmatrix}(y_{r1}-y_{ci})(x_{r1}-x_{ci})+\\+(y_{r2}-y_{ci})(x_{r2}-x_{ci})+\\+(y_{r3}-y_{ci})(x_{r3}-x_{ci})\end{bmatrix};$$

$$a_{13} = \frac{\partial^2 F}{\partial z_{ci}\partial x_{ci}} = 8\begin{bmatrix}(z_{r1}-z_{ci})(x_{r1}-x_{ci})+\\+(z_{r2}-z_{ci})(x_{r2}-x_{ci})+\\+(z_{r3}-z_{ci})(x_{r3}-x_{ci})\end{bmatrix};$$

$$a_{21} = \frac{\partial^2 F}{\partial x_{ci}\partial y_{ci}} = 8\begin{bmatrix}(x_{r1}-x_{ci})(y_{r1}-y_{ci})+\\+(x_{r2}-x_{ci})(y_{r2}-y_{ci})+\\+(x_{r3}-x_{ci})(y_{r3}-y_{ci})\end{bmatrix};$$

$$a_{22} = \frac{\partial^2 F}{\partial y_{ci}^2} = 4\begin{bmatrix}(x_{r1}-x_{ci})^2+3(y_{r1}-y_{ci})^2+\\+(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2+\\+(x_{r2}-x_{ci})^2+3(y_{r2}-y_{ci})^2+\\+(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2+\\+(x_{r3}-x_{ci})^2+3(y_{r3}-y_{ci})^2+\\+(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix};$$

$$a_{23} = \frac{\partial^2 F}{\partial z_{ci}\partial y_{ci}} = 8\begin{bmatrix}(z_{r1}-z_{ci})(y_{r1}-y_{ci})+\\+(z_{r2}-z_{ci})(y_{r2}-y_{ci})+\\+(z_{r3}-z_{ci})(y_{r3}-y_{ci})\end{bmatrix},$$

$$a_{31} = \frac{\partial^2 F}{\partial x_{ci}\partial x_{ci}} = 8\begin{bmatrix}(x_{r1}-x_{ci})(z_{r1}-z_{ci})+\\+(x_{r2}-x_{ci})(z_{r2}-z_{ci})+\\+(x_{r3}-x_{ci})(z_{r3}-z_{ci})\end{bmatrix};$$

$$a_{32} = \frac{\partial^2 F}{\partial y_{ci}\partial z_{ci}} = 8\begin{bmatrix}(y_{r1}-y_{ci})(z_{r1}-z_{ci})+\\+(y_{r2}-y_{ci})(z_{r2}-z_{ci})+\\+(y_{r3}-y_{ci})(z_{r3}-z_{ci})\end{bmatrix};$$

$$a_{33} = \frac{\partial^2 F}{\partial z_{ci}^2} = 4\begin{bmatrix}(x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+\\+3(z_{r1}-z_{ci})^2-\left[v_n(t_{r1}-t_{ci})\right]^2+\\+(x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+\\+3(z_{r2}-z_{ci})^2-\left[v_n(t_{r2}-t_{ci})\right]^2+\\+(x_{r3}-x_{ci})^2+(y_{r3}-y_{ci})^2+\\+3(z_{r3}-z_{ci})^2-\left[v_n(t_{r3}-t_{ci})\right]^2\end{bmatrix},$$

and also introduce the increment for the n+1-st and n-th approximants:

$$\Delta x_{ci} = (x_{ci})_{n+1} - (x_{ci})_n;$$

$$\Delta y_{ci} = (y_{ci})_{n+1} - (y_{ci})_n;$$

$$\Delta z_{ci} = (z_{ci})_{n+1} - (z_{ci})_n.$$

Thus, system (4) will be transformed to an approximant system, which will be a compact representation (5) and rapidly convergent at conditions of boundedness of the second derivatives or coefficients:

$$\begin{cases} \dfrac{\partial F}{\partial x_{ci}} + a_{11}\Delta x_{ci} + a_{12}\Delta y_{ci} + a_{13}\Delta z_{ci} = 0; \\ \dfrac{\partial F}{\partial y_{ci}} + a_{21}\Delta x_{ci} + a_{22}\Delta y_{ci} + a_{23}\Delta z_{ci} = 0; \\ \dfrac{\partial F}{\partial z_{ci}} + a_{31}\Delta x_{ci} + a_{32}\Delta y_{ci} + a_{33}\Delta z_{ci} = 0. \end{cases} \tag{6}$$

It is easy to simplify it by the method of exclusion if Δx_{ci} is first excluded:

$$\begin{cases} a_{21}\dfrac{\partial F}{\partial x_{ci}} - a_{11}\dfrac{\partial F}{\partial y_{ci}} + (a_{12}a_{21} - a_{11}a_{22})\Delta y_{ci} + \\ +(a_{13}a_{21} - a_{11}a_{23})\Delta z_{ci} = 0; \\ a_{31}\dfrac{\partial F}{\partial y_{ci}} - a_{21}\dfrac{\partial F}{\partial z_{ci}} + (a_{31}a_{22} - a_{21}a_{32})\Delta y_{ci} + \\ +(a_{23}a_{31} - a_{21}a_{33})\Delta z_{ci} = 0. \end{cases} \tag{7}$$

Next, exclude Δy_{ci}:

$$(a_{31}a_{22} - a_{21}a_{32})\left(a_{21}\frac{\partial F}{\partial x_{ci}} - a_{11}\frac{\partial F}{\partial y_{ci}}\right) -$$
$$-(a_{12}a_{21} - a_{11}a_{22})\left(a_{31}\frac{\partial F}{\partial y_{ci}} - a_{21}\frac{\partial F}{\partial z_{ci}}\right) +$$
$$+\begin{bmatrix} (a_{31}a_{22} - a_{21}a_{32})(a_{13}a_{21} - a_{11}a_{23}) - \\ -(a_{12}a_{21} - a_{11}a_{22})(a_{23}a_{31} - a_{21}a_{33}) \end{bmatrix}\Delta z_{ci} = 0.$$

The solution of the latter makes it possible to consistently calculate increments which can be generalized for the n-th approximation:

$$\Delta z_{nci} = -\begin{bmatrix} (a_{31}a_{22} - a_{21}a_{32})\left(a_{21}\dfrac{\partial F}{\partial x_{ci}} - a_{11}\dfrac{\partial F}{\partial y_{ci}}\right) - \\ -(a_{12}a_{21} - a_{11}a_{22})\left(a_{31}\dfrac{\partial F}{\partial y_{ci}} - a_{21}\dfrac{\partial F}{\partial z_{ci}}\right) \end{bmatrix} \times$$
$$\times\begin{bmatrix} (a_{31}a_{22} - a_{21}a_{32})(a_{13}a_{21} - a_{11}a_{23}) - \\ -(a_{12}a_{21} - a_{11}a_{22})(a_{23}a_{31} - a_{21}a_{33}) \end{bmatrix}^{-1}; \tag{8}$$

$$\Delta y_{nci} = -\begin{bmatrix} a_{21}\dfrac{\partial F}{\partial x_{ci}} - a_{11}\dfrac{\partial F}{\partial y_{ci}} + \\ +(a_{13}a_{21} - a_{11}a_{23})\Delta z_{nci} \end{bmatrix}(a_{12}a_{21} - a_{11}a_{22})^{-1}; \tag{9}$$

$$\Delta x_{nci} = -\frac{1}{a_{11}}\left(\frac{\partial F}{\partial x_{ci}} + a_{12}\Delta y_{nci} + a_{13}\Delta z_{nci}\right). \tag{10}$$

Thus, the solution of the system will be reversed for the following approximation:

$$z_{n+1,ci} = z_{nci} + \Delta z_{nci}$$

or calculated as a recurrent quantity:

$$z_{n+1,ci} = z_{nci} - \begin{bmatrix} (a_{31}a_{22} - a_{21}a_{32})\left(a_{21}\dfrac{\partial F}{\partial x_{ci}} - a_{11}\dfrac{\partial F}{\partial y_{ci}}\right) - \\ -(a_{12}a_{21} - a_{11}a_{22})\left(a_{31}\dfrac{\partial F}{\partial y_{ci}} - a_{21}\dfrac{\partial F}{\partial z_{ci}}\right) \end{bmatrix} \times$$
$$\times\begin{bmatrix} (a_{31}a_{22} - a_{21}a_{32})(a_{13}a_{21} - a_{11}a_{23}) - \\ -(a_{12}a_{21} - a_{11}a_{22})(a_{23}a_{31} - a_{21}a_{33}) \end{bmatrix}^{-1}. \tag{11}$$

Further, for the next coordinate, the following approximation is similarly given as:

$$y_{n+1,ci} = y_{nci} + \Delta y_{nci}$$

or through direct calculation

$$y_{n+1,ci} = y_{nci} - \begin{bmatrix} a_{21}\dfrac{\partial F}{\partial x_{ci}} - a_{11}\dfrac{\partial F}{\partial y_{ci}} + \\ +(a_{13}a_{21} - a_{11}a_{23})\Delta z_{nci} \end{bmatrix}(a_{12}a_{21} - a_{11}a_{22})^{-1}. \tag{12}$$

The third coordinate, also in the following approximation, is calculated by coordinates of the n-th approximation:

$$x_{n+1,ci} = x_{nci} + \Delta x_{nci}$$

or by direct calculation:

$$x_{n+1,ci} = x_{nci} - \frac{1}{a_{11}}\left(\frac{\partial F}{\partial x_{ci}} + a_{12}\Delta y_{nci} + a_{13}\Delta z_{nci}\right). \tag{13}$$

The obtained increments make it possible to check the fulfillment of the inequality constraints in (6) and calculate the relative errors:

$$\varepsilon_x = \frac{\Delta x_{ci}}{x_{ci}}; \quad \varepsilon_y = \frac{\Delta y_{ci}}{y_{ci}}; \quad \varepsilon_z = \frac{\Delta z_{ci}}{z_{ci}}.$$

If the modulus of relative error is greater than or equal to the specified error for all three coordinates, that is ($|\varepsilon_x| \geq [\varepsilon]$ and $|\varepsilon_y| \geq [\varepsilon]$ and $|\varepsilon_z| \geq [\varepsilon]$), it is necessary to move on to the next calculation of coordinates of the sound anomaly point for the next approximation.

4. 2. 2. The solution of the problem of finding coordinates of the sound anomaly by the linear scheme of approximants and a quadratic approximation

Let us expand system (4) and reduce the problem (3) to a nonlinear system of algebraic equations using the method of recurrent approximation [30, 32] and restricting to a linear scheme of approximants and a quadratic approximation:

$$\begin{cases} \frac{\partial F}{\partial x_{ci}} + \left(\frac{\partial^2 F}{\partial x_{ci}^2} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^3}\Delta x_{nci} \right)\Delta x_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial y_{ci}\partial x_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^2\partial x_{ci}}\Delta y_{nci} \right)\Delta y_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial z_{ci}\partial x_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^2\partial x_{ci}}\Delta z_{nci} \right)\Delta z_{n+1,ci} = 0; \\ \frac{\partial F}{\partial y_{ci}} + \left(\frac{\partial^2 F}{\partial x_{ci}\partial y_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^2\partial y_{ci}}\Delta x_{nci} \right)\Delta x_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial y_{ci}^2} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^3}\Delta y_{nci} \right)\Delta y_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial z_{ci}\partial y_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^2\partial y_{ci}}\Delta z_{nci} \right)\Delta z_{n+1,ci} = 0; \\ \frac{\partial F}{\partial z_{ci}} + \left(\frac{\partial^2 F}{\partial x_{ci}\partial z_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^2\partial z_{ci}}\Delta x_{nci} \right)\Delta x_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial y_{ci}\partial z_{ci}} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^2\partial z_{ci}}\Delta y_{nci} \right)\Delta y_{n+1,ci} + \\ + \left(\frac{\partial^2 F}{\partial z_{ci}^2} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^3}\Delta z_{nci} \right)\Delta z_{n+1,ci} = 0. \end{cases} \quad (14)$$

Implementation of such a scheme assumes presence at each iteration of the values of approximation of increments of sought values which are zeroed at the first iteration. Expressions of the third derivatives of the selected functional necessary for the implementation of such a solution are as follows:

$$\frac{\partial^3 F}{\partial x_{ci}^3} = -24\left[(x_{r1} - x_{ci}) + (x_{r2} - x_{ci}) + (x_{r3} - x_{ci}) \right].$$

Next, calculate other derivatives

$$\frac{\partial^3 F}{\partial y_{ci}^2\partial x_{ci}} = -8\left[(x_{r1} - x_{ci}) + (x_{r2} - x_{ci}) + (x_{r3} - x_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial z_{ci}^2\partial x_{ci}} = -8\left[(x_{r1} - x_{ci}) + (x_{r2} - x_{ci}) + (z_{r3} - z_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial x_{ci}^2\partial y_{ci}} = -8\left[(y_{r1} - y_{ci}) + (y_{r2} - y_{ci}) + (y_{r3} - y_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial y_{ci}^3} = -24\left[(y_{r1} - y_{ci}) + (y_{r2} - y_{ci}) + (y_{r3} - y_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial z_{ci}^2\partial y_{ci}} = -8\left[(y_{r1} - y_{ci}) + (y_{r2} - y_{ci}) + (y_{r3} - y_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial x_{ci}^2\partial z_{ci}} = -8\left[(z_{r1} - z_{ci}) + (z_{r2} - z_{ci}) + (z_{r3} - z_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial y_{ci}^2\partial z_{ci}} = -8\left[(z_{r1} - z_{ci}) + (z_{r2} - z_{ci}) + (z_{r3} - z_{ci}) \right];$$

$$\frac{\partial^3 F}{\partial z_{ci}^3} = -24\left[(z_{r1} - z_{ci}) + (z_{r2} - z_{ci}) + (z_{r3} - z_{ci}) \right].$$

Now, redefine coefficients of the equations in the new system (14) as follows:

$$b_{n1} = \frac{\partial F}{\partial x_{ci}}; \quad b_{n2} = \frac{\partial F}{\partial z_{ci}}; \quad b_{n3} = \frac{\partial F}{\partial y_{ci}};$$

$$a_{n+1,11} = a_{n11} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^3}\Delta x_{nci};$$

$$a_{n+1,12} = a_{n12} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^2 x_{ci}}\Delta y_{nci};$$

$$a_{n+1,13} = a_{n13} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^2 x_{ci}}\Delta z_{nci};$$

$$a_{n+1,21} = a_{n21} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^2 y_{ci}}\Delta x_{nci};$$

$$a_{n+1,22} = a_{n22} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^3}\Delta y_{nci};$$

$$a_{n+1,23} = a_{n23} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^2\partial y_{ci}}\Delta z_{nci};$$

$$a_{n+1,31} = a_{n31} + \frac{1}{2}\frac{\partial^3 F}{\partial x_{ci}^2\partial z_{ci}}\Delta x_{nci};$$

$$a_{n+1,32} = a_{n32} + \frac{1}{2}\frac{\partial^3 F}{\partial y_{ci}^2\partial z_{ci}}\Delta y_{nci};$$

$$a_{n+1,33} = a_{n33} + \frac{1}{2}\frac{\partial^3 F}{\partial z_{ci}^3}\Delta z_{nci}. \quad (15)$$

Then, taking into account notations (15), the new system of equations (14) will take a similar form:

$$\begin{cases} b_{n1} + a_{n+1,11}\Delta x_{n+1,ci} + a_{n+1,12}\Delta y_{n+1,ci} + a_{n+1,13}\Delta z_{n+1,ci} = 0; \\ b_{n2} + a_{n+1,21}\Delta x_{n+1,ci} + a_{n+1,22}\Delta y_{n+1,ci} + a_{n+1,23}\Delta z_{n+1,ci} = 0; \\ b_{n3} + a_{n+1,31}\Delta x_{n+1,ci} + a_{n+1,32}\Delta y_{n+1,ci} + a_{n+1,33}\Delta z_{n+1,ci} = 0. \end{cases}$$

but its coefficients will vary from approximation to approximation. It should be noted that subindices n and $n+1$ indicate the approximation number which means that in all values of the functional and its partial derivatives, coordinates of the sound anomaly and their increments are determined namely for these approximations. Thus, the approximate solution of the problem of finding coordinates of the sound anomaly for three microphones according to linear scheme approximants and linear or quadratic approximation can be combined into a single algorithm.

4. 3. Direct solution of the problem of finding the sound anomaly coordinates

To provide a direct solution, consider the original system (1). Simplify the solution of the original nonlinear system (1) by reducing it to a linear one. Based on the properties of difference between squares of two numbers and possible combinations of two numbers of three, show the property of representing the difference of squares as a linear form with respect to each of the three numbers. To do this, subtract the equation written for a similar condition of the other microphone from the equation-condition of the signal reaching each microphone. When realizing this, write a new system of three equations:

$$\begin{aligned}&x_{r2}^2-x_{r1}^2-2(x_{r2}-x_{r1})x_{ci}+\\&+y_{r2}^2-y_{r1}^2-2(y_{r2}-y_{r1})y_{ci}+\\&+z_{r2}^2-z_{r1}^2-2(z_{r2}-z_{r1})z_{ci}-\\&-\left\{v_n^2(t_{r2}-t_{r1})(t_{r2}+t_{r1}-2t_{ci})\right\}=0;\\&x_{r3}^2-x_{r2}^2-2(x_{r3}-x_{r2})x_{ci}+\\&+y_{r3}^2-y_{r2}^2-2(y_{r3}-y_{r2})y_{ci}+\\&+z_{r3}^2-z_{r2}^2-2(z_{r3}-z_{r2})z_{ci}-\\&-\left\{v_n^2(t_{r3}-t_{r2})(t_{r3}+t_{r2}-2t_{ci})\right\}=0;\\&x_{r1}^2-x_{r3}^2-2(x_{r1}-x_{r3})x_{ci}+\\&+y_{r1}^2-y_{r3}^2-2(y_{r1}-y_{r3})y_{ci}+\\&+z_{r1}^2-z_{r3}^2-2(z_{r1}-z_{r3})z_{ci}-\\&-\left\{v_n^2(t_{r1}-t_{r3})(t_{r1}+t_{r3}-2t_{ci})\right\}=0.\end{aligned} \quad (16)$$

It is easy to make sure that now it is linear with respect to the desired coordinates x_{ci}, y_{ci}, z_{ci} of the i-th sound anomaly and t_{ci} is a parameter. Call it a combination-difference form.

4. 3. 1. Algorithm for direct solving a problem of three microphones

The problem is reduced to considering the wave propagation from a sound anomaly with unknown coordinates to the receivers with known coordinates and known time of its registration. It is also stated by the original conditions that the computer system has three microphones. According to these conditions, problem (1) will be represented by a system of three equations after algebraic transformations of the combination-difference form:

$$\begin{cases}b_1+a_{11}x_{ci}+a_{12}y_{ci}+a_{13}z_{ci}=0;\\b_2+a_{21}x_{ci}+a_{22}y_{ci}+a_{23}z_{ci}=0;\\b_3+a_{31}x_{ci}+a_{32}y_{ci}+a_{33}z_{ci}=0,\end{cases} \quad (17)$$

where the coefficients are:

$$b_1=x_{r2}^2-x_{r1}^2+y_{r2}^2-y_{r1}^2+z_{r2}^2-z_{r1}^2-\left\{v_n^2(t_{r2}-t_{r1})(t_{r2}+t_{r1}-2t_{ci})\right\};$$

$$b_2=x_{r3}^2-x_{r2}^2+y_{r3}^2-y_{r2}^2+z_{r3}^2-z_{r2}^2-\left\{v_n^2(t_{r3}-t_{r2})(t_{r3}+t_{r2}-2t_{ci})\right\};$$

$$b_3=x_{r1}^2-x_{r3}^2+y_{r1}^2-y_{r3}^2+z_{r1}^2-z_{r3}^2-\left\{v_n^2(t_{r1}-t_{r3})(t_{r1}+t_{r3}-2t_{ci})\right\};$$

$$a_{11}=-2(x_{r2}-x_{r1});\ \ a_{21}=-2(x_{r3}-x_{r2});$$

$$a_{31}=-2(x_{r1}-x_{r3});$$

$$a_{12}=-2(y_{r2}-y_{r1});\ \ a_{22}=-2(y_{r3}-y_{r2});$$

$$a_{32}=-2(y_{r1}-y_{r3});$$

$$a_{13}=-2(z_{r2}-z_{r1});\ \ a_{23}=-2(z_{r3}-z_{r2});$$

$$a_{33}=-2(z_{r1}-z_{r3}).$$

Reduce its analytic solution after one step after exclusion of x_{ci} to a system of two equations:

$$\begin{cases}a_{21}b_1-a_{11}b_2+(a_{21}a_{12}-a_{11}a_{22})y_{ci}+\\+(a_{21}a_{13}-a_{11}a_{23})z_{ci}=0;\\a_{31}b_2-a_{21}b_3+(a_{31}a_{22}-a_{21}a_{32})y_{ci}+\\+(a_{31}a_{23}-a_{21}a_{33})z_{ci}=0.\end{cases} \quad (18)$$

After exclusion of y_{ci}, one equation with one unknown is obtained:

$$\begin{aligned}&(a_{21}b_1-a_{11}b_2)(a_{31}a_{22}-a_{21}a_{32})-\\&-(a_{31}b_2-a_{21}b_3)(a_{21}a_{12}-a_{11}a_{22})+\\&+\begin{bmatrix}(a_{21}a_{13}-a_{11}a_{23})(a_{31}a_{22}-a_{21}a_{32})-\\-(a_{31}a_{23}-a_{21}a_{33})(a_{21}a_{12}-a_{11}a_{22})\end{bmatrix}z_{ci}=0.\end{aligned}$$

from which coordinates of the sound anomaly source are as follows:

$$z_{ci}=-\begin{bmatrix}(a_{21}b_1-a_{11}b_2)(a_{31}a_{22}-a_{21}a_{32})-\\-(a_{31}b_2-a_{21}b_3)(a_{21}a_{12}-a_{11}a_{22})\end{bmatrix}\times$$
$$\times\begin{bmatrix}(a_{21}a_{13}-a_{11}a_{23})(a_{31}a_{22}-a_{21}a_{32})-\\-(a_{31}a_{23}-a_{21}a_{33})(a_{21}a_{12}-a_{11}a_{22})\end{bmatrix}^{-1},$$

so, write the following using the first of equations (18):

$$y_{ci}=-\begin{bmatrix}a_{21}b_1-a_{11}b_2+\\+(a_{21}a_{13}-a_{11}a_{23})z_{ci}\end{bmatrix}(a_{21}a_{12}-a_{11}a_{22})^{-1},$$

which in turn will yield from the first of equations (18):

$$x_{ci}=-\frac{1}{a_{11}}(b_1+a_{12}y_{ci}+a_{13}z_{ci}).$$

Thus, the direct calculation has given coordinates of sound anomaly according to the data for three microphones. It also should be noted that at such a system setting, its independence from the time of the sound anomaly is observed and accuracy will be determined by the relative positioning of microphones and the distance between them. A conclusion can be drawn from the solution that distance between microphones determines the solution error and that microphones themselves should not be in the same plane and at the same height. It is impossible to ensure the fulfillment of these conditions for three microphones in all cases at an arbitrary location of the sound anomaly. In the cases where the wave propagation time to two microphones is practically the same or the distance between the microphones is practically the same, the error in determining the coordinates is greatest. To reduce it, it is necessary to increase the number of microphones and distance between them and change their position in space.

4. 3. 2. An algorithm of direct solution of the problem for four microphones

The initial system is formed on the basis of equality of distance between the source of sound anomaly and the path passed by a spherical wave for four microphones:

$$\begin{cases} (x_{r1}-x_{ci})^2+(y_{r1}-y_{ci})^2+ \\ +(z_{r1}-z_{ci})^2-[v_n(t_{r1}-t_{ci})]^2=0; \\ (x_{r2}-x_{ci})^2+(y_{r2}-y_{ci})^2+ \\ +(z_{r2}-z_{ci})^2-[v_n(t_{r2}-t_{ci})]^2=0; \\ (x_{r3}-x_{ci})^2+(y_{r3}-y_{ci})^2+ \\ +(z_{r3}-z_{ci})^2-[v_n(t_{r3}-t_{ci})]^2=0; \\ (x_{r4}-x_{ci})^2+(y_{r4}-y_{ci})^2+ \\ +(z_{r4}-z_{ci})^2-[v_n(t_{r4}-t_{ci})]^2=0. \end{cases} \quad (19)$$

To simplify the solution of the original nonlinear system (19), it is necessary to convert it to a linear combination-difference form. To do this, subtract the equation with a similar condition from the equation-condition of the signal reaching each of the microphones. When implementing the above, write:

$$x_{r2}^2-x_{r1}^2-2(x_{r2}-x_{r1})x_{ci}+$$
$$+y_{r2}^2-y_{r1}^2-2(y_{r2}-y_{r1})y_{ci}+$$
$$+z_{r2}^2-z_{r1}^2-2(z_{r2}-z_{r1})z_{ci}-$$
$$-\{v_n^2(t_{r2}-t_{r1})(t_{r2}+t_{r1}-2t_{ci})\}=0;$$

$$x_{r3}^2-x_{r2}^2-2(x_{r3}-x_{r2})x_{ci}+$$
$$+y_{r3}^2-y_{r2}^2-2(y_{r3}-y_{r2})y_{ci}+$$
$$+z_{r3}^2-z_{r2}^2-2(z_{r3}-z_{r2})z_{ci}-$$
$$-\{v_n^2(t_{r3}-t_{r2})(t_{r3}+t_{r2}-2t_{ci})\}=0;$$

$$x_{r1}^2-x_{r3}^2-2(x_{r1}-x_{r3})x_{ci}+$$
$$+y_{r1}^2-y_{r3}^2-2(y_{r1}-y_{r3})y_{ci}+$$
$$+z_{r1}^2-z_{r3}^2-2(z_{r1}-z_{r3})z_{ci}-$$
$$-\{v_n^2(t_{1r2}-t_{r3})(t_{r1}+t_{r3}-2t_{ci})\}=0;$$

$$x_{r4}^2-x_{r3}^2-2(x_{r4}-x_{r3})x_{ci}+$$
$$+y_{r4}^2-y_{r3}^2-2(y_{r4}-y_{r3})y_{ci}+$$
$$+z_{r4}^2-z_{r3}^2-2(z_{r4}-z_{r3})z_{ci}-$$
$$-\{v_n^2(t_{1r4}-t_{r3})(t_{r4}+t_{r3}-2t_{ci})\}=0.$$

Technical implementation of the task can be reduced to a computer system with four microphones. The model of such a system will be represented after transformations by this system of four equations:

$$\begin{cases} b_1+a_{11}x_{ci}+a_{12}y_{ci}+a_{13}z_{ci}+a_{14}t_{ci}=0; \\ b_2+a_{21}x_{ci}+a_{22}y_{ci}+a_{23}z_{ci}+a_{24}t_{ci}=0; \\ b_3+a_{31}x_{ci}+a_{32}y_{ci}+a_{33}z_{ci}+a_{34}t_{ci}=0; \\ b_4+a_{41}x_{ci}+a_{42}y_{ci}+a_{43}z_{ci}+a_{44}t_{ci}=0. \end{cases} \quad (20)$$

In system (20), coefficients are equal to:

$$b_1=x_{r2}^2-x_{r1}^2+y_{r2}^2-y_{r1}^2+z_{r2}^2-z_{r1}^2-v_n^2(t_{r2}^2-t_{r1}^2);$$

$$b_2=x_{r3}^2-x_{r2}^2+y_{r3}^2-y_{r2}^2+z_{r3}^2-z_{r2}^2-v_n^2(t_{r3}^2-t_{r2}^2);$$

$$b_3=x_{r1}^2-x_{r3}^2+y_{r1}^2-y_{r3}^2+z_{r1}^2-z_{r3}^2-v_n^2(t_{r1}^2-t_{r3}^2);$$

$$b_4=x_{r4}^2-x_{r3}^2+y_{r4}^2-y_{r3}^2+z_{r4}^2-z_{r3}^2-v_n^2(t_{r4}^2-t_{r3}^2);$$

$$a_{11}=-2(x_{r2}-x_{r1}); \quad a_{21}=-2(x_{r3}-x_{r2});$$

$$a_{31}=-2(x_{r1}-x_{r3}); \quad a_{41}=-2(x_{r4}-x_{r3});$$

$$a_{12}=-2(y_{r2}-y_{r1}); \quad a_{22}=-2(y_{r3}-y_{r2});$$

$$a_{32}=-2(y_{r1}-y_{r3}); \quad a_{42}=-2(y_{r4}-y_{r3});$$

$$a_{13}=-2(z_{r2}-z_{r1}); \quad a_{23}=-2(z_{r3}-z_{r2});$$

$$a_{33}=-2(z_{r1}-z_{r3}); \quad a_{43}=-2(z_{r4}-z_{r3});$$

$$a_{14}=-2v_n^2(t_{r2}-t_{r1}); \quad a_{24}=-2v_n^2(t_{r3}-t_{r2});$$

$$a_{34}=-2v_n^2(t_{r1}-t_{r3}); \quad a_{44}=-2v_n^2(t_{r4}-t_{r3}).$$

Its analytic solution in a single step is obtained by successive exclusions: the unknown coordinate x_{ci} is excluded first:

$$a_{21}b_1-a_{11}b_2+(a_{21}a_{12}-a_{11}a_{22})y_{ci}+$$
$$+(a_{21}a_{13}-a_{11}a_{23})z_{ci}+(a_{21}a_{14}-a_{11}a_{24})t_{ci}=0;$$

$$a_{31}b_2-a_{21}b_3+(a_{31}a_{22}-a_{21}a_{32})y_{ci}+$$
$$+(a_{31}a_{23}-a_{21}a_{33})z_{ci}+(a_{31}a_{24}-a_{21}a_{34})t_{ci}=0;$$

$$a_{41}b_3-a_{31}b_4+(a_{41}a_{32}-a_{31}a_{42})y_{ci}+$$
$$+(a_{41}a_{33}-a_{31}a_{43})z_{ci}+(a_{41}a_{34}-a_{31}a_{44})t_{ci}=0.$$

The following exclusion of the coordinate y_{ci} gives:

$$(a_{31}a_{22}-a_{21}a_{32})(a_{21}b_1-a_{11}b_2)-$$
$$-(a_{21}a_{12}-a_{11}a_{22})(a_{31}b_2-a_{21}b_3)+$$
$$+\begin{bmatrix}(a_{31}a_{22}-a_{21}a_{32})(a_{21}a_{13}-a_{11}a_{23})- \\ -(a_{21}a_{12}-a_{11}a_{22})(a_{31}a_{23}-a_{21}a_{33})\end{bmatrix}z_{ci}+$$
$$\begin{bmatrix}(a_{31}a_{22}-a_{21}a_{32})(a_{21}a_{14}-a_{11}a_{24})- \\ -(a_{21}a_{12}-a_{11}a_{22})(a_{31}a_{24}-a_{21}a_{34})\end{bmatrix}t_{ci}=0;$$

$$(a_{41}a_{32}-a_{31}a_{42})(a_{31}b_2-a_{21}b_3)-$$
$$-(a_{31}a_{22}-a_{21}a_{32})(a_{41}b_3-a_{31}b_4)+$$
$$\begin{bmatrix}(a_{41}a_{32}-a_{31}a_{42})(a_{31}a_{23}-a_{21}a_{33})- \\ -(a_{31}a_{22}-a_{21}a_{32})(a_{41}a_{33}-a_{31}a_{43})\end{bmatrix}z_{ci}+$$
$$\begin{bmatrix}(a_{41}a_{32}-a_{31}a_{42})(a_{31}a_{24}-a_{21}a_{34})- \\ -(a_{31}a_{22}-a_{21}a_{32})(a_{41}a_{34}-a_{31}a_{44})\end{bmatrix}t_{ci}=0.$$

Enter notations:

$$C_1=(a_{31}a_{22}-a_{21}a_{32})(a_{21}b_1-a_{11}b_2)-$$
$$-(a_{21}a_{12}-a_{11}a_{22})(a_{31}b_2-a_{21}b_3);$$

$$C_2=\left(a_{41}a_{32}-a_{31}a_{42}\right)\left(a_{31}b_2-a_{21}b_3\right)-$$
$$-\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{41}b_3-a_{31}b_4\right);$$

$$D_1=\begin{bmatrix}\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{21}a_{13}-a_{11}a_{23}\right)-\\-\left(a_{21}a_{12}-a_{11}a_{22}\right)\left(a_{31}a_{23}-a_{21}a_{33}\right)\end{bmatrix};$$

$$D_2=\begin{bmatrix}\left(a_{41}a_{32}-a_{31}a_{42}\right)\left(a_{31}a_{23}-a_{21}a_{33}\right)-\\-\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{41}a_{33}-a_{31}a_{43}\right)\end{bmatrix};$$

$$Q_1=\begin{bmatrix}\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{21}a_{14}-a_{11}a_{24}\right)-\\-\left(a_{21}a_{12}-a_{11}a_{22}\right)\left(a_{31}a_{24}-a_{21}a_{34}\right)\end{bmatrix};$$

$$Q_2=\begin{bmatrix}\left(a_{41}a_{32}-a_{31}a_{42}\right)\left(a_{31}a_{24}-a_{21}a_{34}\right)-\\-\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{41}a_{34}-a_{31}a_{44}\right)\end{bmatrix}.$$

Then the system will be simplified to a system of two equations with two unknowns:

$$\begin{cases}C_1+D_1z_{ci}+Q_1t_{ci}=0;\\C_2+D_2z_{ci}+Q_2t_{ci}=0;\end{cases}\qquad(21)$$

from which the event registration time and coordinates of the sound anomaly source are obtained:

$$C_1D_2-C_2D_1+\left(Q_1D_2-Q_2D_1\right)t_{ci}=0;$$

$$t_{ci}=\left[C_2D_1-C_1D_2\right]\left(Q_1D_2-Q_2D_1\right)^{-1};$$

$$z_{ci}=\begin{cases}-\dfrac{C_1+Q_1t_{ci}}{D_1}\quad\forall\,D_1\neq 0,\\-\dfrac{C_2+Q_2t_{ci}}{D_2}\quad\text{if }\ D_1=0.\end{cases}\qquad(22)$$

If the denominator of the fraction becomes zero, then the coordinate of the sound source (x_{ci}, y_{ci}, z_{ci}) is calculated by the expression obtained in solution of the second equation of system (21):

$$z_{ci}=-\frac{C_2+Q_2t_{ci}}{D_2},$$

$$y_{ci}=-\begin{pmatrix}a_{21}b_1-a_{11}b_2+\\+\left(a_{21}a_{13}-a_{11}a_{23}\right)z_{ci}+\\+\left(a_{21}a_{14}-a_{11}a_{24}\right)t_{ci}\end{pmatrix}\left(a_{21}a_{12}-a_{11}a_{22}\right)^{-1};$$

$$x_{ci}=-\frac{1}{a_{11}}\left(b_1+a_{12}y_{ci}+a_{13}z_{ci}+a_{14}t_{ci}\right).$$

Conditions when the coefficient D_1 is zero:

$$D_1=\begin{bmatrix}\left(a_{31}a_{22}-a_{21}a_{32}\right)\left(a_{21}a_{13}-a_{11}a_{23}\right)-\\-\left(a_{21}a_{12}-a_{11}a_{22}\right)\left(a_{31}a_{23}-a_{21}a_{33}\right)\end{bmatrix};$$

$$D_1=a_{31}a_{22}a_{21}a_{13}-a_{21}a_{32}a_{21}a_{13}-$$
$$-a_{31}a_{22}a_{11}a_{23}+a_{21}a_{32}a_{11}a_{23}-$$
$$-a_{21}a_{12}a_{31}a_{23}+a_{11}a_{22}a_{31}a_{23}+$$
$$+a_{21}a_{12}a_{21}a_{33}-a_{11}a_{22}a_{21}a_{33}=0$$

or

$$a_{22}=a_{23};\ \ a_{32}=a_{33};\ \ a_{13}=a_{12}.$$

Thus, parameter D_1 turns to zero for individual values of coordinates of microphone locations which in turn blocks the operation of the algorithm unless it is corrected by the logical transition to another equation of system (21).

5. Conducting the numerical experiment

The data on conditions for conducting the numerical experiment characterizing coordinates of the four microphones are given in Table 1. Coordinates for the numerical experiment were selected based on recommendations given in [15].

Table 2 presents additional parameters for conducting the experiment: the time of recording an abnormal event by microphones. Besides, coordinates of the event and its absolute error E, iteration number n, and total time τ of the calculation obtained during the experiment are presented.

Table 1

Coordinates of microphones of the computerized sound anomaly recording system

Microphone number	Microphone coordinates		
	X_i, m	Y_i, m	Z_i, m
1	85	90	5
2	50	20,000	5
3	15,000	35	5
4	19,000	1,700	5

The experiment was conducted on a computerized system capable of recording audio anomalies with four microphones. This variant of the system from many numerical experiments was chosen based on the need to simultaneously determine three coordinates and time of the event. The simulation results are presented in Table 2. Column 1 shows the experiment number. Columns 2, 3, 4, 5 show the time of recording the audio event. Coordinates of the calculated sound anomaly are presented in columns 6, 7, 8, respectively. In the course of the numerical experiment, calculations were performed until the absolute error of 0.01 m was reached which is reflected in Table 2.

The calculations were continued to provide an absolute error of 0.0001 m. These two values of n and N (total number of iterations) are given in the ninth column by first and second numbers, respectively, separated by a slash. The total calculation time τ is the time for which the error reaches a value less than the specified value of 0.0001 m. This parameter is shown in Table 2 after the second slash in the ninth column.

The highest amount of spent time 98, 91, 158, 105 ms was in experiments 11, 16, 17, 20, respectively. In order to reduce cell size in Table 2 when error values were less than 0.01 m, the value was written as 0. It should be noted that even the largest of these values is sufficiently small (0.158 s) and capable of satisfying the requirements for operation of AMS when solving the list of problems [1, 9, 10, 15, 16, 24–29] based on the idea of determining coordinates of sound anomaly sources. Thus, summarizing the results of numerical simulation, it should be added that the greater the distance between microphones, the smaller the number of iterations in the process of finding the final result for the chosen calculation accuracy. The placement of microphones along the perimeter of the scanned area (a square) is optimal.

Table 2

Parameters of conditions and results of the numerical experiment

Experi-ment No.	Time of event recording by i-th microphone, s				Coordinate/relative error, m/m			Iteration/total iteration number/calculatiom time
	1	2	3	4	X_{ci}/E	Y_{ci}/E	Z_{ci}/E	n/N/τ, 1/ms
1	65.369	65.579	32.76	24.567	20000/–0.14	10000/–0.4	2.99/0.01	12/17/20
2	80.703	55.687	59.836	53.777	19000/0	20000/0	2.9/0.1	8/16/50
3	65.362	29.240	60.482	59.929	10000/0	20000/0	3.06/–0.06	7/29/73
4	58.571	2.7917	71.658	75.432	1000/0	20000/0	2.99/0.01	10/13/37
5	58.585	80.960	14.965	3.5871	20000/0	1000/0	2.99/0.01	13/17/10
6	39.172	39.438	34.182	38.190	9000/0	10000/0	2.99/0.01	10/15/9
7	61.975	46.325	43.977	40.814	15000/0	15000/0	2.94/0.06	7/10/3
8	18.454	45.583	35.466	45.134	4000/0	5000/0	2.98/0.02	12/16/24
9	46.135	20.676	52.892	56.747	5000/0	15000/0	2.99/0.01	9/14/34
10	43.363	60.196	14.883	17.605	14000/0	5000/0	2.97/0.03	8/11/5
11	29.259	63.028	14.965	26.528	10000/0	1000/0	2.98/0.02	10/14/98
12	29.246	29.519	50.499	58.249	1000/0	10000/0	2.99/0.01	18/22/53
13	32.514	52.896	20.707	28.170	10000/0	5000/0	3/0	8/11/4
14	32.507	32.788	41.486	47.828	5000/0	10000/0	3/0	9/12/71
15	52.623	52.855	29.284	27.076	15000/0	10000/0	2.99/0.01	8/10/3
16	52.619	32.724	46.367	47.192	10000/0	15000/0	2.99/0.01	10/19/91
17	60.265	14.546	65.619	67.710	5000/0	20000/0	2.99/0.01	13/33/158
18	73.105	43.933	58.671	55.047	15000/0	20000/0	2.97/0.03	8/11/63
19	60.276	73.349	20.707	10.133	20000	5000/0	2.99/0.01	12/17/11
20	73.108	60.440	46.367	39.195	20000/0	15000/0	2.99/0.01	12/17/105

When the microphone placement is compact, the values of their coordinates of the same name must not be the same. In addition, the elevation of placement is always greater than the vertical coordinate of the sound anomaly. Compliance with these requirements reduces both the number of iterations and the total time spent in calculating the sound anomaly coordinates.

6. Discussion of results obtained in studying the applicability of indirect and direct methods for determining the sound anomaly coordinates

The obtained results show that the constructed model of recording the sound anomaly of a single point of the spherical wave's front makes it possible to make analytical solutions both by direct and indirect methods. Solutions of the problem of finding coordinates of sound anomaly according to the linear approximant scheme and by linear and quadratic approximations that were constructed and represented by expressions (11)–(13) and (14), (15) are analytic recurrent expressions. Their analyticity is a major advantage over the results of the studies determining coordinates of the sound anomaly by numerical methods [9, 10, 15, 16, 27–29]. Thanks to such property as analyticity, it is relatively simple to build an algorithm of calculating coordinates of a sound anomaly. The difference between the current results and the results obtained in [27–29] can be explained by the proposed elimination of irrationality from the model composition in the course of its formation, departure from the methods of [30, 31] and application of the recurrent approximation method (RAM) [33–35] for the solution. The data of numerical experiments given in Tables 1 and 2 also demonstrate stable algorithm operation and good enough convergence. The number of iterations (10–17), accuracy selected and adjusted and total calculation time (maximum 0.158 s) privilege the suggestions obtained.

The advantage of these results over those obtained by direct and indirect numerical methods [30–32] based on the Newton-Kantorovich method includes the advantages of the RAM itself. This method eliminates the main problem of optimization when gradient drops when approaching the optimum point leading to a step-like growth in the algorithm of root search.

The results of numerical experiments show that the greatest error is made by systems in which coordinates of microphones and sound sources are almost identical or coincide. As expected from the analysis of expressions (11)–(13) and (14), (15) and is obvious from the analyticity of solutions can be explained by the following causes. First, under these conditions, coefficients of the equations reduce their values to almost zero or turn to zero. Secondly, the main determinants of intermediate systems tend to small quantities, and the difference between values of the sought coordinates increases sharply from iteration to iteration since it is inversely proportional to the result tending to zero because of the operator action which slows down convergence of solutions.

The proposed transformation of a system of nonlinear equations to a linear combination-difference form (16) has made it possible to obtain a simple algorithm of finding coordinates. However, since it has been reduced to the solution of a system of algebraic equations of the first order, it has limitations on the principal determinant of the system. The latter leads to a limitation on the design parameters of the microphone system, that is, the distance and location which must be taken into account when implementing it.

As evidenced by the results of the numerical experiment (Table 2), the way out of such a position implies the spatial separation of microphones and an increase in their number combined with selective data sampling to calculate the sound anomaly coordinates. The idea of redundancy [27] and selection can also produce new positive results when combined with the solutions obtained. Therefore, the proposed solution to the problem with four microphones is one of the advantages of using the proposed solutions. The conclusion drawn from an analysis of the results obtained in the numerical experiment, regardless of the methods used, is as follows. In contrast to the proposals of studies [7, 14], in order to reduce the error, it is necessary to provide spatial separation of microphones between themselves and relative to the plane of two of them. This conclusion is important for practitioners and the implementation of systems because it reduces the number of microphones and improves the accuracy of the coordinate determination.

However, accuracy is not regulated for direct methods. Its value is determined by the accuracy of calculating the coefficients and the items that do not contain unknowns. In the cases of the orientation of the sound source to the microphones when distances become equal, the right parts approach zero. In this case, because of the error of recording the time of the leading wave edge, the solutions lose the property of unambiguity and contain an error of uncertain value which is unadjustable.

Applying the approximate method to coordinate search by solving the minimization problem with the involvement of RAM makes it possible to build simple algorithms of analytical solutions. Their implementation to solve the problems of numerical experiments quickly gives practically accurate values of coordinates being the feature and advantage of using the proposed method of solution. Thus, the constructed model does not contain the flaws produced by the irrationality of the equation systems and the limitations of Newton's method regarding the absence of not simple roots and the functional oscillations.

During the numerical experiments with the proposed algorithms based on the problem solutions by direct methods, it was found that the system coefficients turn to zero at a certain coincidence of parameters. Applying the methods of logical analysis and rules of inference to the construction of algorithms makes it possible to avoid division by infinitesimal values or zero. This approach reduces the number of iterations which in turn reduces the overall search time. Simultaneous application of the offered algorithms of the search for the sound anomaly coordinates and mapping tools obviously forms the basis for the construction of automated emergency monitoring and control subsystems [1, 15, 16].

This study drawback is its focusing on a simplified model for which the fact of sound anomaly is considered as an instant impulse without interferences and the tasks are related to constructing the model and the aspects of constructing solutions and algorithms of determining the anomaly coordinates. However, real sound series contain an aggregate of resulting sound images along with interferences. The choice of types of sounds of the amplitude-frequency description of anomaly sounds and characteristic points on them enables the application of the obtained models and methods of determining coordinates for the surveillance systems. Due to this, the main drawback, as well as the task of further improvement of the system, is the necessity of developing a method of severance of the anomaly image from the aggregate spectrum and recording the moment of time. The solution to this problem forms the direction of improvement and implementation of AMS for recording and mapping of events of general purpose.

7. Conclusions

1. The idea of propagation of spherical acoustic waves from point sources the front of which is recorded as instantaneous impulses by spatially separated microphones makes it possible to construct a mathematical model that does not contain irrationality. The relation between coordinates of the sound anomaly and the speed of sound propagation in air, the distance between microphones, their number and location are represented by recurrent systems that adjust the set accuracy.

2. The solution of the problem of finding coordinates of the sound anomaly by indirect methods for three microphones obtained by a linear scheme of approximants and a linear and quadratic approximation was reduced to an analytic recurrent form. Their implementation provides simple algorithms for calculating coordinates of a sound anomaly that are rapidly convergent with a specified or adjustable accuracy and the number of iterations.

3. Conversion of the system of nonlinear equations to the proposed linear combination-difference form makes it possible to obtain a simple algorithm of searching by methods of direct solution of the problem of calculating coordinates of a sound anomaly in a single iteration for three and four microphones.

4. The numerical experiment with determining coordinates of a sound anomaly with a specified accuracy has demonstrated the workability of the algorithms built for this model by indirect and direct methods. The ability of the algorithms to controlled adjustment of accuracy reduces the number of iterations which is especially important for the cases of worse convergence. The maximum total coordinate calculation time was 0.158 s which satisfies the AMS requirements to speed capability. The algorithms are simple, fast converging, with adjustable accuracy and a short calculation time to complete convergence.

References

1. Nazirov, E. K., Nazirova, T. A. (2018). Emergency notification system «ACEH». Scientific Bulletin of UNFU, 28 (1), 140–144. doi: https://doi.org/10.15421/40280128
2. Seah, C. E., Hwang, I. (2009). Stochastic Linear Hybrid Systems: Modeling, Estimation, and Application in Air Traffic Control. IEEE Transactions on Control Systems Technology, 17 (3), 563–575. doi: https://doi.org/10.1109/tcst.2008.2001377
3. Camacho, E. F., Ramirez, D. R., Limon, D., Muñoz de la Peña, D., Alamo, T. (2010). Model predictive control techniques for hybrid systems. Annual Reviews in Control, 34 (1), 21–31. doi: https://doi.org/10.1016/j.arcontrol.2010.02.002
4. SPOTLITE – Electro-Optical Small-Arms Fire Detection System. Available at: https://defense-update.com/20060726_spotlite.html
5. PILAR – Ground Version. Available at: http://metravib.acoemgroup.com/defence/catalog/PILAR---GROUND-VERSION
6. Yong, J., Wang, D.-Y. (2015). Impact of noise on hearing in the military. Military Medical Research, 2 (1), 6. doi: https://doi.org/10.1186/s40779-015-0034-5
7. Rascon, C., Meza, I. (2017). Localization of sound sources in robotics: A review. Robotics and Autonomous Systems, 96, 184–210. doi: https://doi.org/10.1016/j.robot.2017.07.011
8. Boomerang III. Available at: https://www.raytheon.com/capabilities/products/boomerang
9. Yang, X., Xing, H., Ji, X. (2018). Sound Source Omnidirectional Positioning Calibration Method Based on Microphone Observation Angle. Complexity, 2018, 1–15. doi: https://doi.org/10.1155/2018/2317853
10. Kakaya tehnika pozvolit vyigrat' voynu s terroristami. Available at: http://www.autoconsulting.com.ua/article.php?sid=30908
11. Nguyen, L., Valls Miro, J., Qiu, X. (2019). Multilevel B-Splines-Based Learning Approach for Sound Source Localization. IEEE Sensors Journal, 19 (10), 3871–3881. doi: https://doi.org/10.1109/jsen.2019.2895854
12. L'vov, A. V., Agapov, M. N., Tishchenko, A. I. (2010). Triangulyatsionnaya sistema opredeleniya koordinat istochnika zvuka. Polzunovskiy vestnik, 2, 159–162.
13. Risoud, M., Hanson, J.-N., Gauvrit, F., Renard, C., Lemesre, P.-E., Bonne, N.-X., Vincent, C. (2018). Sound source localization. European Annals of Otorhinolaryngology, Head and Neck Diseases, 135 (4), 259–264. doi: https://doi.org/10.1016/j.anorl.2018.04.009
14. Döbler, D., Heilmann, G., Ohm, M. (2010). Automatic detection of microphone coordinates. 3rd Berlin Beamforming Conference. Available at: http://www.bebec.eu/Downloads/BeBeC2010/Papers/BeBeC-2010-15.pdf
15. Byelozyorov, Zh. O. (2016). Analysis of the algorithm for calculating the coordinates of the shot and its practical implementation on the basis of mobile device in cooperation with the UAV. Naukovi pratsi ChNU im. Petra Mohyly. Seriya: Kompiuterni tekhnolohiyi, 287 (275), 34–40.
16. Palahin, O. V., Vasiukhin, M. I., Kasim, A. M., Ivanyk, Yu. Yu., Dolynnyi, V. V. (2015). Metody ta zasoby pobudovy dynamichnykh stsenariyiv u navihatsiynykh heoinformatsiynykh systemakh. Perspektyvy rozvytku avtomatyzovanykh system upravlinnia viyskamy ta heoinformatsiynykh system: zbirnyk materialiv naukovo-praktychnoi konferentsiyi. Lviv: ASV, 185–200.
17. Engel, J., Sturm, J., Cremers, D. (2012). Camera-based navigation of a low-cost quadrocopter. 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems. doi: https://doi.org/10.1109/iros.2012.6385458
18. Bilenko, O. I., Hunko, H. L. (2015). Vyznachennia parametriv zvuku postrilu, yaki vplyvaiut na vykonannia spetsyfichnykh vohnevykh zavdan sylamy bezpeky ta pidliahatymut rehlamentatsiyi. Perspektyvy rozvytku ozbroiennia ta viyskovoi tekhniky sukhoputnykh viysk. Zbirnyk tez dopovidei Mizhnarodnoi naukovo-tekhnichnoi konferentsiyi. Lviv, 14–15.
19. Krainyk, Y., Darnapuk, Y. (2019). Configurable Description of FPGA-based Control System for Sensor Processing. 2019 XIth International Scientific and Practical Conference on Electronics and Information Technologies (ELIT). doi: https://doi.org/10.1109/elit.2019.8892313
20. Trounov, A. N. (1984). Application of sensory modules for adaptive robots. IFS Publication. Robot Vision and Sensory Control, 285–294.
21. Musiyenko, M. P., Zhuravska, I. M., Kulakovska, I. V., Kulakovska, A. V. (2016). Simulation the behavior of robot sub-swarm in spatial corridors. 2016 IEEE 36th International Conference on Electronics and Nanotechnology (ELNANO). doi: https://doi.org/10.1109/elnano.2016.7493090
22. Zhuravska, I., Kulakovska, I., Musiyenko, M. (2018). Development of a method for determining the area of operation of unmanned vehicles formation by using the graph theory. Eastern-European Journal of Enterprise Technologies, 2 (3 (92)), 4–12. doi: https://doi.org/10.15587/1729-4061.2018.128745
23. Zhuravska, I. Musiyenko, M., Tohoiev, O. (2019). Development the Heat Leak Detection Method for Hidden Thermal Objects by Means the Information-Measuring Computer System. Proceedings of the Second International Workshop on Computer Modeling and Intelligent Systems (CMIS-2019), 2353, 350–364. Available at: http://ceur-ws.org/Vol-2353/paper28.pdf
24. Kryvoruchko, A. V. (2012). Ohliad ta porivnialnyi analiz tekhnichnykh system vyiavlennia pozytsiyi snaipera. Suchasna spetsialna tekhnika, 3, 75–81. 2012. Available at: http://elar.naiau.kiev.ua/jspui/bitstream/123456789/2468/1/%d0%9a%d1%80%d0%b8%d0%b2%d0%be%d1%80%d1%83%d1%87%d0%ba%d0%be%20%d0%90.%20%d0%92..pdf
25. Babichev, V. I., Shigin, A. V., Morozov, V. I., Golomidov, B. A., Larin, D. V., Larin, A. V. et. al. (2011). Pat. No. 2453790 RF. Sposob strel'by artilleriyskimi snaryadami s zakrytyh ognevyh pozitsiy. No. 2011104932/28; declareted: 10.02.2011; published: 20.06.2012, Bul. No. 17. Available at: http://www.freepatent.ru/patents/2453790

26. Gorshkov, A. I. (2001). Pat. No. 2215972 RF. Sistemy navedeniya. declareted: 28.09.2001; published: 10.11.2003. Available at: http://www.freepatent.ru/patents/2215972
27. Tkachenko, V. N., Khashan, T. S., Manuilenko, R. I. (2013). Extreme formulation of the problem of determining the origin of the sound source in information redundancy conditions. Iskusstvenniy intellekt, 3, 462–469.
28. Trembach, B. R., Kochan, R. V. (2016). Analiz pokhybky vymiriuvannia kuta napriamu na tsil rozpodilenoiu systemoiu zvukovoi artyleriyskoi rozvidky. Vymiriuvalna tekhnika ta metrolohiya, 77, 177–182.
29. Kochan, R., Trembach, B., Kochan, O. (2019). Methodical error of targets bearing by sound artillery intelligence system. Measuring Equipment and Metrology, 80 (3), 10–14. doi: https://doi.org/10.23939/istcmtm2019.03.010
30. Taringoo, F., Caines, P. E. (2010). Gradient-geodesic HMP algorithms for the optimization of Hybrid systems based on the geometry of switching manifolds. 49th IEEE Conference on Decision and Control (CDC). doi: https://doi.org/10.1109/cdc.2010.5717541
31. Banitalebi, A., Ahmad, R., Abd Aziz, M. I. (2012). A direct method for optimal control problem. International Journal of Pure and Applied Mathematics, 81 (1), 33–47.
32. Dal Bianco, N., Bertolazzi, E., Biral, F., Massaro, M. (2018). Comparison of direct and indirect methods for minimum lap time optimal control problems. Vehicle System Dynamics, 57 (5), 665–696. doi: https://doi.org/10.1080/00423114.2018.1480048
33. Trunov, A. (2017). Theoretical predicting the probability of electron detachment for radical of cell photo acceptor. 2017 IEEE 37th International Conference on Electronics and Nanotechnology (ELNANO). doi: https://doi.org/10.1109/elnano.2017.7939776
34. Trunov, A., Malcheniuk, A. (2018). Recurrent network as a tool for calibration in automated systems and interactive simulators. Eastern-European Journal of Enterprise Technologies, 2 (9 (92)), 54–60. doi: https://doi.org/10.15587/1729-4061.2018.126498
35. Trunov, A. (2017). Recurrent Approximation in the Tasks of the Neural Network Synthesis for the Control of Process of Phototherapy. Chap. 10. Computer Systems for Healthcare and Medicin. Denmark, 213–248.
36. Trunov, A. (2018). Transformation of operations with fuzzy sets for solving the problems on optimal motion of crewless unmanned vehicles. Eastern-European Journal of Enterprise Technologies, 4 (4 (94)), 43–50. doi: https://doi.org/10.15587/1729-4061.2018.140641
37. Trunov, A. (2016). Recurrent approximation as the tool for expansion of functions and modes of operation of neural network. Eastern-European Journal of Enterprise Technologies, 5 (4 (83)), 41–48. doi: https://doi.org/10.15587/1729-4061.2016.81298
38. Trunov, A. (2019). Forming a методology for transforming a model as the basis for expanding its informativeness. Eastern-European Journal of Enterprise Technologies, 5 (4 (101)), 34–43. doi: https://doi.org/10.15587/1729-4061.2019.181866