Викладена ефективна чисельна методика розв'язання задач про вільні коливання ізотропних пологих оболонок з застосуванням методу сплайн-апроксимації невідомих функцій по одному з координатних напрямків. З використанням запропонованої методики були досліджені резонансні частоти коливань циліндричних оболонок та оболонок двоякої кривизни як з квадратним, так і з прямокутним планом. Розрахунки проводились та порівнювались по двом теоріям: класичній (Кірхгоффа-Лява) та уточненій (Тимошенка-Міндліна). Встановлювалась залежність власних частот коливань від співвідношення товщини оболонок і їх розмірів в плані. Виявлено, що обчислені в уточненій постановці частоти вільних коливань пологих оболонок мають менші значення, ніж відповідні частоти, обчислені в класичній постановці. Зі збільшенням товщини оболонок різниця у значеннях відповідних частот зростає. Отримані результати розрахунків порівнювались з частотами, розрахованими аналітично шляхом розкладання невідомих функцій в ряди Фур'є. Порівняння дало змогу визначити оптимальну область застосування кожної з теорій. Встановлено, що частоти вільних коливань тонких пологих оболонок доцільніше розраховувати в класичній постановці. Розрахунок частот нетонких оболонок (при співвідношенні товщини до найменшого розміру в плані h/a≥0,05) при будь-яких геометричних параметрах оболонок доцільніше проводити в уточненій постановці. Отримані результати підтвердили теоретичні припущення щодо важливості врахування кутів повороту спочатку прямолінійного елемента, викликаних поперечними зсувами, при обчисленнях власних частот коливань нетонких оболонок. Підтверджено універсальність та високу точність методу сплайн-апроксимації

Ключові слова: вільні коливання, пологі оболонки, класична теорія Кірхгоффа-Лява, уточнена теорія Тимошенка-Міндліна

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DISCOVERING A PATTERN IN THE FREE VIBRATIONS OF GENTLY SLOPING SHELLS OF DIFFERENT GEOMETRY IN THE CLASSIC AND REFINED STATEMENTS

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1. Introduction

The isotropic gently sloping shells, rectangular in plan, are widely used in many spheres of human activity, from aviation to construction. The process of the development and implementation of design and engineering solutions predetermines a significant increase in the requirements for the parameters of strength and reliability of the developed structures and mechanisms. There is a need to calculate the mechanical characteristics of structural elements, including shells of different shapes, which implies determining their resonance oscillation frequencies.

An effective numerical procedure has been proposed for solving the problems on the free oscillations of gently sloping isotropic shells in order to perform calculations involving the application of a spline-approximation method of unknown functions. The initial system of differential equations in the partial derivatives was reduced, through the spline-collocation along one of the coordinate directions, to the boundary value problem of eigenvalues for a system of ordinary differential equations with variable coefficients. The resulting one-dimensional problem was solved by a stable numerical discrete orthogonalization method in a combination with an incremental search method.

When calculating the thin gently sloping shells, it is advisable to construct a computation algorithm based on the classic theory by Kirchhoff-Love. It implies the introduction of a series of simplifications to the initial equations of the elasticity theory, which, in this case, exert an insignificant influence on the calculation results but considerably simplify the resulting equations. When calculating the non-thin shells, it is expedient to take into consideration the turning angles of an initial rectilinear element caused by the transverse offsets. This clarification is taken into consideration in the theory of the Timoshenko-Mindlin type.

It is a relevant issue to establish the limits of the use of each theory in the calculation of the natural oscillation frequencies of gently sloping shells. At low thickness, it is more practical to perform calculations in a classic statement. However, for the case of relatively thick shells, the computation in the classic statement leads to significant errors. Therefore, starting at a certain value of the thickness, it is more expedient, in terms of accuracy of the obtained results, to compute the natural oscillation frequencies of gently sloping shells in the refined statement. Determining such a limit of the shell thickness would make it possible to properly choose between the classic and refined statements when investigating the natural oscillation frequencies of gently sloping shells, which could improve accuracy.

2. Literature review and problem statement

Paper [1] reports the results of studying the rectangular plates of different thicknesses using a spline-approximation method. A given method was subsequently used by the authors of article [2] to investigate the natural oscillation frequencies of orthotropic rectangular plates of different thicknesses. In this case, the calculations were carried out both in the classical statement and in the refined statement. The cited article considered plates. Then, the spline approximation and discrete orthogonalization methods were used to study the stressed state of orthotropic shells in the refined statement. Thus, work [3] considered the shells of different curvature; paper [4] – shells of different thicknesses. The use of B-splines to solve the static problems for plates with parameters that change in two coordinate directions was reported in study [5]. However, the above papers considered the problems exclusively on the stressed-strained state of plates and shells. The problems on the free oscillations of the shells were solved in article [6]; the branched meridian method was used. This method combines a Fourier method, an incremental search method, and an orthogonal sweep method. Paper [7] examined the vibration of thin-walled systems consisting of coaxial combinations of spin shells of various shapes with Toro-elliptical elements in the classic statement. In this case, in order to solve the appropriate two-dimensional problems on eigenvalues, the authors employed a numerical-analytical procedure, which includes the method of separation of a Fourier variable, an incremental search method, and the orthogonal sweep method with a solution to the Cauchy problem. Study [8] proposes two approaches to investigating the free and forced axisymmetric oscillations of a cylindrical shell. They are based on three-dimensional elasticity theory and a division of the original cylindrical shell with cross-sectional concentric circles into several uniaxial cylindrical shells. The cited papers address various methods for solving the problems on free oscillations, however, the calculations are performed for thin shells. The oscillation frequencies of relatively thick shells are considered in the refined statement in study [9]. In this case, the authors examined the orthotropic shells of variable thickness and the calculations were carried out using a spline-approximation method. All the above works, except [2], report studies based only on the classic statement or only in the refined statement, without comparing the results of calculations of the same objects in both theories. Article [10] proposed an approach to determine the natural oscillation frequencies of shells of different geometry and relative thickness. The shells were made from isotropic, orthotropic, and anisotropic materials. The approach implies building a mathematical model based on the classic theory by Kirchhoff-Love, the improved theory by Timoshenko-Mindlin, the theory of spatial elasticity. Although the cited work employed different theories, the authors, however, did not resolve the issues related to the influence of the shell thickness on the accuracy of calculations in various theories. We are not aware of the studies that would establish the dependence of the natural oscillation frequencies of gently sloping shells on the ratio of their thickness and size in the plan when applying both theories. This allows us to argue that it is advisable to conduct relevant calculations that could make it possible to identify the boundaries of the application of each theory. This study aims to establish a dependence of the natural oscillation frequencies of isotropic gently sloping shells on the ratio of their thickness and dimensions in the plan when performing calculations based on the classic (by Kirchhoff-Love) and refined (by Timoshenko-Mindlin) theories. The results to be obtained would make it possible to choose between the classic and refined statements in studying the natural oscillation frequencies of gently sloping shells, which could improve the accuracy of computations.

3. The aim and objectives of the study

The aim of this study is to determine the character of the behavior of resonance oscillation frequencies of a gently sloping isotropic shell depending on the ratio of its thickness and minimum size in the plan. This would make it possible to establish the boundaries of application of the classic (by Kirchhoff-Love) and refined (by Timoshenko-Mindlin) theories.

To accomplish the aim, the following tasks have been set:

– to establish the influence of the number of collocation points on the result of calculating the frequency of free oscillations of the gently sloping isotropic shells rectangular in plan when applying a spline-approximation method;

– to investigate the frequencies of free oscillations of isotropic gently sloping shells with a square (a/b=1) plan and two cases of a rectangular plan (a/b=2 and b/a=2), where a and b are the dimensions in the plan. To consider, for each case, both the cylindrical shells (l_x/a=0.1 and l_y/a=0) and the shells of double curvature (l_x/a=0.1 and l_y/a=0.1), where l is the arrow of lifting. To perform calculations at different values of the shell thickness: from h/a=0.01 (thin shells) to h/a=0.11 (non-thin shells) with an increment of 0.02. All calculations are to be carried out according to two theories: classic and refined;

– to compute the frequencies of free oscillations of gently sloping shells by decomposing the unknown functions into a Fourier series, using a mathematical apparatus of the refined theory; to compare the analytical frequencies with those obtained by the proposed procedure and to define the boundary of application of each theory.

4. Materials and methods to study the natural oscillation frequencies of isotropic gently sloping shells

4. 1. Examined objects

The object of our study: isotropic gently sloping shells, rectangular in plan, of constant thickness. The characteristics of the shells' material: $E=2.016\cdot10^{11}$ Pa, $\nu=0.3$, $\rho=$ $=7{,}800$ kg/m^3.

We have studied the cylindrical shells and shells of double curvature, rectangular in plan, which meet the flatness condition

$$l \le \frac{\min(a,b)}{5},$$

where l is the arrow of lifting, a and b are the dimensions in the plan. For each value, the shell aspect ratio: $a/b=1$ – square plan, $a/b=2$ and $b/a=2$ – rectangular plan, two types of geometric parameters were calculated: the cylindrical shell ($l_x/a=0.1$ and $l_y/a=0$) and the shell of double curvature ($l_x/a=0.1$ and $l_y/a=0.1$).

The thickness of each of the examined shells changed discretely, the ratio of the shell thickness to the smallest size in plan varied from $h/a=0.01$ (thin shells) to $h/a=0.11$ (non-thin shells).

The calculations were performed at the hinge fastening of the shell's sides.

4. 2. Initial ratios

Let us represent the shell displacement in the following form:

– classic theory (by Kirchhoff-Love):

$$u_x(x,v,z,t)=u(x,y,t)+z\vartheta_x(x,y,t),$$
$$u_y(x,v,z,t)=v(x,y,t)+z\vartheta_y(x,y,t),$$
$$u_z(x,v,z,t)=w(x,y,t); \tag{1}$$

– refined theory (by Timoshenko-Mindlin):

$$u_x(x,v,z,t)=u(x,y,t)+z\psi_x(x,y,t),$$
$$u_y(x,v,z,t)=v(x,y,t)+z\psi_y(x,y,t),$$
$$u_z(x,v,z,t)=w(x,y,t), \tag{2}$$

where x, y, z are the coordinates of shell points; u_x, u_y, u_z are the corresponding displacements; u, v, w are the displacements of coordinate surface points in the directions x, y, z; ϑ_x, ϑ_y are the angles of rotation of a normal element relative to the coordinate axes without taking into consideration the transverse offsets; ψ_x, ψ_y are the complete turning angles of a rectilinear element.

According to (1) and (2), the expressions for deformations are written in the form:

– classic theory (by Kirchhoff-Love):

$$e_x(x,y,z,t)=\varepsilon_x(x,y,t)+z\kappa_x(x,y,t),$$
$$e_y(x,y,z,t)=\varepsilon_y(x,y,t)+z\kappa_y(x,y,t),$$
$$e_{xy}(x,y,z,t)=\varepsilon_{xy}(x,y,t)+z2\kappa_{xy}(x,y,t),$$
$$e_{xz}(x,y,z,t)=0,\ e_{yz}(x,y,z,t)=0; \tag{3}$$

– refined theory (by Timoshenko-Mindlin):

$$e_x(x,y,z,t)=\varepsilon_x(x,y,t)+z\chi_x(x,y,t),$$
$$e_y(x,y,z,t)=\varepsilon_y(x,y,t)+z\chi_y(x,y,t),$$
$$e_{xy}(x,y,z,t)=\varepsilon_{xy}(x,y,t)+z2\chi_{xy}(x,y,t),$$
$$e_{xz}(x,y,z,t)\cong\gamma_x(x,y,t),\ e_{yz}(x,y,z,t)\cong\gamma_x(x,y,t), \tag{4}$$

where γ_x, γ_y are the angles of rotation caused by transverse offsets; ε_x, ε_y, ε_{xy} are the components of tangential deformation defining the internal geometry of the coordinate surface; κ_x, κ_y, $2\kappa_{xy}$ are the components of bending deformation in a classic theory; χ_x, χ_y, $2\chi_{xy}$ are the components of bending deformation in the refined theory, characterizing the bending and twisting of the coordinate surface.

The equations describing free transverse oscillations in the gently sloping shells in the theories by Kirchhoff-Love and by Timoshenko-Mindlin take the form:

– classic theory (by Kirchhoff-Love):

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{yx}}{\partial y}=0,\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,$$
$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-k_1N_x-k_2N_y+\rho h\frac{\partial^2 w}{\partial t^2}=0,$$
$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y}-Q_x=0,\ \frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=0; \tag{5}$$

– refined theory (by Timoshenko-Mindlin):

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{yx}}{\partial y}=0,\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,$$
$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-k_1N_x-k_2N_y+\rho h\frac{\partial^2 w}{\partial t^2}=0,$$
$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y}-Q_x+\rho\frac{h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}=0,$$
$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y+\rho\frac{h^3}{12}\frac{\partial^2\psi_y}{\partial t^2}=0. \tag{6}$$

We set, on the shell's contours $x=0,a$ and $y=0,b$, the boundary conditions, which are determined through the displacements and rotation angles. Below are the expressions for boundary conditions at $x=const$($x=0$, $x=a$) and at hinge fastening:

– classic theory (by Kirchhoff-Love):

$$\frac{\partial u}{\partial x}=v=w=\frac{\partial^2 w}{\partial x^2}=0; \tag{7}$$

– refined theory (by Timoshenko-Mindlin):

$$\frac{\partial u}{\partial x}=v=w=\frac{\partial\psi_x}{\partial x}=\psi_y=0. \tag{8}$$

Similar conditions are set on the contours y=const, making the following substitutions in equations (7) and (8): $x\to y$, $u\to v$, $\psi_x\to\psi_y$.

4. 3. Solving procedure

The solutions to the systems of equations (5) and (6) will be derived in the form: – classic theory (by Kirchhoff-Love):

$$u(x,y)=\sum_{i=0}^{N}u_i(x)\varphi_i(y),$$
$$v(x,y)=\sum_{i=0}^{N}v_i(x)\chi_i(y),$$
$$w(x,y)=\sum_{i=0}^{N}w_i(x)\psi_i(y); \tag{9}$$

– refined theory (by Timoshenko-Mindlin):

$$u(x,y)=\sum_{i=0}^{N}u_i(x)\varphi_{1,i}(y),$$

$$v(x,y)=\sum_{i=0}^{N}v_i(x)\varphi_{2,i}(y),$$

$$w(x,y)=\sum_{i=0}^{N}w_i(x)\varphi_{3,i}(y),$$

$$\psi_x(x,y)=\sum_{i=0}^{N}\psi_{xi}(x)\varphi_{4,i}(y),$$

$$\psi_y(x,y)=\sum_{i=0}^{N}\psi_{yi}(x)\varphi_{5,i}(y), \tag{10}$$

where $u_i(x)$, $v_i(x)$, $w_i(x)$, $\psi_{xi}(x)$, $\psi_{yi}(x)$ (i=0,...,N) are the desired functions; $\varphi_i(y)$, $\chi_i(y)$, $\varphi_{ji}(y)$ (j=1...5) are the functions built using the third degree B-splines N≥4, $\psi_i(y)$ are the functions built using the fifth degree B-splines N≥6, satisfying the boundary conditions on the contours y=0 and y=b.

Substituting (9) and (10) into equations (5) and (6), respectively, we require that they should be satisfied at the assigned collocation points $\xi_k\in[0,b]$, k=0,...,N. After all the transformations, we derive a system of N+1 linear differential equations with respect to u_i, v_i, w_i in the classic theory and u_i, v_i, w_i, ψ_{xi}, ψ_{yi} in the refined theory.

The problem on eigenvalues for the systems of ordinary differential equations was solved by the stable numerical method of orthogonalization in a combination with the incremental search method.

5. Results of studying the free oscillations of isotropic gently sloping shells of different geometry

5. 1. Determining the impact of the number of collocation points on the result of the computation of the natural oscillation frequencies of the examined shells

In order to determine the influence of the number of the N collocation points on the results of computing the natural oscillation frequencies of the studied shells, we calculated the first four oscillation frequencies ϖ_i at the different number of collocation points, from 10 to 22 in an increment of 2. The calculation was performed for two gently sloping shells, rectangular in plan: a cylindrical shell with a ratio of thickness to the smallest size in the plan of h/a=0.07, and a shell of double curvature with the ratio h/a=0.11. The computation was conducted in the refined statement. The computation results are given in Table 1.

Given the results obtained, all subsequent calculations were performed at N=18 collocation points, which is a tradeoff between the relative accuracy of calculations and the cost of machine time for computation.

5. 2. Results of calculating the resonance oscillation frequencies of the examined shells using the proposed method

The results of our calculations of the resonance oscillation frequencies of the investigated shells using the proposed method of spline approximation for the case of hinge fastening of all sides are given in Table 2 – for the case of the classic statement, and in Table 3 – for the case of the refined statement.

Table 1

Oscillation frequencies of gently sloping shells computed at different number of the collocation points

Shell plan	Shell type	$\bar{\varpi}_i=\omega_i a\sqrt{\rho(1-\nu^2)/E}$							
		i	N						
			10	12	14	16	18	20	22
a/b=2	l_x/a=0.1 l_y/a=0 h/a=0.07	1	0.5920	0.5852	0.5818	0.5801	0.5791	0.5785	0.5782
		2	0.8154	0.8105	0.8081	0.8068	0.8062	0.8056	0.8055
		3	1.2709	1.2679	1.2664	1.2657	1.2652	1.2649	1.2648
		4	1.7906	1.7352	1.7086	1.6946	1.6866	1.6819	1.6790
b/a=2	l_x/a=0.1 l_y/a=0.1 h/a=0.11	1	0.4695	0.4689	0.4686	0.4685	0.4684	0.4683	0.4683
		2	0.7607	0.7526	0.7486	0.7465	0.7452	0.7445	0.7440
		3	1.1830	1.1415	1.1222	1.1119	1.1060	1.1024	1.1002
		4	1.2270	1.2268	1.2267	1.2267	1.2267	1.2266	1.2266

Table 2

Frequencies of the free oscillations of shells, rectangular in plan (a/b=2), computed in the classic statement

Shell plan	Shell type	$\bar{\varpi}_i=\omega_i a\sqrt{\rho(1-\nu^2)/E}$						
		i	h/a					
			0.01	0.03	0.05	0.07	0.09	0.11
a/b=2	lx/b=0.1 ly/b=0	1	0.2187	0.3701	0.4671	0.5831	0.7087	0.8398
		2	0.2206	0.3906	0.6003	0.8198	1.0430	1.2679
		3	0.3105	0.5676	0.9332	1.3015	1.6708	2.0404
		4	0.4302	0.8089	1.2621	1.7322	2.2085	2.6878
	lx/b=0.1 ly/b=0.1	1	0.5039	0.6329	0.6941	0.7769	0.8752	0.9844
		2	0.6000	0.8504	1.0845	1.2197	1.3797	1.5566
		3	0.6604	0.9840	1.2891	1.7520	2.2241	2.7006
		4	0.6661	1.0419	1.5442	2.0816	2.6326	3.1903

Table 3

Frequencies of the free oscillations of shells, rectangular in plan (a/b=2), computed in the refined statement

Shell plan	Shell type	$\bar{\omega}_i=\omega_i a\sqrt{\rho(1-\nu^2)/E}$						
		i	h/a					
			0.01	0.03	0.05	0.07	0.09	0.11
a/b=2	l_x/a=0.1 l_y/a=0	1	0.2312	0.3764	0.4682	0.5791	0.6980	0.8195
		2	0.2314	0.3955	0.5970	0.8062	1.0135	1.2154
		3	0.3178	0.5685	0.9205	1.2652	1.5959	1.9093
		4	0.4534	0.8535	1.2648	1.6866	2.0956	2.4813
	l_x/a=0.1 l_y/a=0.1	1	0.5312	0.6331	0.6860	0.7584	0.8438	0.9373
		2	0.6878	0.8902	1.0740	1.1958	1.3361	1.4864
		3	0.6809	0.9825	1.2764	1.6784	1.9321	2.1902
		4	0.7498	1.0711	1.4451	1.6847	2.0836	2.4612

5. 3. Results of computing the frequencies of the free oscillations of gently sloping shells by decomposing the unknown functions into a Fourier series

Table 4 gives the first four dimensionless frequencies of the free oscillations of shells, square in plan, $\bar{\omega}_i$, which were calculated analytically, using the decomposition of the unknown functions in the refined statement into a Fourier series:

$$u=\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty} a_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$

$$v=\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty} b_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b},$$

$$w=\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty} c_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$

$$\psi_x=\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty} d_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$

$$\psi_y=\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty} e_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}. \quad (11)$$

Table 4

The first four dimensionless frequencies of the free oscillations of shells $\bar{\omega}_i$, calculated analytically using the decomposition of the unknown functions in the refined statement into a Fourier series (11)

Shell plan	Shell type	$\bar{\omega}_i=\omega_i a\sqrt{\rho(1-\nu^2)/E}$						
		i	h/a					
			0.01	0.03	0.05	0.07	0.09	0.11
a/b=1	l_x/a=0.1 l_y/a=0	1	0.2032	0.4024	0.4578	0.5284	0.6070	0.6891
		2	0.2920	0.4437	0.7036	0.9557	1.1930	1.4129
		3	0.3710	0.7216	0.9058	1.1140	1.3243	1.5265
		4	0.4309	0.7639	1.1525	1.5282	1.8738	2.1841
	l_x/a=0.1 l_y/a=0.1	1	0.7358	0.7511	0.7803	0.8212	0.8708	0.9266
		2	0.7469	0.8426	1.0011	1.1882	1.3827	1.5730
		3	0.7469	0.8427	1.0011	1.1883	1.3829	1.5733
		4	0.7673	0.9903	1.3095	1.6456	1.9667	2.2609

The charts (Fig. 1) shows the deviation (in percentage) in the values of the oscillation frequencies of shells, computed by using the method of spline-approximation in the classic (dashed line) and refined (solid line) statements from those analytically calculated in the refined statement. Thus, we compare the convergence of frequencies computed in the classic and refined statements with those analytically calculated in the refined statement. Red color denotes the first frequencies, orange color – second, green – third, blue – fourth.

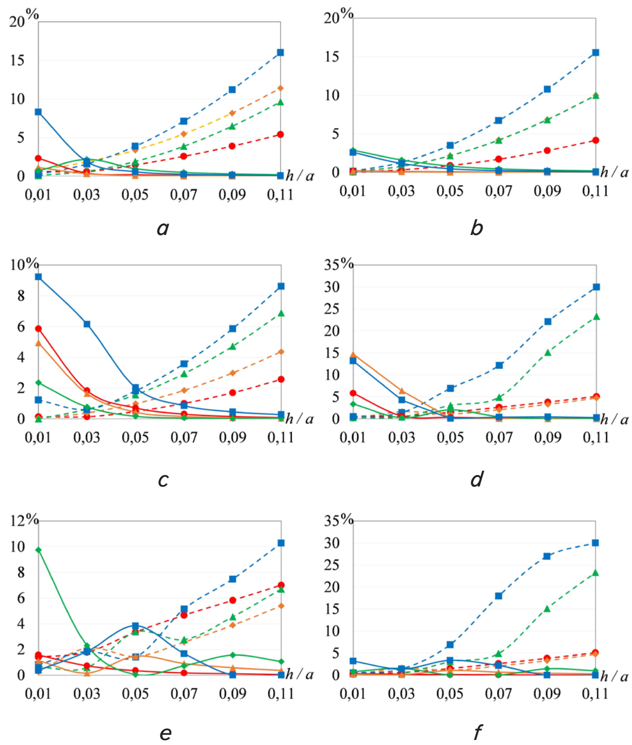

Fig. 1. Deviations (percentage) in the values of the resonance oscillation frequencies of gently sloping shells, calculated using the spline approximation method in the classic (dashed line) and refined (solid line) statements, from those analytically computed in the refined statement: a – cylindrical shell (l_x/a=0,1; l_y/a=0), square in plan (a/b=1); b – shell of double curvature (l_x/a=0,1; l_y/a=0,1), square in plan (a/b=1); c – cylindrical shell (l_x/b=0,1; l_y/b=0), rectangular in plan (a/b=2); d – shell of double curvature (l_x/b=0,1; l_y/b=0,1), rectangular in plan (a/b=2); e – cylindrical shell (l_x/a=0,1; l_y/a=0), rectangular in plan (b/a=2); f – shell of double curvature (l_x/a=0,1; l_y/a=0,1), rectangular in plan (b/a=2)

All cases of the geometric parameters of the examined shells (Fig. 1) demonstrate a similar pattern: with the increasing ratio of shell thickness and minimum size in the plan, the accuracy of computation in the classic statement decreases, and that in the refined statement grows.

6. Discussion of results of studying the resonance oscillations of isotropic gently sloping shells

The data from Table 1 indicate that an increase in the number of collocation points leads to a decrease in the frequency values, and the computation accuracy improves.

The magnitude of change in the values is different for different frequencies and varies from 0.02 % to almost 7 %. Thus, increasing the number of collocation points from 10 to 22 could improve the computation accuracy by 7 %; but, at the same time, the cost of machine time for calculating each frequency dramatically and significantly increases. Our analysis of computation results makes it possible to assert that already at 16 collocation points the accuracy of calculations achieves acceptable values, and the frequencies themselves differ from those calculated at 22 points by not larger than 1 %. Therefore, a reasonable number of collocation points in terms of computing accuracy and the cost of machine time is 16 or 18.

Analyzing the data given in the form of charts (Fig. 1), one notes a peculiarity typical for almost all cases of the geometric parameters of the shells. At low relative thickness, the difference between the analytically calculated frequencies and those computed in the classic statement is less than the corresponding difference to the frequencies calculated in the refined statement. This indicates that at a small shell thickness relative to the dimensions in the plan, the computation of frequencies in the classic statement makes it possible to obtain more accurate results.

With an increase in shell thickness, at a particular h/a ratio (different for different cases of geometric parameters), the difference between the analytically calculated frequencies and those calculated in the refined statement becomes less than the corresponding difference at computation in the classic statement. With an increase in shell thickness, this difference continues to decrease, which indicates the improved accuracy of the corresponding calculations. Therefore, starting at a certain thickness value, it is more expedient to carry out computation in the refined statement.

The analysis of our results (Fig. 1) indicates that the boundary value is the ratio of the thickness to the smallest size in plan of h/a=0.05. Therefore, the natural oscillation frequencies of gently sloping shells with the ratio $h/a \leq 0.05$ should be calculated, in terms of accuracy, in the classic statement while for thicker shells – in the refined statement.

Determining the shell thickness limit makes it possible to choose between the classic and refined statements when studying the natural oscillation frequencies of gently sloping isotropic shells, rectangular in plan, which would improve the computation accuracy. In the future, it is advisable to conduct a study of the character of the behavior of the resonance oscillation frequencies of orthotropic gently sloping shells depending on the ratio of their thickness and minimum size in the plan.

7. Conclusions

1. We have devised a numerical procedure for solving the problems on the free oscillations of gently sloping isotropic shells, rectangular in plan, with the application of a spline-approximation method of unknown functions and a numerical method of discrete orthogonalization in a combination with the incremental search method. We have computed the frequencies of free oscillations of the gently sloping shells both in the classical and refined statements with the different number of collocation points, from 10 to 22, in an increment of 2. It has been determined that the computation results demonstrate a significant dependence on the number of collocation points. In some cases, when increasing the number of collocation points from 10 to 22, the calculation accuracy increases by 7 %. It has been established that a compromise value is 18 collocation points, so all subsequent calculations were conducted at this value.

2. The first four frequencies of the free oscillations of isotropic gently sloping cylindrical shells and shells of double curvature, with a square and rectangular plan, have been investigated with the application of the proposed method of spline approximation. The ratios of the thickness of the studied shells and the smallest size in plan varied from h/a=0.01 to h/a=0.11 in an increment of 0.02. The calculations were carried out based on two theories: classic (by Kirchhoff-Love) and refined (by Timoshenko-Mindlin). The computation results are given in Tables 2, 3. It has been established that at the shell thickness less than h/a=0.04 the frequencies, calculated in the refined statement, accept higher values than the corresponding frequencies calculated in the cl assic statement. However, for shells with a thickness exceeding h/a=0.04 the frequencies computed in the refined statement accept lower values than the corresponding frequencies calculated in the classic statement. With the increase in shells thickness, this difference increases.

3. We have calculated the frequencies of free oscillations of gently sloping shells by expanding the unknown functions into a Fourier series, using a mathematical apparatus of the refined theory. A comparison was performed between the results obtained with the help of the proposed method and those calculated analytically by expanding the unknown functions into a Fourier series. It has been established that at the low relative thickness, the difference between the analytically calculated frequencies and those calculated in the classic statement is less than the corresponding difference from the frequencies computed in the refined statement. However, with the increase in thickness, this difference increases. Instead, the difference between the analytically calculated frequencies and those calculated with the use of the proposed procedure in the refined statement decreases with the increase in the shell thickness. Thus, there is such a value of the shell thickness for which, at thickness smaller than it, it is advisable to use the classic theory, and when the thickness is larger (the ratio of thickness and smallest size in the plan is $h/a \geq 0.05$) – the refined theory should be applied.

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