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Великий об'єм гірських порід із вмістом цінних мінералів переробляється на гірничо-збагачувальних комбінатах у Казахстані. Кульові і стрижневі млини використовуються для їх подрібнення і подальшої переробки.

Кульові млини із зубчастими кільцями в барабанах зазнають інтенсивного зносу зубців через важкий режим роботи млина. Тому це вимагає їх частішої заміни і тривалого простою млина. Шестерні приводу кульового млина зазнають сильної ударної напруги, що знижує ресурс їхньої роботи і млина загалом через зношення.

У статті представлено дослідження з розробки раціональних параметрів евольвентного зачеплення, спрямованих на підвищення вантажопідйомності редуктора, а також на зменшення габаритних розмірів, шуму і вібрації. Для вирішення поставлених завдань моделюються динамічні процеси, пропонується модифікація зубців, а завдання з проектування початкової контури сітки вирішується, коли лінія профілю зубця злегка відхиляється від евольвентної кривої його поверхні.

Виявлено кінематичні й динамічні параметри зубчастої передачі, що впливають на зносостійкість зубців, а також визначено вплив вантажопідйомності в умовах стабільного змащення.

Через складність модифікації ведучого зубчастого колеса великого діаметру пропонується модифікувати тільки зубці ведучого колеса, як на їхніх верхівках, так і на ніжках

Ключові слова: кульовий млин, знос зубців, евольвентне зачеплення, модифікація зубців, лінія профілю зубців

AN INCREASE OF THE LOADING CAPACITY AND RELIABILITY OF GEARS BY METHODS OF OPTIMIZING INVOLUTE GEARING PARAMETERS

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1. Introduction

Creation of rational gearing parameters requires special knowledge in the field of technology and design approaches [1].

Direct Gear Design is used to create gearings that are superior to those developed in accordance with technological standards. Increased load capacity, reduced noise, and lower vibration levels are achieved by the rational tooth shape and high overlapping ratios [2–6].

The design approach is applied in the aviation industry, governed by the industry standards and design tech-

niques [7, 8]. In the automotive industry, the possibilities of this approach are not fully implemented. In machine tool building, shipbuilding, general industrial and civil engineering, the technological approach is preserved as motivated by additional costs of special tools, and there is the lack of guidelines for design and specialists. These problems can be solved by applying a comprehensive approach, i. e. the total impact of both technological and constructive methods, which is achieved by rational parameters of gear teeth.

Particularly, it is essential to design and manufacture heavy loaded large-module gears of ball mills maintained in

concentrating manufactures where these transfers work in difficult conditions of dusty atmosphere.

2. Literature review and problem statement

The quality of gears (accuracy, purpose, technology, and reliability) largely determines the quality of mining machines. Methods to improve the quality of heavy-duty gears imply a single integrated process to cover the entire production cycle from design to manufacture of gears [9–11].

Contact surface breakage is observed only on the tooth legs and heads, where the teeth enter and exit the meshing [12]. Wear appears as micropitting on the idle feet and as limited sticking on the heads. The analysis shows that the reason of wear is not only the increased sliding of the involute tooth profiles, which is consistent with the theory of gear geometry, but also the high level of contact pressure associated with the small radius of curvature of the tooth edge.

The presence of wear areas that are similar to traces of plastic deformation indirectly confirms this. However, according to the classical theory of plastic deformations, at high rates of deformation, the limit of plastic deformations shifts to the area of higher contact stresses [13, 14].

In research paper [15], numerical tests of dynamic behaviour of the spur gear system are discussed. Meshing states and nonlinear dynamics of the gear system are investigated by different mathematical tools. The study demonstrates essential advantages of numerical tests in comparison with analytical research. The article, however, does not address the issue of modifying the shape of an involute gear tooth to enhance the loading capacity and reliability of gears.

In report [16], emphasis is made on the surface roughness impact on the dynamic properties of the gear. A new method is discussed for analysing the dynamics of high-contact-ratio (HCR) gear. It does not address the issue of the tooth surface roughness impact on the loading capacity or reliability of gears.

In study [17], the single tooth stiffness of an involute spur gear was measured experimentally by means of a special test rig. The physical experiments were repeated numerically in the ANSYS Workbench, and the elastic deformations were calculated and compared with experimental data. Experimental and numerical results were found to be generally consistent. Dynamic processes in a gear, including vibration, were not studied during that research.

Article [18] explores the significance of the load distribution along the face width and the load sharing between the pairs when the contact moves from the highest point of tooth contact (HPTC) to the lowest point of tooth contact (LPTC). The results of this study are useful for predicting the crack propagation as well as life calculations for spur gear drives. However, the issue of modifying the shape of an involute gear tooth was not under consideration there.

Research paper [19] is dedicated to multiobjective design optimization of geared transmissions. The efficiency of the proposed method is demonstrated by solving a challenging gear design problem, namely, ease-off topography optimization of a hypoid gear set for maximum efficiency and minimum contact stress. This paper does not address the issue of modifying the shape of an involute gear tooth to increase the loading capacity and reliability of gears.

In research paper [20], the profile modification and its influence on the load sharing and transmission error are de-

scribed. The tip relief of the driving and driven gear profiles as a method of modifying the gear tooth was studied. The relief of the tooth leg, however, was not considered in that context.

Research paper [21] proposes a novel spur gear dynamical model to evaluate the influence of the tooth pitting and spalling on the vibration responses of a gear transmission. This model is validated by comparison with responses obtained from the experimental test rig. This paper does not address the issue of modifying the shape of an involute gear tooth to increase the loading capacity and reliability of gears.

The provided literature review confirms the importance of studies dedicated to increasing the loading capacity and reliability of gears by methods of optimizing involute gearing parameters, including modification of the gear tooth shape. Besides, it demonstrates that some aspects of the problem have not been fully studied yet.

Due to the noted contradictions between the existing theories and the results of experiments, the types of wear and destruction of the surface of the stems and heads of gear teeth require a different approach to calculating the gears of ball mills.

3. The aim and objectives of the study

The aim of the study is to justify research methods for developing rational parameters of heavy-duty gears of a ball mill.

To achieve this aim, the following objectives are accomplished:

- to provide simulation and research of the parameters of manufacturing heavy-duty gears on the basis of the AEROFLANK software package and to provide calculation of the gear drive parameters of the ball mill drive in the AEROFLANK program,
- to establish numerical solution of the problem of determining the dynamic and kinematic parameters of gearing in the software complex AEROFLANK before modification;
- to conduct a numerical experiment under the modification of teeth with the radii of 11 μm and 22 μm .

4. Simulation and research of parameters of manufacturing heavy-duty gears on the basis of AEROFLANK software package

The theoretical basis of the new approach to calculating gears is, firstly, the direct synthesis of geometry, which is not at the first stage of calculation bound to the original contour, and secondly, the functional calculation of the course of change in the rigidity of the gearing, stresses, loads, thickness of the lubricant layer, temperature at each tooth, as well as linear and torsional vibrations of the wheel body.

The new approach is implemented by the AEROFLANK computer simulation system [22].

The software package AEROFLANK, developed at the P. I. Baranov Central Institute of Aviation Motors (Moscow, Russia), consists of the following subsystems:

- 1) geometry of cylindrical gear wheels with a modified initial contour (blunted edges of teeth, undercut bases, border areas, etc.);
- 2) geometry of cylindrical gear wheels with asymmetrical teeth;
- 3) geometry of cylindrical chevron gear wheels;
- 4) bevel geometry with circular teeth and arch gears;

- 5) tolerances according to ISO, DIN and GOST standards;
- 6) tolerances for the shape of the profile modification and tooth direction;
- 7) key elements of gear grinding machine setting, including preparation of control files *.pfl;
- 8) drawing up graphs of tooth shape coefficients and graphs of specific tooth stiffness;
- 9) strength calculations according to GOST 21354-87, ISO 6336, and DIN 3990;
- 10) construction of the kinematic error by the method of solving the inverse problem of the gearing theory and calculation of the kinematic error, taking into account the deformation of teeth;
- 11) subsystems of solving integral and differential equations;
- 12) subsystems of calculating stress fields with a deep analysis of the depth of contact stress occurrence;
- 13) simulation of a course of change of contact and bending pressure depending on the input of teeth in gearing before an output of it and the distribution of pressure throughout the whole width of the teeth;
- 14) simulation of changes in the thickness of the lubricant layer between the contact surfaces of each tooth and the temperature at each point of contact, taking into account the characteristics of the oil;
- 15) simulation of torsional and radial vibrations as well as their spectral composition;
- 16) simulation of dynamic forces acting on the bearings;
- 17) calculation of stationary and bifurcation amplitude-frequency characteristics of the above mentioned processes;
- 18) direct calculations based on the data of measuring the errors of the wheel teeth profile.

5. Calculation of the gear drive parameters of the ball mill drive in the AEROFLANK software package

This section of the article describes the application of the AEROFLANK software package to calculate the gearing of a ball mill, the initial data of which are presented in Table 1.

Table 1

Gear parameters		
Technical parameters		
Gear wheel	Gear	Wheel
Number of teeth	20	260
Working module, mm	16	16
Angle of engagement, degree	20°	20°
Tooth angle	5°11'39"	5°11'39"
Coverage factor	2,023	1,502
Toothed ring width	600	600

The initial degree of accuracy of transmission was 7–6–6 according to GOST 1643-81 – Basic norms of interchangeability. Gear cylindrical gears. Tolerances [23].

The ball mill speed was 19 rpm, with the drive power of 800 kW.

Fig. 1 shows the window of the AEROFLANK software package.

Fig. 2 shows the designed initial gearing in the AEROFLANK program.

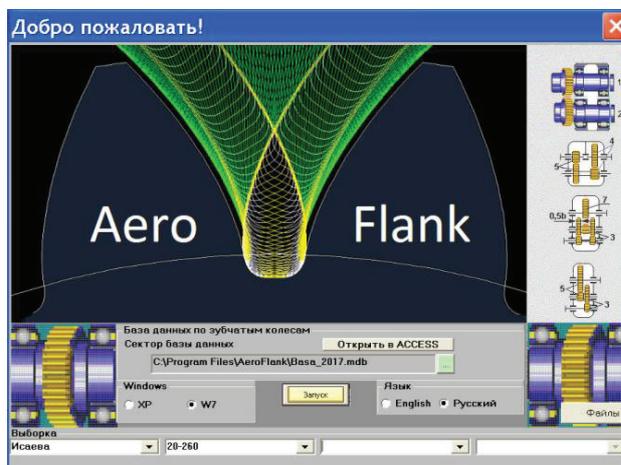


Fig. 1. The window of the AEROFLANK software package

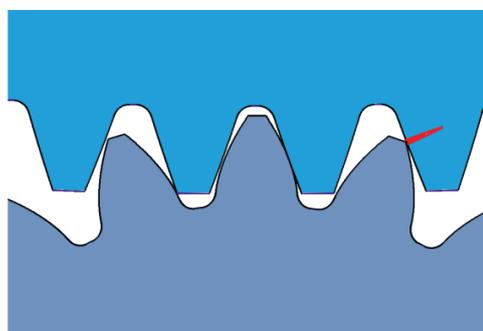


Fig. 2. Initial gearing in the AEROFLANK program

Fig. 3 shows the shape of the wheels' bodies and the shape of the teeth; yellow, indicates that there is a problem in this type of meshing, from the entry of the teeth into the meshing to the exit from it there are load fluctuations – this is a consequence of dynamic processes, excited by the variable rigidity of the teeth and errors in the profile of the teeth of the wheels.

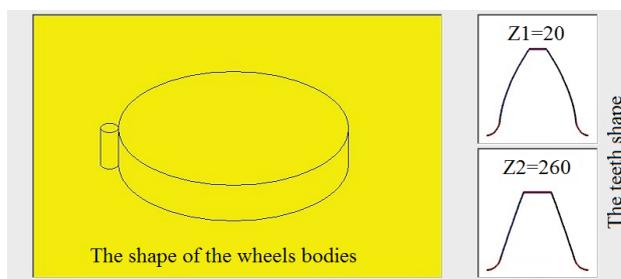


Fig. 3. The toothed shape

In order to evaluate the resistance of the tooth transmission, it is necessary to calculate the stress-strain state.

Fig. 4 shows the result of the stress distribution depending on the actual loads on the master and slave wheels.

The software enables to simulate places of fatigue cracks origin in the form of peaks located along the width of the teeth (Fig. 5).

Fig. 6, 7 show the calculated shape of the teeth in two-pair and single-pair gearing.

Fig. 8 show the total contact between the two teeth in grey, the driving wheel profile in green, the driven wheel profile in blue, and the straight line between the designated tooth profiles indicates an ideal transmission without errors.

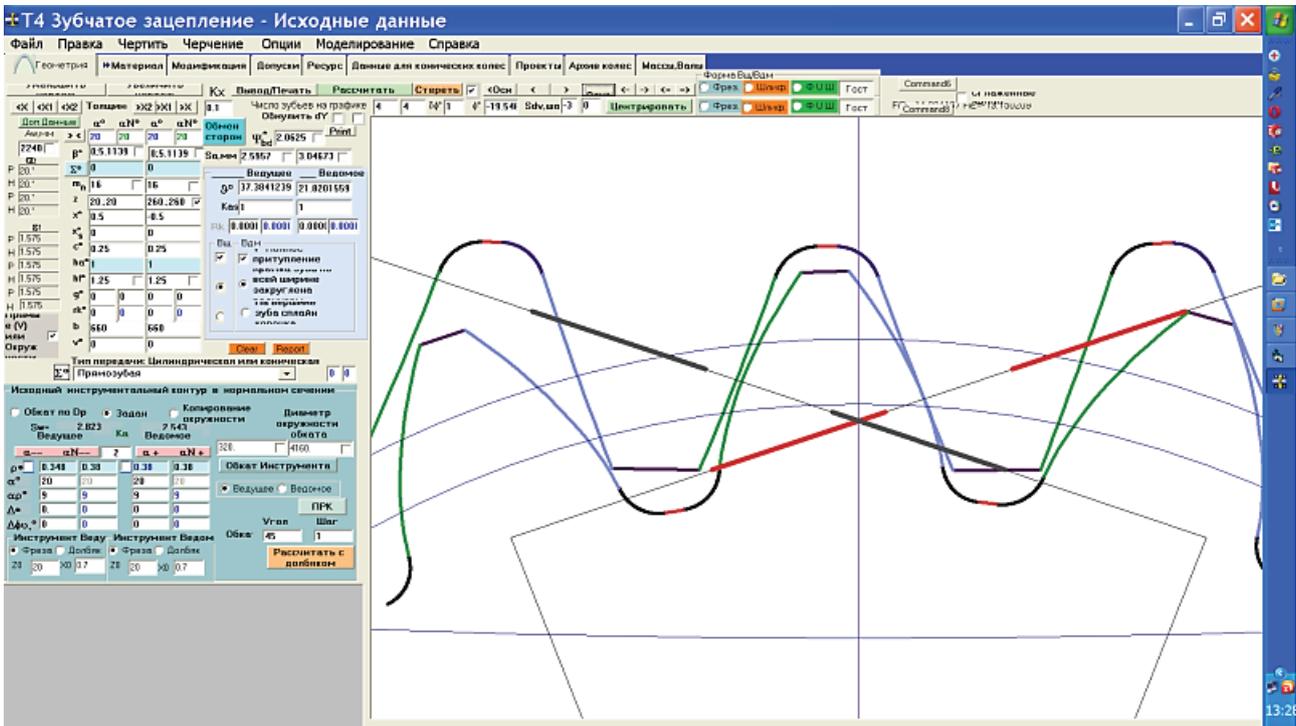


Fig. 7. A structural diagram of single pair teeth in the AEROFLANK program

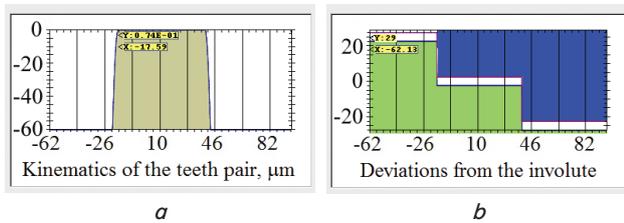


Fig. 8. Results of calculating gear kinematics: *a* – kinematics of a pair of teeth; *b* – deviations from the involute

The concentration points correspond to the contact of the tooth tips, and stress peaks correspond to bending stresses of 40 MPa (for the driving wheel), with contact stresses of 5,000 MPa. This is the reason for the failures shown in Fig. 9.

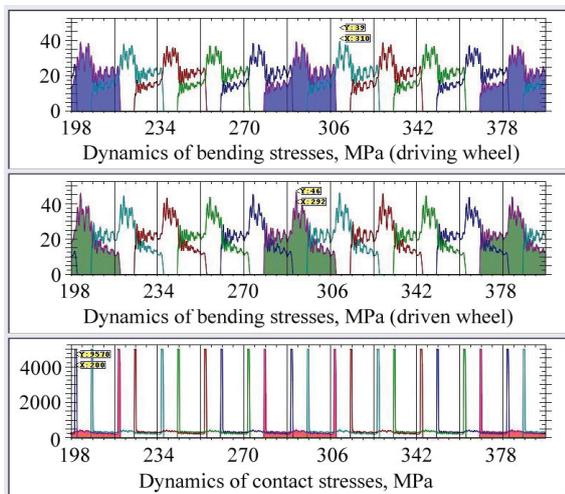


Fig. 9. Results of calculating bending and contact stress dynamics

Fig. 10, 11 show the result of calculating the transmission vibration power (sheet 1) and (sheet 2).

When gear elements are connected, the load on the teeth creates a dynamic oscillating process, which is caused by the presence of the edge contact. It has been determined that the main load is carried by outposts and their supports. In the course of operation, the edge contact additionally increases the development of mechanical wear and tear, which can lead to failure of the gear transmission and to a reduction in the reliability of the drive as a whole. The edge contact and, as a consequence of edge impact, reduces the impact of dynamic influences on the gearing.

One of the promising directions of gear development is the use of modifications of gear profiles. There are two ways of modification – creation of a barrel shape and flanking of the profile.

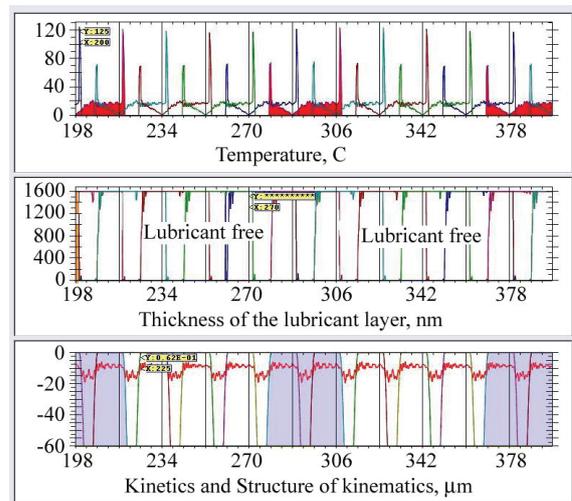


Fig. 10. Results of calculating the transmission capacity (sheet 1)

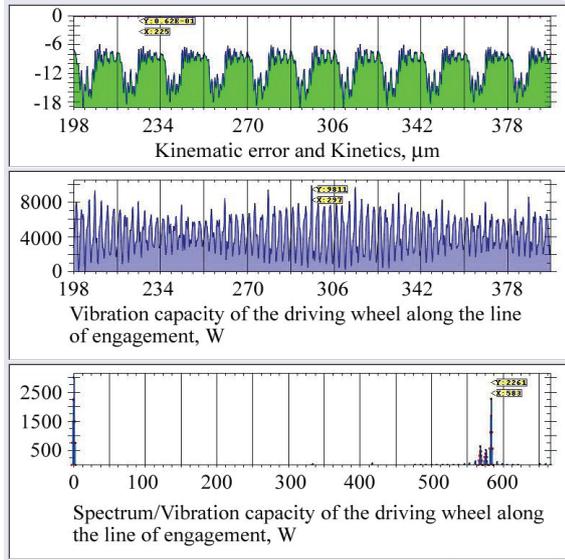


Fig. 11. Results of calculating the transmission capacity (sheet 2)

6. Numerical determination of dynamic and kinematic parameters of gearing before modification

In order to solve the problem of improving the quality of gearing in the AEROFLANK program, the parameters specified in section 5 were introduced.

Fig. 12–14 show the dynamics of bending and contact stresses of the leading and driven wheels before modification (large peaks of stresses, highlighted in red), the calculation of temperature, the thickness of the lubricant layer and the structure of kinematics, the calculation of the kinematic error, and the vibration capacity of the leading wheel on the line of meshing in the AEROFLANK program are visible.

Numerical tests to improve the quality of the gear transmission of the ball mill by creating a barrel shape and flanked profile have shown that the shape of the teeth must be changed so that the edges are removed from the adjacent profile and so that the contact stress is not greater than in the meshing pole.

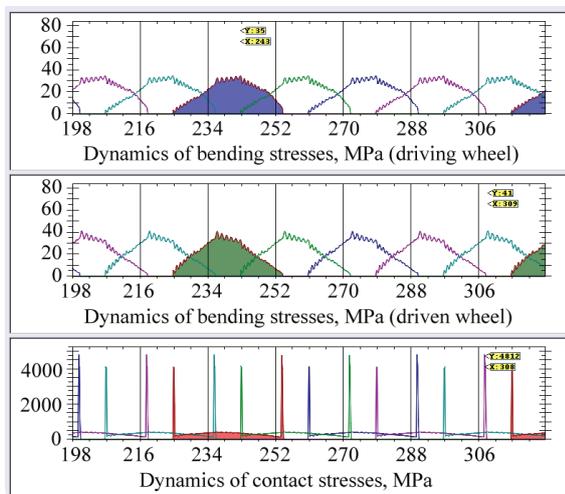


Fig. 12. Dynamics of bending and contact stresses of master and slave wheels

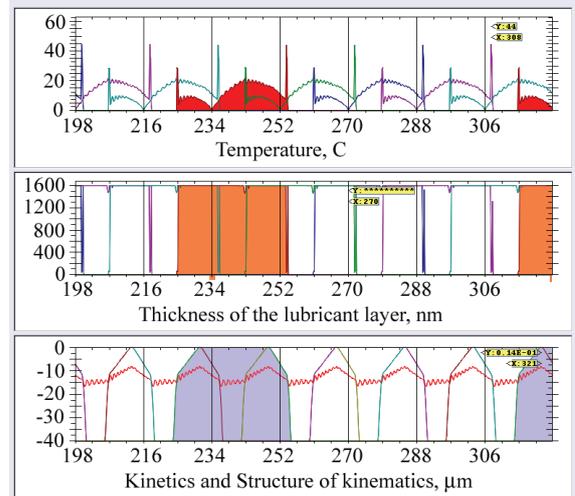


Fig. 13. Calculation of the temperature, lubricant layer thickness and kinematics structure in the AEROFLANK program

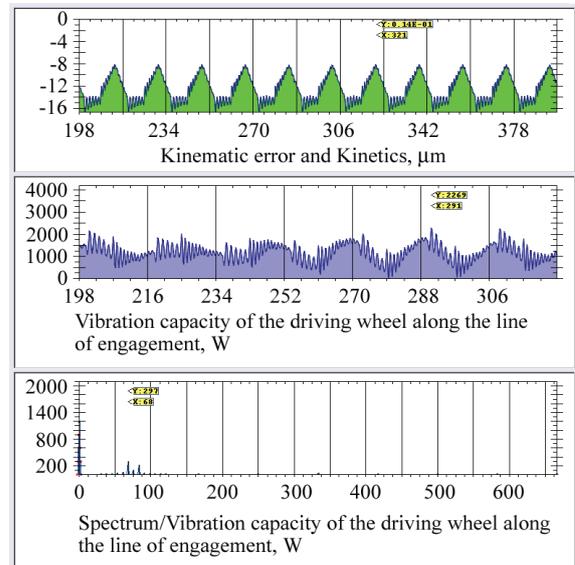


Fig. 14. Calculation of the kinematic error and driving wheel vibration capacity on the meshing line in the AEROFLANK program

7. Numerical tests on the modification of teeth with the radii of 11 μm and 22 μm

The calculation of the modified transmission begins with the entry of these parameters as well as the modification parameters by the type of the barrel shape and flanking as a tooth head cut with a radius of 11 μm, as is shown in Fig. 15.

Experimental calculations of gearing in the AEROFLANK program with modification of 11 μm were carried out, and the results are presented in Fig. 16–18. Fig. 20 shows the estimates of the transmission capacity. After the introduction of the modification of teeth (instead of the involute profile, which allows removal of the tooth tip), the stress concentration is eliminated.

The disadvantage is that it is difficult to modify a wheel of a large diameter. As the method of solving the problem, we suggest modifying not the driven wheel of a large diameter but the teeth of the driving wheel, both at their tops and legs.

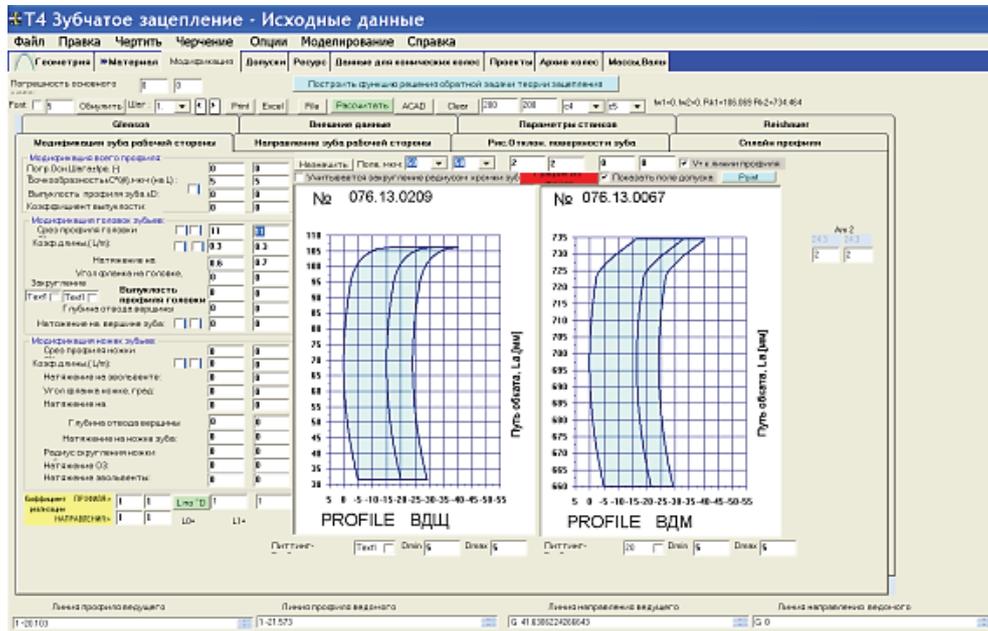


Fig. 15. Input data for modified transmission

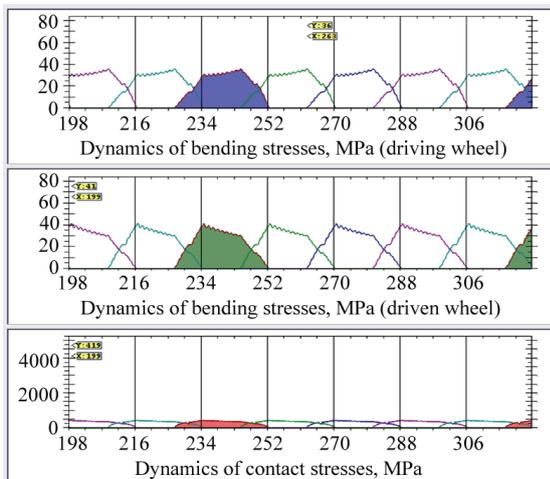


Fig. 16. Calculation of the bending stress dynamics (master and slave) and contact stress dynamics in the AEROFLANK program

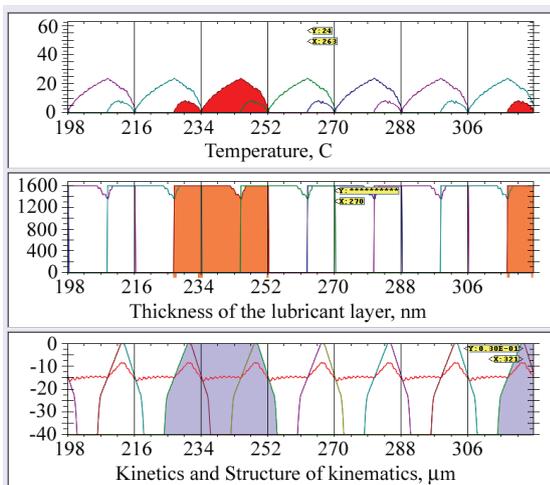


Fig. 17. Calculation of the temperature, lubricant layer thickness and kinematics structure in the AEROFLANK program

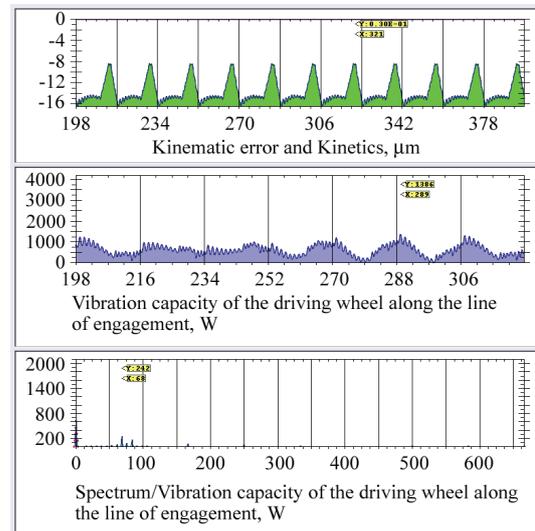


Fig. 18. Calculation of the kinematic error, vibration capacity of the driving wheel on the meshing line in the AEROFLANK program

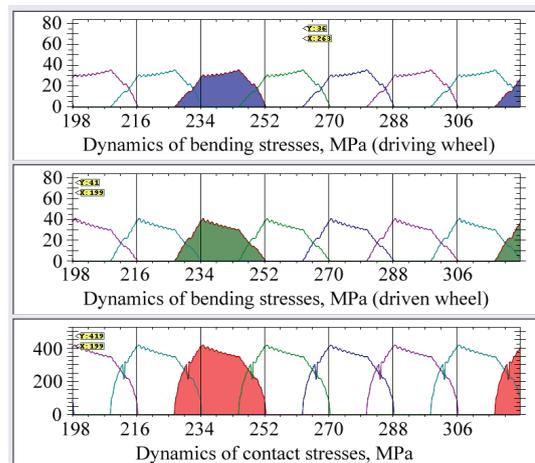


Fig. 19. Calculation of the bending stress dynamics of the master and slave wheels in the AEROFLANK program

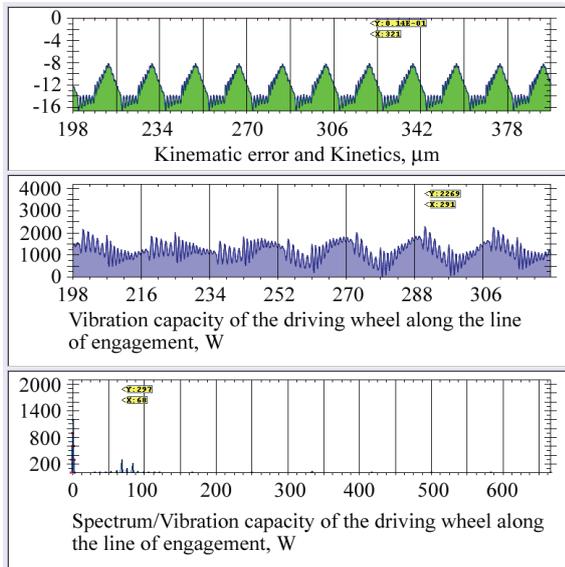


Fig. 20. Results of calculating the vibration capacity

The vibration capacity diagrams of the modified and not modified wheel show that there is an increase in resistance to vibration of the wheel with the changed profile by almost 1.5 times.

Further we carried out an experiment on flanking a tooth of a leading gear of 22 μm.

The initial data for flanking is the cut of the head profile and the tooth leg of the driving wheel of 22 μm, as in Fig. 21.

After the numerical calculation of the results of bending and contact pressures on the thickness of lubricating layer and also on the vibropower of the driving gear wheel, Fig. 22–25 were obtained as follows.

The results shown in the diagrams lead to a conclusion that the flanking of the teeth of a driving wheel of 22 μm decreases the contact pressure to 400 MPa, lowers the bending pressure of the driving wheel to 30 MPa, as well as decreases the temperature to 24 degrees and vibrability of the driving wheel on the line of gearing to 1,200 W.

Thus, flanking is recommended as a method to modify the involute gearing.

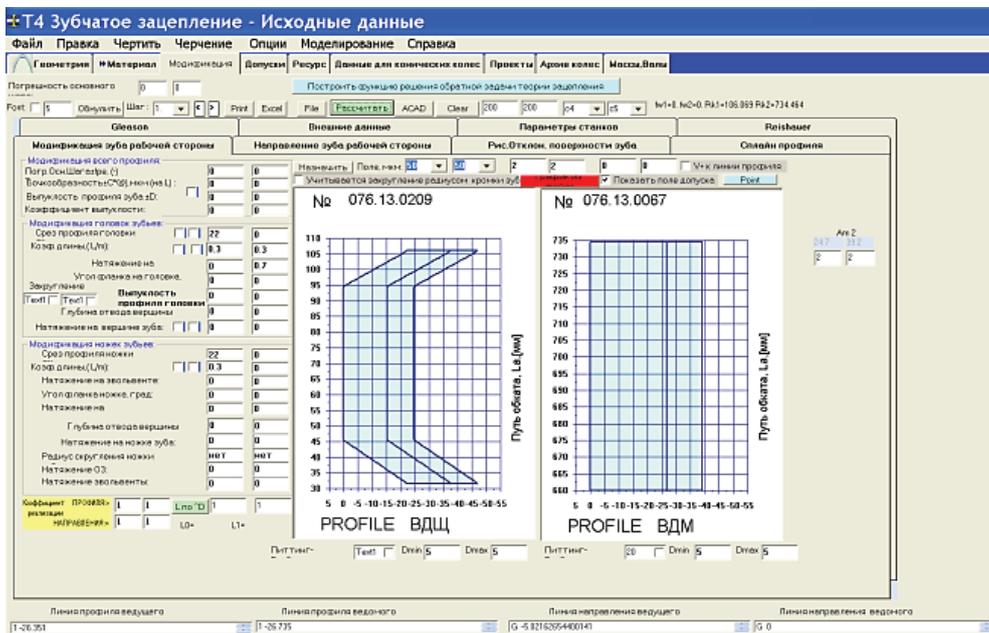


Fig. 21. The entry data window for the 22 μm drive wheel modification

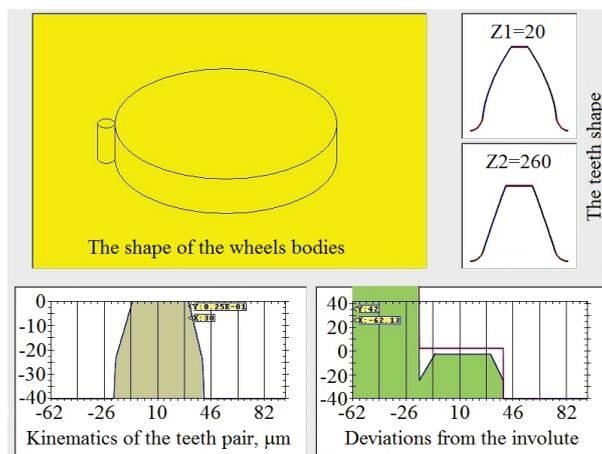


Fig. 22. Deviations from the involute of the driving wheel

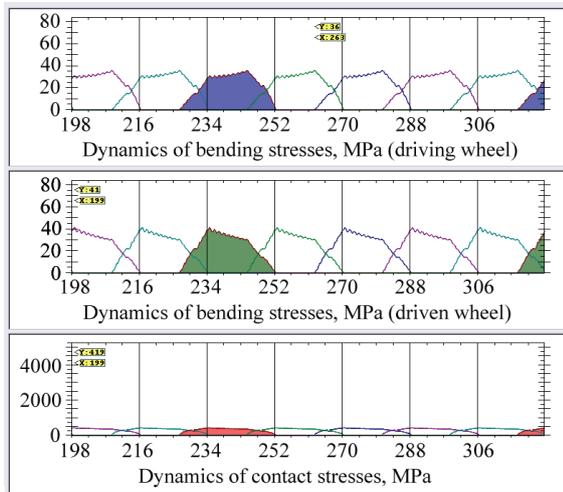


Fig. 23. Stress results for 22 μm of the drive wheel modification

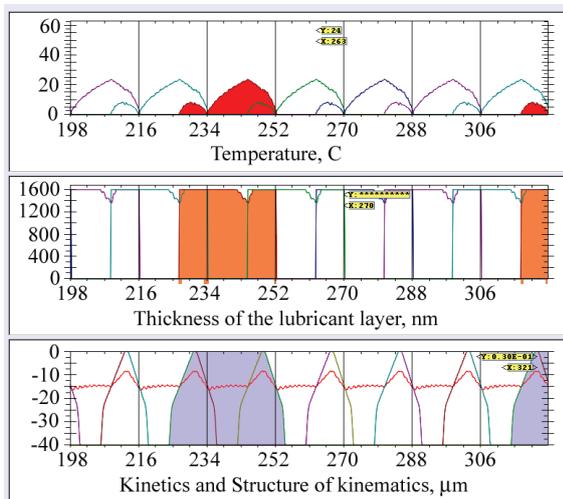


Fig. 24. Results of calculating the kinetic parameters of gearing with 22 μm of the driving wheel modification

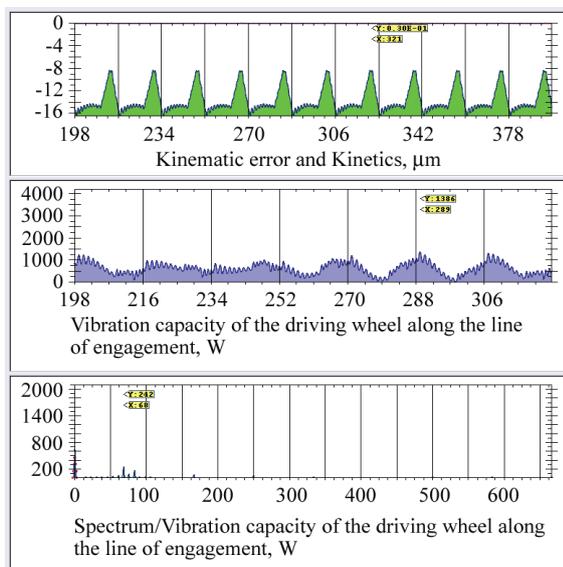


Fig. 25. Results of calculating the vibration capacity with 22 μm of the driving wheel modification

8. Discussion of the results of increasing the load capacity and reliability of gears by the methods of optimizing the involute gearing parameters

Using the AEROFLANK gear direct synthesis system has helped find a profile line in which the teeth contact only through the layer of a lubricant throughout the entire gearing line, which will increase the wear resistance of the ball mill gear. In the course of numerical experiments, it was proven that the designed gear train helps reduce several times the power loss on the vibration generation as well as decrease contact and bending stresses (Fig. 12, 16, 23, Sections 6 and 7).

Applying standard methods for calculating the geometric parameters of gears does not always allow providing the required quality features. In particular, it is not always possible to avoid edge contact in the engagement, which in some cases leads to significant wear during the initial period of transmission operation and deterioration of performance.

The choice of gear parameters due to the condition of minimizing the edge interaction as one of the main calculation criteria is justified by domestic and foreign studies and also by the set of experimental data.

In this regard, a system for selecting engagement parameters for heavy loaded ball mill gears (made up of wheels cut with a standard gear-cutting tool) based on the condition of minimizing the edge interaction of the teeth is proposed. In this case, flanking is recommended as a method of modifying the involute engagement.

On the basis of the carried out numerical experiments for modifications of 11 μm and 22 μm and the comparison of the results in the software package AEROFLANK as to the criterion of loading capacity, it is recommended to choose double flanking of a tooth head and leg with 22 μm as a modification parameter.

After modifying the involute profile of the teeth by removing the vertices of the teeth, the stress concentration is eliminated.

The disadvantage of this method is the complexity of modifying a gearing wheel of a large diameter. To solve this problem, we suggest modifying not the driven wheel of a large diameter but the teeth of the driving wheel, both at their tops and legs.

9. Conclusion

1. We have implemented modelling and research of the manufacturing parameters of heavy-duty gears on the basis of the software package AEROFLANK. The computer simulation of the ball mill gear drive makes it possible to visualize the stress distribution on the tooth surface and thus to determine ways for further modifying of the teeth shape to reduce stress at critical points of the teeth and to improve the reliability of the gear drive.

2. The numerical solution of the problem was aimed at increasing the quality of gearing. It is established that one of the promising directions of gearing development is the modification of the gear profile, which shows that the shape of the teeth should be changed so that the edges surfaces could be removed from the adjacent profile and so that the contact stresses could not exceed those in the meshing pole.

3. Computational experiments, with an 11 μm modification and a 22 μm modification, as well as calculations of the bending stress dynamics (of the driving and driven wheels)

in the AEROFLANK program have shown that with the modification of 11 μm , contact stress dynamics from the level of 4500 MPa decreased to the level of 450 MPa, and the mo-

dification with 22 μm reduces contact stresses to 400 MPa, lowers bending stresses to the level of 30 MPa, and decreases vibration to 1,200 W.

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