На підприємствах машинобудування, особливо на підшипникових заводах, і металургії, що здійснюють обробку металів, щомісяця утворюються тисячі тонн металовмісних шламів. В підшипниковому виробництві щорічно утворюється до 10,0 тисяч тонн шламів, які містять до 90 % металевої фракції. На теперішній час шлами практично не переробляються, а вивозяться в спеціальні захоронення або на звалища, погіршуючи екологію.

Для підвищення однорідності металевого порошку на стадії миття шламів необхідно здійснювати поділ твердих частинок за щільністю.

Для вирішення цього питання застосовується комплексна система екологічно безпечної технології утилізації шліфувальних шламів, де у процесі руху частинок шламу в потоці миючого розчину відбувається їх миття та розділення по щільності. Дослідження траєкторії руху твердих частинок шламу дозволяє встановити режимні параметри мийного жолобу, які забезпечують ефективне розділення частинок по щільності. Це дає можливість визначити висоту встановлення роздільника в потоці розчину та отримати більш значний відсоток однорідних металевих частинок.

Основними параметрами миючого жолоба, що можуть забезпечити ефективне розділення металевих і абразивних частинок, є: довжина, ширина жолоба, рівень потоку розчину, витрати розчину, витрати потоку рідини через бічну стінку жолоба, витрати потоку розчину через форсунки, кількість форсунок та відстань між ними.

На основі проведених теоретичних досліджень та побудованої математичної моделі, що описує рух металевих і абразивних частинок в миючому розчині, розроблено програму на мові С++ в середовищі С++ Builder 6.

Розроблена програма дає можливість моделювати траєкторії руху металевих і абразивних частинок в потоці миючого розчину в жолобі. В режимі випадкових параметрів частинок діаметр металевих частинок вибирається з діапазону 18–500 мкм, абразивних – 31–200 мкм

Ключові слова: шліфувальний шлам, металеві і абразивні частинки, траєкторія руху, мийний жолоб

MODELING THE TRAJECTORY OF MOTION OF METALLIC AND ABRASIVE PARTICLES IN A WASHING GUTTER

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1. Introduction

A considerable amount of metal waste is formed in grinding sludge as a result of the mechanical machining of ball-bearing steels at enterprises in the machine-building industry [1].

Environmental pollution with solid, liquid and gaseous wastes from production and consumption remains the most pressing environmental problem of social and economic priority.

In order to solve this problem, it is necessary to improve equipment and technology through engineering measures and raising work efficiency in the environmental field [2].

Composition analysis of sludges resulting from abrasive metal processing shows that metal particles removed from these sludges can be used as raw material for powder metallurgy because their sizes are within $5 \cdot 10^{-6}$ m to $600 \cdot 10^{-6}$ m. The content of metal particles in sludges of abrasive metal processing reaches 60–80 %.

Recycling of production wastes is a huge reserve of not only saving natural raw materials but also enhancing production efficiency and improvement of the ecological status of enterprises and the region territory [3].

Authors of [4] proposed a technology implying the washing of original sludge with a flow of solution of synthetic detergents to separate metal particles from abrasive ones due to differences in their densities. Metal and abrasive particles are dried separately and the purified solution is reused in a closed water circulating system.

To separate particles by density, it is necessary to impart a certain speed of horizontal movement to them and install a horizontal separator so that it can divide the solution flow into two flows.

For effective separation of metal and abrasive particles by their densities, it is necessary to study the trajectory of solids' motion in the flow of detergent solution. This will make it possible to establish mode parameters of the gutter and ensure effective separation of solid sludge particles and

subsequently use the metal particles in powder metallurgy and the abrasive particles in the production of abrasive tools and the construction industry. Processing of waste sludge will reduce the technogenic load on the environment. The use of secondary resources and their subsequent utilization will result in energy and resource savings. It is an urgent scientific and practical problem posed before industrialized countries for today.

2. Literature review and problem statement

Many authors of scientific and research papers [5–12] have devoted their active search to solve the problem of recycling grinding sludge with subsequent use of its components as secondary raw materials.

A technology of recycling grinding sludge whereby it is subjected to milling was developed in [5]. Cellulose was added to the sludge with its subsequent briquetting in a press under high pressure and sending the ready briquettes for melting. In the process of briquetting, lubricoolants were separated from the sludge. The addition of cellulose to the sludge was effective in maintaining the briquette shape. However, the issues of separation of metal and abrasive particles remain unresolved which limits the use of the final product. The environment is polluted with lubricant vapors when melting the briquettes.

A method of recycling sludge resulting from grinding high-strength 40X10C2M steel can be considered a variant of overcoming these obstacles. The sludge from grinding 40X10C2M steel is processed in a ball mill at a temperature of 60 to 80 °C for one hour followed by washing with detergents, magnetic separation and annealing in a reducing medium [6]. The use of this method involves high energy consumption and detergent vapors pollute the environment.

A method of recycling stainless steel sludge for using the metal powder in powder metallurgy was proposed in [7]. In this process, the sludge is subjected to milling, flotation, and settling of metal particles. However, flotation does not remove all pollutants since its effectiveness depends on the substance hydrophobicity. Additional costs are required for introducing reagents that improve foam quality and enhance the hydrophobicity of contaminants.

Authors of [8] have developed a method of recycling grinding sludges in which sludge is mixed with sodium salt, washed in a reactor, and finally filtered to separate the solid component. After filtering, the solid component is washed again for the final separation of surfactants and briquetted to minimize moisture content. Oil is separated from the detergent solution with a cloth filter. However, the problem of separating solids using the difference in their densities remains unresolved. This product is not a homogeneous metal suitable for use in powder metallurgy.

The authors of [9] have developed a technology of extraction of metal powder from sludges of abrasive metal processing. It includes a series of operations according to the classical scheme of processing and additional processing by milling and roll briquetting with subsequent annealing in a SUOL-044/12-M2-U42 electric furnace in hydrogen medium and a pass-through furnace of STN–2.45...2.5 type in an endogas atmosphere [10]. Unfortunately, the considerable laboriousness of the process does not make it possible to use it at most industrial enterprises. The material is dehydrated in an atmosphere of expensive natural gas and annealed at high temperatures of 850–900 °C with the supply of hydrogen into the furnace which makes the process environmentally dangerous.

The method of processing metal-containing sludges developed in [11] includes washing out lubricating and cooling liquids (LCL) from sludge, drying, milling in a white mill, sieving solids through a 0.63 mm mesh sieve and subsequent magnetovibrational separation. The disadvantages of this method include high power consumption connected with milling in a white mill and during magnetovibrational separation of the entire amount of sludge, poor quality of ShKh15 powder. The use of an active solution of soda detergent endangers the environmental safety of production.

A method of obtaining metal powder from sludge was considered in [12] according to which the crude sludge of tool production is loaded into a vacuum furnace chamber and the air is evacuated with a slow rise of temperature to 100 °C. Water is separated by distillation, then 80–85 % of oil is separated by vacuum distillation at a temperature of 300–350 °C. The resulting pasty product is heated to 850–900 °C and kept for 3–6 hours. Isothermal annealing at 700 °C is followed by milling and a 2–3-fold magnetic separation. Drawbacks of this method include considerable complexity of processing and high cost of equipment. Oil is distilled in a closed chamber for 3–4 hours and continuity of the process is not ensured at all its stages.

The literature analysis has shown that the abovementioned methods of reclaiming grinding sludge do not ensure uniformity of metal powder. The methods of producing metal powder do not provide mathematical models of the process of separation of solid sludge particles.

Mathematical models of the motion of solids in a fluid flow are proposed in [13–16].

The authors of [13] use a mathematical model developed on the basis of a Lagrangian-Eulerian approach to simulate the motion of a large number of solid particles in a liquid. Despite the benefits of Eulerian and Lagrangian methods, namely the ability to observe the interphase boundary, these models are complex in implementing and have a high demand for computational resources. The high-speed proportional collision between particles and with walls described in [13] does not affect the overall flow of particles in the fluid. It can be neglected when modeling high-speed flows. A significant increase in calculation time for a large number of solid particles is the model disadvantage.

A mathematical model of the motion of solid particles in a fluid flow is presented and its solution by the finite element method using conjugate gradient algorithms is described in [14]. The proposed method was used for three-dimensional modeling of sedimentation of two spherical particles in a flow and settling of 504 particles in a closed two-dimensional box. The authors use the method of dummy domains based on Lagrange multipliers for direct numerical modeling a flow with solid particles. However, the model does not take into account the effect of additional fluid flow in a perpendicular direction on motion and settling of particles.

The authors of [15] propose a mathematical model based on a dummy domain for modeling solid particles of any shape moving freely in a fluid flow. Numerical implementation of the model uses the method of finite volume on a related Cartesian grid together with the method of reduced step for variable flows with a small Mach number. However, the use of a non-uniform grid leads to an increase in computation time and additional requirements for the computer system resources.

A mathematical model of the motion of a solid particle in a unidirectional horizontal fluid flow is proposed in [16]. Analytical dependences of the trajectory of motion of a particle whose coordinates are obtained by the Lagrange method are presented. The model study was conducted for one particle sizing from 0.15 to 0.5 mm. The fact that the model describes only laminar fluid motion and does not simultaneously account for the motion of a large number of particles of different masses and densities is its disadvantage.

Analysis of published data makes it possible to conclude that the existing methods of reclaiming grinding sludge do not enable the obtaining of homogeneous metal powder. The above mathematical models consider the motion of solid particles in a unidirectional laminar fluid flow. Modeling of the trajectory of motion of solid particles having different densities in a joint motion of two fluid flows, that is, horizontal and vertical, remains unresolved. In view of the above, modeling of the process of separation of metal and abrasive particles in such conditions is an important problem of further improvement of the processes of reclaiming grinding sludges.

3. The aim and objectives of the study

The study objective is to model the trajectory of motion of metal and abrasive particles in a washing solution. This will make it possible to determine the mode parameters: flow rates of horizontal and vertical flows of solution through the nozzles located in the bottom gutter part, their number and distance between them. The determined parameters will enable the separation of solid sludge particles by their densities and obtaining a homogeneous metal powder.

To achieve this objective, the following tasks were solved:
– to construct a mathematical model for describing the trajectory of motion of metal and abrasive particles in the joint motion of two solution flows: horizontal and vertical;
– based on the constructed mathematical model, develop a computer program for visualization and numerical study of the trajectory of motion of metal and abrasive particles in the solution flow;
– to determine mode parameters of the gutter (flow rates in horizontal and vertical flows of solution through nozzles located in the bottom part of the gutter, their number and distance between them).

4. Constructing a mathematical model to describe the trajectory of metal and abrasive particles

The abrasive processing sludge consists of 60–85 % solid metal particles, 25 % abrasive tool particles and 10 % lubricating fluid [17]. The size (diameter) of solid abrasive particles is from 70 to 200 μm, density $\rho_a = 2,400$ kg/m^3. The size of metal particles is from 60 to 470 μm, density $\rho_m = 7,800$ kg/m^3 [18].

The density of the detergent solution is $\rho_p = 1,006.5$ kg/m^3 increases in the process of sludge washing by 7–10 % as concentration of oils in the washing process does not exceed 5–15 g/l.

To construct a mathematical model of the trajectory of solids in the washing solution, the following assumptions were made:
– mutual influence of solid particles in sludge is very small due to smallness of their mass and volume compared

to the flow rate of the detergent solution and length of the washing gutter;
– assume the density of the detergent solution constant due to the continuous cleaning from oils which is provided by technology [4].
– assume values of particle mass and size during washing are constant (the volume fraction of oil per particle is about 10 %).

At equal masses, the abrasive particle has a larger volume, so it is subjected to a greater ejecting force. In the process of movement of the sludge particles in the flow of detergent solution, separation of solid particles by their density takes place. By placing a separator at the end of the gutter, it is possible to obtain separation of metal particles from abrasive particles.

To separate metal and abrasive particles, a device consisting of a gutter, a conveyor feeding sludge, a divider, nozzle groups creating vertical flows was used. The schematic view of this device is presented in Fig. 1.
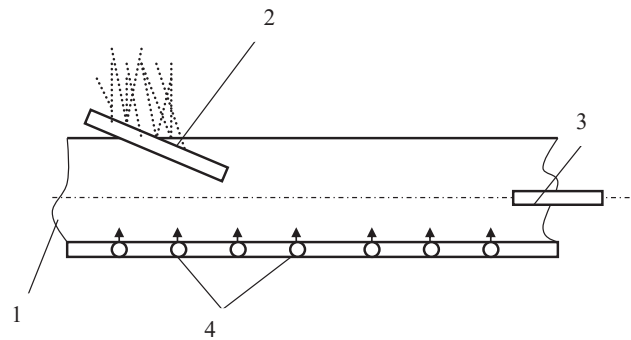

Fig. 1. Schematic view of washing gutter: gutter (1); conveyor (2); separator (3); nozzle groups that create vertical jets (4)

The following equations are used to describe the process under study. Dynamics of the fluid (detergent solution) is described by the Navier-Stokes equation:

$$\frac{\partial \vec{v}}{\partial t} = -\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} + \nu \Delta \vec{v} - \vec{\nabla} p', \tag{1}$$

$$\vec{\nabla} \cdot \vec{v} = 0, \tag{2}$$

where $\vec{v}$ is the speed of the detergent solution; $\nu = \mu/\rho$ is the coefficient of kinematic viscosity; $p' = p/\rho$ is the pressure normalized to density.

Assume the environment to be incompressible, that is, condition (2) holds.

The trajectory of motion of solid (metal and abrasive) particles in the moving flow of detergent solution will be described by the system of two nonlinear differential equations of second order:

$$\ddot{x} = \frac{F_{cx}}{m}, \tag{3}$$

$$\ddot{y} = \frac{1}{m}\left(F_a - mg + F_{cy}\right), \tag{4}$$

where $F_{cx} = 6\pi r \eta \left(v_x - \dot{x}\right)$ and $F_{cy} = 6\pi r \eta \left(v_z - \dot{y}\right)$ are the projection of the Stokes force acting on the particle from the side of the solution flow, that is:

$$\vec{F}_c = 6\pi r \eta \vec{v}, \tag{5}$$

where r is the radius of the particle; η is the coefficient of dynamic viscosity; $\vec{v}$ is the speed of motion of a particle relative to the solution, $\dot{x}$ and $\dot{y}$ are the derivatives of coordinates of the particle or projection of speed of the particle relative to the gutter, v_x, v_y are the projections of speed of the solution surge on the particle.

In determining resistance force from the solution on a particle moving in it, the Reynolds criterion exerts a significant influence:

$$\mathrm{Re} = \frac{Vd}{\mu}, \tag{6}$$

where V is the speed of relative motion, m/s; d is the characteristic size, m; μ is the dynamic viscosity, kg/(m·s).

When moving a particle in a flow of detergent solution, the speed of relative motion does not exceed 0.1 m/s with the Reynolds number Re≈1. This makes it possible to apply the Stokes formula (5) to the force acting on the particle from the side of the solution flow.

Beside the Stokes force, the particle is also affected by force of gravity and Archimedes force $F_a = \rho_s gV$, where ρ_s is the solution density (1,000 kg/m³); V is the particle volume.

The Eulerian approach was applied here, that is, the solution flow surges on the fixed particle with speed $\vec{v}$.

Using equations (3), (4), it is possible to obtain the coordinate value for each particular particle at the gutter exit through their computer implementation. The washing solution flow rate Q, the particle mass m, its density ρ are parameters of the model.

4. 1. A method of solving equations describing the motion of detergent solution

To solve the system of equations (1), (2), the method of splitting by physical factors is used taking into account the turbulent nature of the motion of the washing solution, namely the statement of uneven turbulence of the solution and localization of Reynold's number. This change in the model has resulted in the rejection of the constant coefficient of kinematic viscosity.

Let us consider the scheme of splitting by physical factors developed for the problems of the hydrodynamics of the flow of a viscous incompressible fluid [19, 20].

Equations (1), (2) include an unknown pressure field. To find it, the condition of incompatibility (2) must be used:

$$\vec{\nabla} \cdot \vec{v} = 0,$$

which completes equation (1) to a complete system of equations. As can be seen, we do not have an explicit equation for finding the pressure. This fact complicates the process of finding it. However, when solving hydrodynamic problems in natural variables, the pressure field plays an extremely important role. Namely, it ensures that the condition of medium incompressibility is satisfied and if this condition is not satisfied exactly, the liquid quickly becomes effectively compressible which immediately leads to a divergence in computations.

The basic idea of the method of detachment by physical factors implies the separation of finding pressure from the calculation of speed at each time step of the process. Namely, let us detach the last summand in formula (1) according to the general scheme of detachment. In this case, the detachment scheme takes the following form:

$$\tilde{\vec{v}} - \vec{v}^n = \tau\left(-\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} + \nu\Delta\vec{v}\right),$$

$$\vec{v}^{n+1} - \tilde{\vec{v}} = -\tau\vec{\nabla}\,p'. \tag{7}$$

The last equation includes an unknown pressure field. To find it, use the condition of incompressible medium (2) which results in:

$$\vec{\nabla} \cdot \vec{v}^{n+1} = 0. \tag{8}$$

Note that the intermediate speed $\tilde{\vec{v}}$ may not satisfy the condition of solenoidality (2) since the pressure field was not taken into account when finding it. Applying the operator $\vec{\nabla}$ to the right and left sides of the last of formulas (7) and considering relation (8), the pressure field equation is found:

$$\Delta p' = \vec{\nabla} \cdot \tilde{\vec{v}}/\tau, \tag{9}$$

which is a Poisson equation. Next, concretize the method of calculating the right-hand sides in the formulas of our detachment scheme. Assuming they are calculated at the n-th time layer, we arrive at a scheme explicit by time. As a result, the detachment scheme takes the following form:

$$\text{I } \tilde{\vec{v}} = \vec{v}^n + \tau\left(-\left(\vec{v}^n \cdot \vec{\nabla}\right)\vec{v}^n + \nu\Delta\vec{v}^n\right);$$

$$\text{II } \Delta p' = \vec{\nabla} \cdot \tilde{\vec{v}}/\tau; \tag{10}$$

$$\text{III } \vec{v}^{n+1} = \tilde{\vec{v}} - \tau\vec{\nabla}p'.$$

The finite-difference analog is constructed on a uniform chess grid in the Cartesian coordinates when $\vec{v} = u\vec{e} + w\vec{e}$:

$$\text{I } \tilde{u} = u + \tau\left[-\frac{\partial u^2}{\partial x} - \frac{\partial(uw)}{\partial z} + \frac{\partial}{\partial x}\left(\nu^x\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu^z\frac{\partial u}{\partial z}\right)\right],$$

$$\tilde{w} = w + \tau\left[-\frac{\partial w^2}{\partial z} - \frac{\partial(uw)}{\partial x} + \frac{\partial}{\partial x}\left(\nu^x\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu^z\frac{\partial w}{\partial z}\right)\right];$$

$$\text{II } \tilde{D} = \frac{\partial\tilde{u}}{\partial x} + \frac{\partial\tilde{w}}{\partial z}, \tag{11}$$

$$\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial z^2} = \frac{\tilde{D}}{\tau};$$

$$\text{III } u^{n+1} = \tilde{u} - \tau\frac{\partial p'}{\partial x},$$

$$w^{n+1} = \tilde{w} - \tau\frac{\partial p'}{\partial z}.$$

There are no components of the external volumetric force $\vec{F}$ in equations (1), (2). The fact is that, as a rule, the force of gravity $\vec{F} = \rho\vec{g}$, acts as $\vec{F}$ and is directed opposite to the z axis. Thanks to it, the static pressure of a column of liquid $p_{col} = -\rho gz + \text{const}$ is created in the liquid volume. In the case of the flat liquid surface, this pressure component does not affect the dynamics of fluid motion but is intended to compensate for the action of gravity force $-\vec{\nabla}p_{col} = -\rho\vec{g}$. Therefore, instead of full pressure, its dynamic component can be substituted in the Navier-Stokes equation and the force of gravity omitted. Thus, the variable $\tilde{p}$ in equations (11) is only connected with the dynamic pressure component $\tilde{p} = p_g/\rho$.

When writing a convective summand, a zero summand $\vec{v}\cdot(\vec{\nabla}\vec{v})$, was added to it because of speed non-divergence $\vec{\nabla}\cdot\vec{v}=0$, and the transformations were made.

When writing a diffusion summand, the coefficient of kinematic viscosity n is entered under the derivative sign. It has a vector character. The difference analog of equations (11) on the chess grid is:

$$\text{I } \tilde{u}_{i',j} = u_{i',j} - \frac{\tau}{2\Delta x}\left(u_{i'+1,j}^2 - u_{i'-1,j}^2\right) -$$

$$- \frac{\tau}{\Delta z}\left[(uw)_{i',j'} - (uw)_{i',j'-1}\right] +$$

$$+ \tau\left\{\frac{\begin{bmatrix}\nu_{i',j}^x\left[u_{i'+1,j} - 2u_{i',j} + u_{i'-1,j}\right]/\Delta x^2 + \\ \begin{bmatrix}\nu_{i',j}^z\left(u_{i',j+1} - u_{i',j}\right) - \\ -\nu_{i',j'-1}^z\left(u_{i',j} - u_{i',j-1}\right)\end{bmatrix}/\Delta z^2\end{bmatrix}}{}\right\};$$

$$\tilde{w}_{i,j'} = w_{i,j'} - \frac{\tau}{2\Delta z}\left(w_{i,j'+1}^2 - w_{i,j'-1}^2\right) -$$

$$- \frac{\tau}{\Delta x}\left[(uw)_{i',j'} - (uw)_{i'-1,j'}\right] +$$

$$+ \tau\left\{\frac{\begin{bmatrix}\nu_{i',j'}^x\left(w_{i+1,j'} - w_{i,j'}\right) - \\ -\nu_{i'-1,j'}^x\left(w_{i,j'} - w_{i-1,j'}\right)\end{bmatrix}/\Delta x^2 + \\ +\nu_{i,j'}^z\left[w_{i,j'+1} - 2w_{i,j'} + w_{i,j'-1}\right]/\Delta z^2}{}\right\}; \quad (12)$$

$$\text{II } \tilde{D}_{i,j} = \left(\tilde{u}_{i',j} - \tilde{u}_{i'-1,j}\right)/\Delta x + \left(\tilde{w}_{i,j'} - \tilde{w}_{i,j'-1}\right)/\Delta z,$$

$$\left(p'_{i+1,j} - 2p'_{i,j} + p'_{i-1,j}\right)/\Delta x^2 +$$
$$+ \left(p'_{i,j+1} - 2p'_{i,j} + p'_{i,j-1}\right)/\Delta z^2 = \tilde{D}_{i,j}/\tau;$$

$$\text{III } u_{i',j}^{n+1} = \tilde{u}_{i',j} - \frac{\tau}{\Delta x}\left(p'_{i+1,j} - p'_{i,j}\right),$$

$$w_{i,j'}^{n+1} = \tilde{w}_{i,j'} - \frac{\tau}{\Delta z}\left(p'_{i,j+1} - p'_{i,j}\right).$$

where

$$(uw)_{i',j'} = \frac{1}{4}\left(u_{i',j} + u_{i',j+1}\right)\left(w_{i,j'} + w_{i+1,j}\right). \quad (13)$$

The necessary stability conditions at the first scheme stage (12):

$$\nu > \frac{\tau}{2}\max\left(u^2, w^2\right), \quad (14)$$

$$\nu > \max\left(\frac{\Delta x^2}{4}\frac{\partial u}{\partial x}, \frac{\Delta z^2}{4}\frac{\partial w}{\partial z}\right), \quad (15)$$

are obtained for the case of the constant coefficient of kinematic viscosity [19]. The first of them gives a time step constraint, and the second gives a coordinate step constraint.

To solve the pressure equation, the iteration method shall be used. This method does not depend on the form of the rated operating conditions which is its great advantage.

To obtain an iterative scheme, rewrite equation (12, II) in an equivalent form:

$$p'_{i,j} = p'_{i,j} - \omega\left(\tilde{D}_{i,j}/\tau - \Delta p'_{i,j}\right). \quad (16)$$

The parameter w in this formula is the parameter of convergence of the iteration scheme. The basic idea of this method is that, given any initial pressure field $p_{i,j}^{(0)}$, all its further values $p_{i,j}^{(1)}$, $p_{i,j}^{(2)}$, ..., $p_{i,j}^{(k)}$ were calculated proceeding from the previous ones using the right-hand term of formula (16):

$$p_{i,j}'^{(k+1)} = p_{i,j}'^{(k)} - \omega\left(\tilde{D}_{i,j}/\tau - \Delta p_{i,j}'^{(k)}\right). \quad (17)$$

The calculations are terminated when the following condition is satisfied for a predetermined value of ε:

$$\left|p_{i,j}'^{(k)} - p_{i,j}'^{(k-1)}\right| < \varepsilon, \quad \forall\, i,j. \quad (18)$$

The practical incompressibility of the fluid is achieved at a sufficiently small e which is manifested in non-divergence of its motion.

The condition of stability of the iterative process (17):

$$\omega \le d^2/2, \quad (19)$$

where $d = \min(\Delta r, \Delta z)$.

The effective kinematic viscosity is determined from the formula:

$$\nu' = \nu + \frac{\Delta}{\text{Re}_\Delta}v_\Delta, \quad (20)$$

where $\nu = \mu/\rho$; μ is the dynamic viscosity; Re_Δ is the Reynolds grid number and v_Δ is the speed within the cell of the computational domain.

The difference analog (20) on the chess grid can be realized as follows:

$$\nu_{i',j}^r = \nu + \frac{\Delta r}{\text{Re}_\Delta}\left|u_{i',j}\right|, \quad (21)$$

$$\nu_{i,j'}^z = \nu + \frac{\Delta z}{\text{Re}_\Delta}\left|w_{i,j'}\right|. \quad (22)$$

The values of viscosity coefficients in the grid corners $\nu_{i',j'}^r$ and $\nu_{i',j'}^z$ are the arithmetic means of the neighbor coefficients $\nu_{i',j}^r$ $\nu_{i',j}^z$:

$$\nu_{i',j'}^r = \frac{1}{2}\left(\nu_{i',j}^r + \nu_{i',j+1}^r\right), \quad (23)$$

$$\nu_{i',j'}^z = \frac{1}{2}\left(\nu_{i,j'}^z + \nu_{i+1,j'}^z\right). \quad (24)$$

It is known from numerical experiments that the Reynolds grid number $\text{Re}_\Delta = 2$ provides the calculation stability.

4. 2. A method of solving equations describing the trajectories of solid particles in a flow of detergent solution

Let us divide the time axis into segments with a step $dt = 0.001$. Let the components of the particle speed are vx^n and vy^n at the n-th iteration and its coordinates be x^n and y^n. Then, according to the Euler method, the speed components and the particle coordinates have the following form at the next step for the system (3), (4):

$$\begin{cases} vx^{n+1} = vx^n + \frac{6\pi r\eta}{m}\left(v_x - vx^n\right)dt, \\ vy^{n+1} = vy^n + \frac{1}{m}\left(\begin{array}{l}\rho_{\text{liquid}}gV - mg + \\ + 6\pi r\eta\left(v_y - vy^n\right)\end{array}\right)dt. \end{cases} \quad (25)$$

$$\begin{cases} x^{n+1} = x^n + vx^n dt, \\ y^{n+1} = y^y + vy^n dt. \end{cases} \quad (26)$$

This system of equations allows one to describe trajectories of any number of particles. Therefore, specifying initial values of speed and coordinates of the particles ($vx^0=0$, $vy^0=0$, $x^0=0$, $y^0=H$), the position of individual particles can be found at any time.

4. 3. Determination of the calculation domain and boundary conditions

The vertical section along the gutter is chosen as the calculation domain. The calculation domain takes the following form a rectangle. Using a uniform chess grid, divide the calculation domain into elementary cells and surround it with a layer of extralimital cells. This auxiliary layer of cells is necessary for localizing initial and boundary conditions. Denote the gutter length by L, m; width by d, m; solution level by H, m; solution flow rate by Q, m^3/s, solution flow rate through the side gutter wall by Q_x, m^3/s; flow rate through nozzles by Q_y.

At the left boundary, the fluid flows at a horizontal speed:

$$v_0^x = \frac{Q_x}{Hd} \ \text{m/s.} \quad (26)$$

The rate of fluid outflow through the nozzles:

$$v_0^y = \frac{Q_y}{dwN}, \ \text{m/s,} \quad (27)$$

where dw is the diameter of the nozzles; N is the number of nozzles.

Hence, we have the following boundary conditions for speed.

Left boundary:

$$v_\perp = v_0^x,$$

$v_\parallel = 0$ is taken into account if the flow is horizontal, with no vertical components.

Boundary conditions at the gutter bottom part have two options:
– above the openings:

$$v_\parallel = 0,$$

$v_\perp = v_0^y$ flow through the nozzles is strictly vertical;
– in other bottom parts:

$$\frac{\partial v_\parallel}{\partial \bar{n}} = 0: \text{ free slip condition;}$$

$$v_\perp = 0: \text{ the no-leakage condition.}$$

Right boundary:

$$v_\perp = V_{\text{liquid}},$$

$$\frac{\partial v_\parallel}{\partial \bar{n}} = 0: \text{ condition of free liquid outflow.}$$

The surface of the liquid:

$$v_\perp = 0,$$

$$\frac{\partial v_\parallel}{\partial \bar{n}} = 0.$$

Boundary conditions of the second kind were taken for pressure at all boundaries.

5. Development of a computer program for modeling the trajectory of motion of solid particles in a solution flow

Based on the above theory and the constructed mathematical model describing the motion of metallic and abrasive particles in a detergent solution, the C++ software was developed in the C++ Builder 6 programming environment. Its interface is shown in Fig. 2.
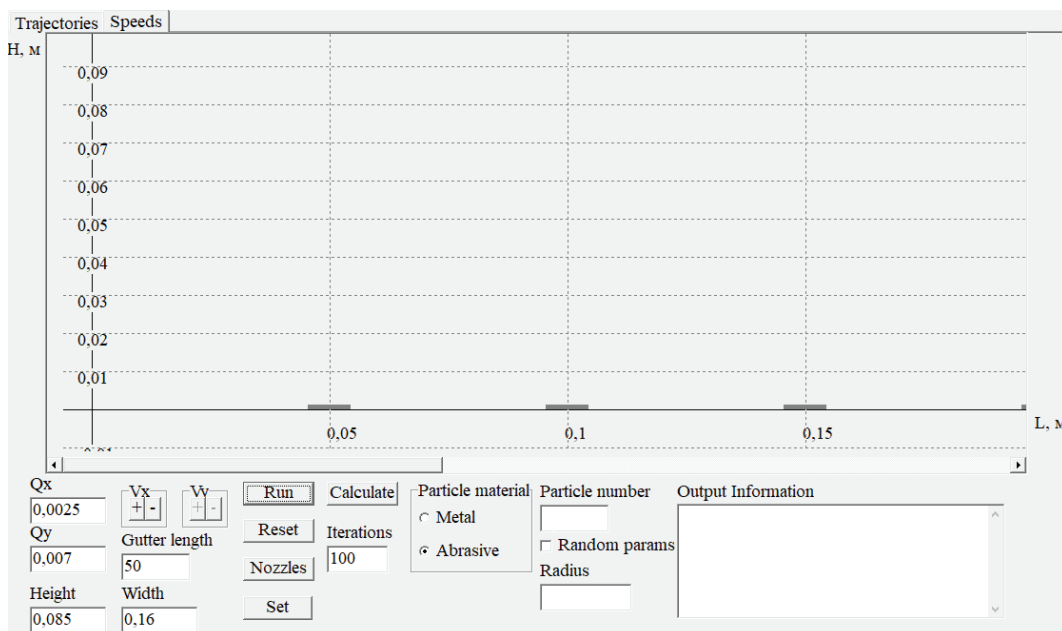

Fig. 2. Software interface

Let us consider a gutter of a variable length $L=0.5$ m, the solution flow level set during in the experiment $H=0.085$ m, the gutter width $d=0.16$ m. The nozzles have a form of openings with diameter $dw=0.001$ m. The position of nozzles on the gutter length can be changed by clicking the Nozzles button. The required configuration can be set in the window shown in Fig. 3.

Particle size (radius) in micrometers, particle material (metal or abrasive), the flow rate of the solution (horizontal and vertical) in m³/s can be specified in the main program window. The particles are considered spherical in shape. The Speeds tab allows one to observe changes in the speed of the washing solution.

The model adequacy was verified by examining the trajectory of the motion of one particle in solution. The size (diameter) was 18 to 500 μm for metal particles and 31 to 200 μm for abrasive particles.

To demonstrate the trajectory of motion, a particle with a radius of 100 μm was taken in the following examples. If there is no vertical flow, the particle trajectory is as shown in Fig. 4.

Next, direct vertical flow of fluid with $Q_y=0.001$ m³/s through three nozzles located at distances of 50, 100, 150 mm from the beginning of the gutter. Speed distribution and particle trajectory can be observed in Fig. 5, 6, respectively.
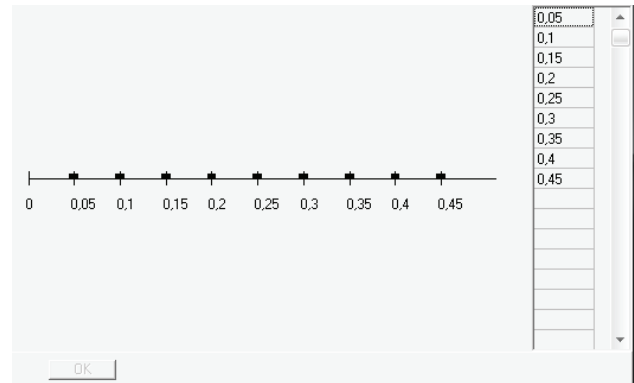

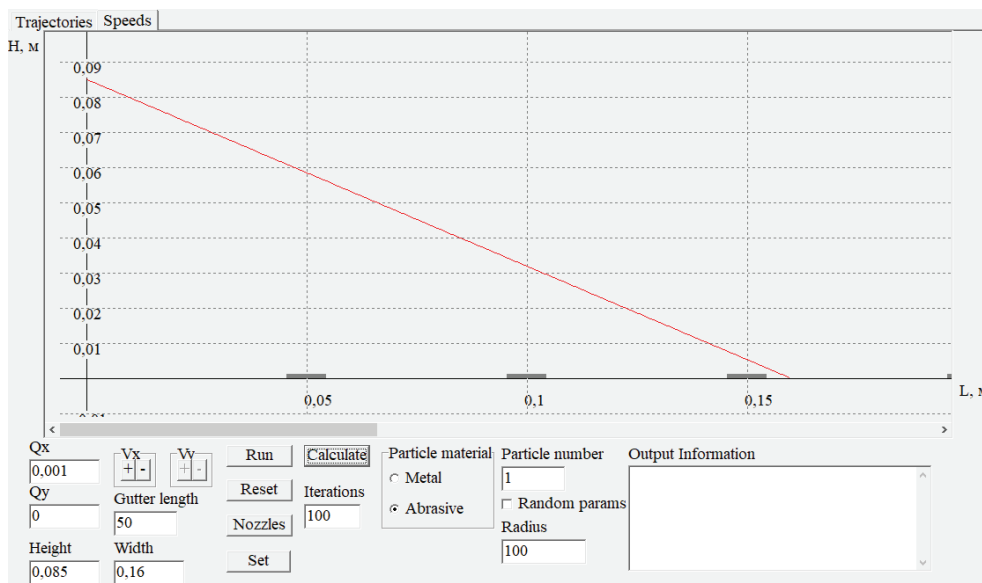Fig. 3. The window of setting the nozzle configuration

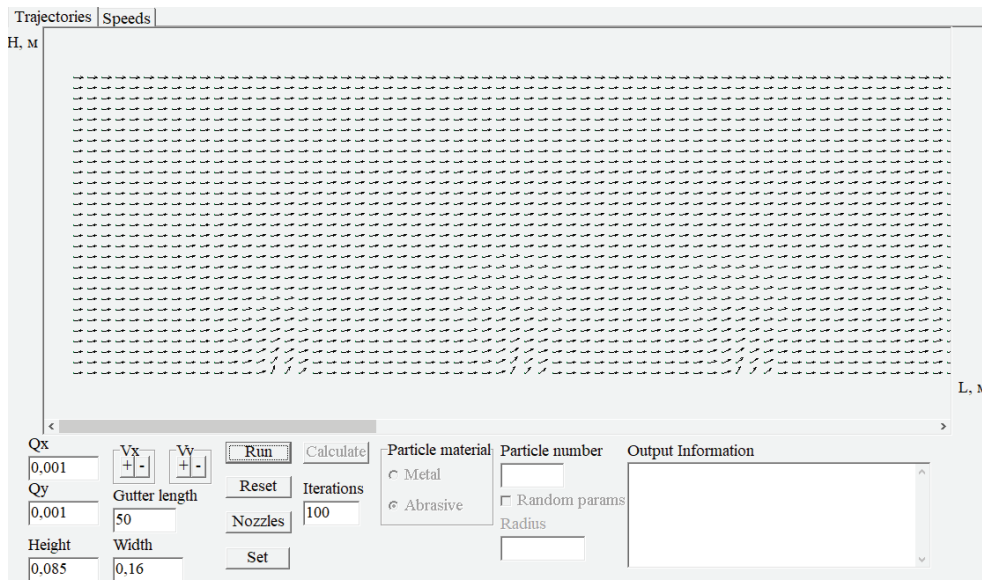Fig. 4. Trajectory of an abrasive particle (of radius $r=100$ μm) at $Q_x=0.001$ m³/s, $Q_y=0$ m³/s

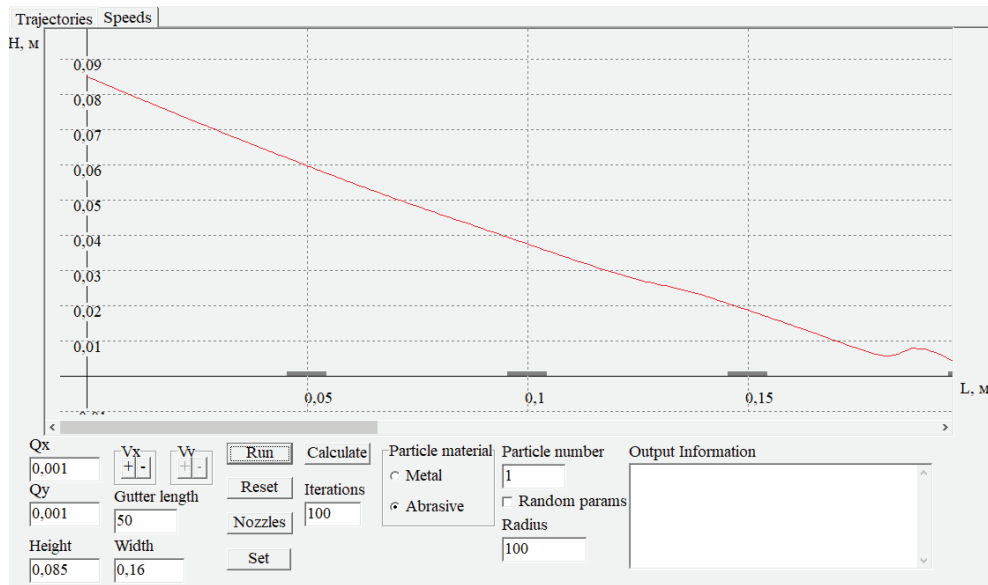Fig. 5. Distribution of speeds at $Q_x=0.001$ m³/s, $Q_y=0.001$ m³/s

Fig. 6. Trajectory of an abrasive particle (radius $r = 100$ μm) at $Q_x = 0.001$ m^3/s, $Q_y = 0.001$ m^3/s

Let us change the parameters of the task. Now, $Q_x = 0.0025$ m^3/s, $Q_y = 0.007$ m^3/s, the nozzles are spaced by 50 mm from each other along the entire length of the gutter. Trajectories of metal and abrasive particles having the same radius of 100 μm can be observed in Fig. 7. The heavier metal particle quickly falls after the start of the movement, moves at a short distance from the gutter bottom and the abrasive particle moves to the end of the gutter without falling.

The developed program has a calculation mode when it is possible not to specify the material and size (radius) of the particle. In this case, values are set randomly by the program, it is necessary just to indicate the number of particles whose trajectories will be found. The speed field for 500 particles will be calculated in 300 iterations leaving all other parameters unchanged.

Fig. 8 shows the trajectories of motion of metal and abrasive particles of random sizes.

Distribution of particles in the gutter is shown in the lower right corner of the program window:
– abrasive particles fallen: 18;
– abrasive particles reached the gutter end: 217;
– metal particles fallen: 219;
– metal particles reached the gutter end: 46;
– number of particles in the gutter: 0.

Percentage of metal particles on the gutter bottom: 82.64 % and percentage of abrasive particles reaching the gutter end: 92.34 %.

Reduce the horizontal flow to $Q_x = 0.001$ m^3/s at all previous conditions. The results are shown in Fig. 9:
– abrasive particles fallen: 54;
– abrasive particles reached the gutter end: 155;
– metal particles fallen: 255;
– metal particles reached the gutter end: 37;
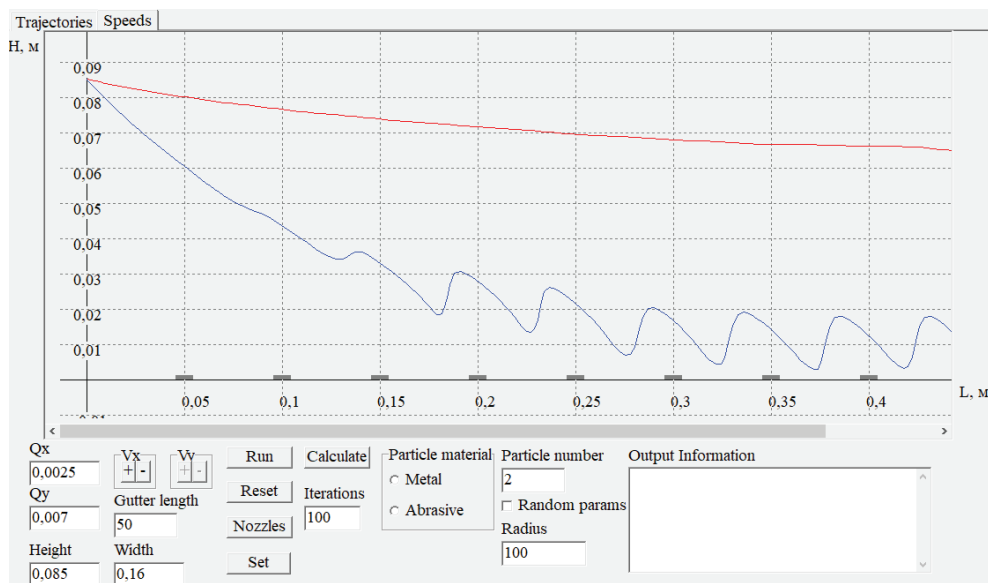– number of particles in the gutter: 29.

Fig. 7. Trajectories of two particles (radius $r = 100$ μm) at $Q_x = 0.0025$ m^3/s, $Q_y = 0.007$ m^3/s: — abrasive particle; — metal particle
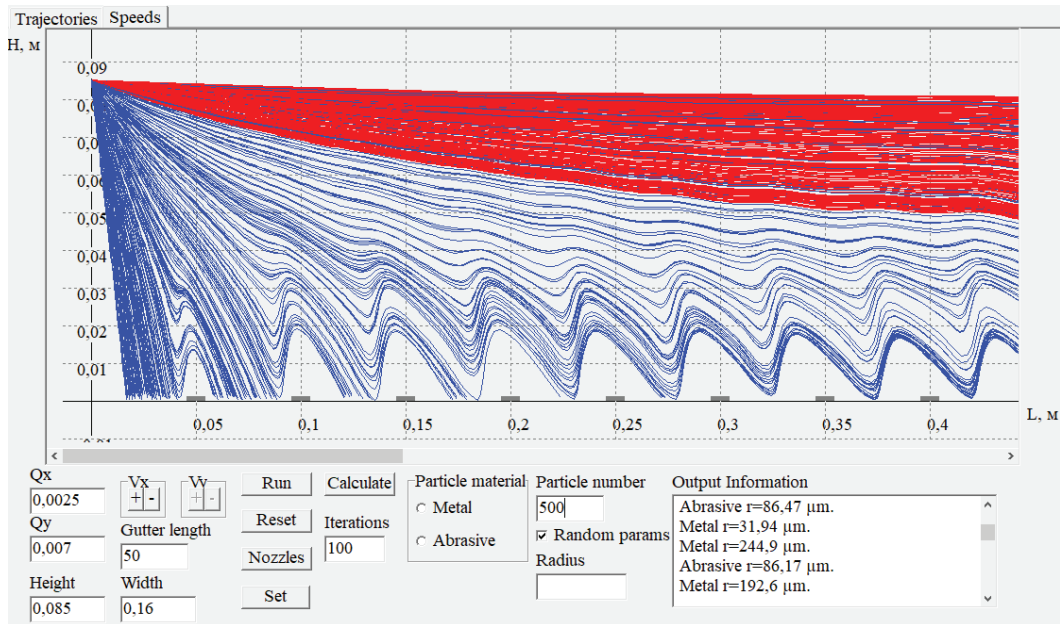
Fig. 8. Trajectories of five hundred particles with random sizes at Q_x=0.0025 m^3/s, Q_y=0.007 m^3/s:
——— − abrasive particle; ——— − metal particle
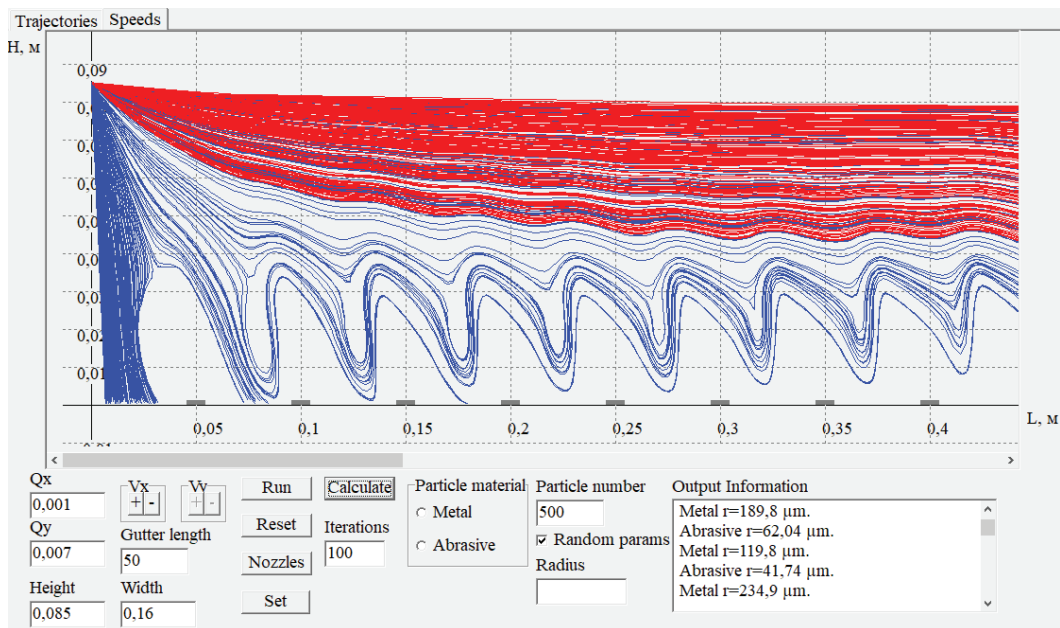
Fig. 9. Trajectories of five hundred particles with random sizes at Q_x=0.001 m^3/s, Q_y=0.007 m^3/s:
——— − an abrasive particle; ——— − a metal particle

As the horizontal flow decreases, fluid motion becomes more turbulent and some particles get in the vortex and move in circles. To prevent the program from falling into the «eternal» cycle of calculating the trajectory of a particle moving in a vortex, a limit on the number of iterations for trajectory calculation is set: 50,000 iterations. Under these conditions, the number of particles that did not fall to the bottom and did not reach the gutter end was 29.

If an objective is to completely separate abrasive particles from metal ones, then it is necessary to set Q_x=0.0025 m^3/s, Q_y=0.009 m^3/s. The calculation results are presented in Fig. 10. Percentage of metal particles at the gutter bottom:

77.96 %, abrasive particles reaching the gutter end: 100 %. Though the number of metal particles found at the gutter end has grown, they have the smallest mass. That is, only heavy metal particles will remain at the bottom under such conditions. The mass fraction of metallic particles at the gutter bottom is 99.92 %.

Therefore, the program makes it possible to simulate trajectories of motion of metal and abrasive particles in the solution flow in the gutter. In the case of random particle parameters, the size (diameter) is selected from the range of 18–500 µm for metal particles and 31–200 µm for abrasive particles. Distribution is uniform. The probability that the particle will be metallic or abrasive is the same.
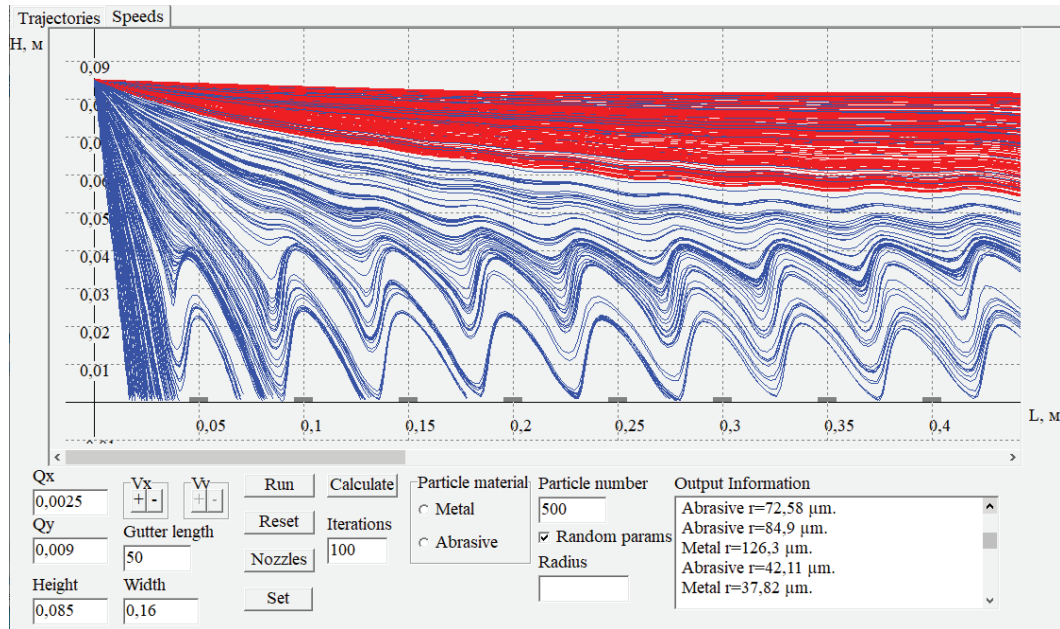
Fig. 10. Trajectories of five hundred particles with random parameters at $Q_x = 0.0025$ m^3/s, $Q_y = 0.009$ m^3/s:
—— — an abrasive particle; —— — a metal particle

6. Discussion of the results obtained in modeling trajectory of motion of metal and abrasive particles in a flow of detergent solution

A mathematical model has been developed to describe the trajectory of motion of metal and abrasive particles in a joint motion of two fluid flows: horizontal and vertical.

Navier-Stokes equations were used to describe the motion of the washing solution in which the turbulent nature of the fluid motion was taken into account by means of a two-parameter turbulence model which is an algebraic expression.

Newton's equations were used to describe the trajectory of the motion of solid particles. The equations take into account the forces acting on a solid particle in the solution flow, namely gravity, Archimedes, resistance and Stokes forces.

This has made it possible to determine trajectories of motion of solid particles of different sizes and densities in the flow of detergent solution.

The developed computer program enables visualization and numerical study of the trajectory of motion of metal and abrasive particles in the solution flow.

The program was developed on the basis of a mathematical model describing trajectories of motion of solid particles in a joint (horizontal and vertical) flow of solution. The computer program was implemented in the C++ language in the C++ Builder 6 programming environment. The software interface makes it possible to set initial flow rates of the solution, size, amount and material of solid particles, number of nozzles located in the bottom part of the gutter and distance between them (Fig. 2–10). The window of visualization of the trajectory of the motion of solid particles is the main interface object.

Studies were carried out for 18–500 μm metal particles and 31–200 μm abrasive particles. The gutter length was from 0.5 to 2.0 m. The horizontal flow of the washing solution varied from 0.001 to 0.0025 m^3/s and the vertical flow of the solution varied from 0 to 0.009 m^3/s.

The stability of solving equations of the obtained mathematical model was ensured by the introduction of a turbulent component for the effective coefficient of kinematic viscosity in the Navier-Stokes equations and selection of turbulence parameters.

The mathematical model and its software implementation make it possible to determine the gutter mode parameters (the flow rate of horizontal and vertical flows of solution through the nozzles located in the bottom part of the gutter, their number and distance between them) for real production conditions.

In the future, it is necessary to elaborate industrial designs of continuously operating equipment for the separation of solid particles by their density in order to study technological and structural parameters of the process equipment.

Further development and refinement of the mathematical model may imply the use of solid-phase transfer equations or equations with average-mass speeds describing the dispersed medium instead of Newton's equations. In this case, the model gets much more complicated and requires other methods of visualizing trajectories of solid particles.

Another way to substantiate the study results is to accomplish this task with the help of well-known numerical simulation programs, such as OpenFoam, which requires interface knowledge and certain skills of its use.

7. Conclusions

1. A mathematical model was developed to describe the trajectory of motion of metal and abrasive particles in the joint motion of two fluid flows: horizontal and vertical. The mathematical model features the possibility of its implementation in any software environment. To solve it, the method of partitioning by physical factors and the Euler method were used.

2. A computer program based on a mathematical model was used to visualize and numerically investigate trajectories of particle motion in a detergent solution flow. A numerical

study was conducted for 18–500 µm metal particles and 31–200 µm abrasive particles with the help of this program. Separation of solid particles of grinding sludge by their density in the process of moving in the solution flow provides a 56–60 % extraction of metal particles and a 40–44 % extraction of abrasive particles with equivalent diameters of 100–500 µm from the sludge.

3. The developed design and mode parameters of the washing gutter that have ensured effective separation of metal particles from abrasive ones in a flow of detergent solution were as follows: the gutter length $L=0.5$ m, horizontal flow rate $Q_x=0.0025$ m^3/s, vertical flow rate $Q_y=0.009$ m^3/s, the nozzle spacing: 50 mm; distance between the leftmost nozzle and the gutter beginning: 50 mm.

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