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NON-TRADITIONAL CALCULATIONS OF ELEMENTARY MATHEMATICAL OPERATIONS: Part 1. MULTIPLICATION AND DIVISION

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Abstract

Different computational systems are a set of functional units and processors that can work together and exchange data with each other if required. In most cases, data transmission is organized in such a way that enables for the possibility of connecting each node of the system to the other node of the system. Thus, a computer system consists of components for performing arithmetic operations, and an integrated data communication system, which allows for information interaction between the nodes, and combines them into a single unit. When designing a given type of computer systems, problems might occur if:

– computing nodes of the system cannot simultaneously start and finish data processing over a certain time interval;

– procedures for processing data in the nodes of the system do not start and do not end at a certain time;

– the number of computational nodes of the inputs and outputs of the system is different.

This article proposes an unconventional approach to constructing a mathematical model of adaptive-quantum computation of arithmetic operations of multiplication and division using the principle of predetermined random self-organization proposed by Ashby in 1966, as well as the method of the dynamics of averages and of the adaptive system of integration of the system of logical-differential equations for the dynamics of number-average states of particles S1, S2 of sets. This would make it easier to solve some of the problems listed above.

Keywords: calculus systems, computational systems, processor, functional nodes of the systems, interaction between functional nodes of systems, execution time of arithmetic operations, adaptive-quantum calculations of arithmetic operations.

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1. Introduction

The main advantage of positional calculus systems, in comparison with non-position systems, is the simplicity of performing arithmetic operations. Positional calculus systems face a problem of performing operations in a particular bit regardless of the performance of operations in all other bits. The solution to this problem is ensured by special tools. The systems of residual classes are not positional systems. They do not have problems inherited from the above-mentioned positional calculus systems. Residual class systems have the following disadvantages:

1. The complexity of performing operations of division and rounding.

2. The absence of attributes that indicate that the number exceeds the maximum allowable value.

3. The lack of visual link between the value of the number represented in the systems of the residual class, without its transformation into the positional number system.

That is why the computational systems of residual classes are not used in modern computer engineering. Then, there is a question. Why do we need to create non-traditional methods for calculation of elementary arithmetic operations if they are well established in modern computer machinery? A large class of software-controlled electronic computing machines has been developed at present. This class of computing machines is fully satisfied with the classical methods for implementation of mathematical operations. The class of computers and systems was complemented specifically by reconfigurable computers [1–11], computer data transmission systems [5, 12–16] and others. Modern level of computer technologies has created a huge class of computing systems that contains a lot of processors, functional nodes of the system (including computational nodes), which can interact with each other if required. Such computer systems, in the course of data processing, often face a problem of synchronizing the interaction between computing nodes of the system. The time for calculating arithmetic operations by classical methods using the computational nodes of the system is typically (with rare exceptions) different for different operations and data. In particular, this is due to the fact that the execution time of arithmetic operations by classical methods is strictly determined. When designing strictly synchronous computational systems, the following problems arise: Computational nodes of a system cannot simultaneously start and finish data processing over a predetermined time interval. Data processing procedures in the nodes of the system do not start and do not end at certain time. The number of inputs and outputs of computational nodes of the system is different. The introduction given in Section 1 provides an overview of some of the problems that occur when designing strictly synchronous computer systems. Section 2 deals with models of the non-traditional approach to calculating arithmetic operations of multiplication and division. The results of non-traditional quantum computation of arithmetic operations of multiplication and division using the mathematical model given in Section 2 are presented in Section 3.

2. Models of non-traditional approach to calculating arithmetic operations of multiplication and division

2. 1. Non-traditional Mechanical Model for Calculating Arithmetic Operations. Multiplication and Division

A non-traditional mechanical model for quantum computation of arithmetic operations of multiplication and division in the initial state is shown in **Fig. 1.**

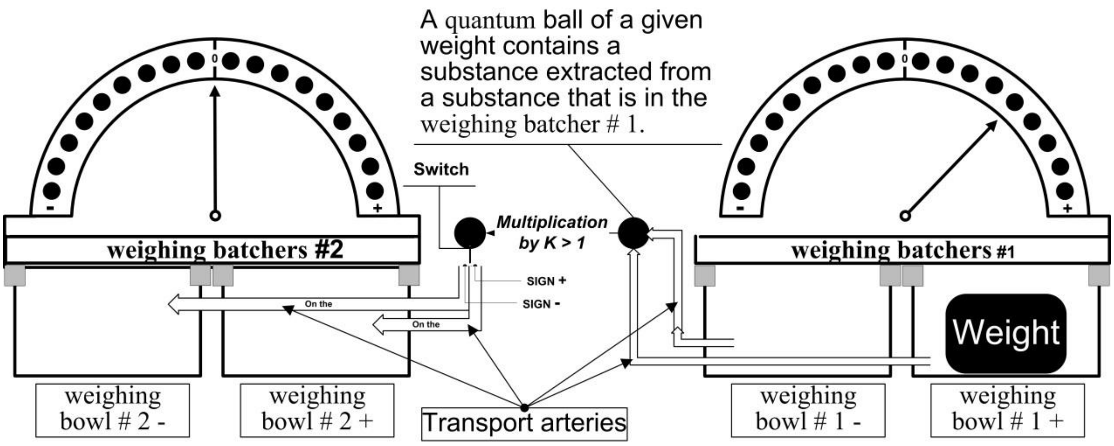

Fig. 1. Non-traditional mechanical model for quantum computation of arithmetic operations of multiplication and division

A non-traditional mechanical model includes weighing dispensers 1 and 2. Weighing dispensers 1 and 2 have polar pointers of weight. Scales of weighing dispensers, shown in **Fig. 1−4**, exhibit the weight placed on them. The weight of the load of weighing dispensers 1 and 2 is represented by a real number with a floating point. The mechanical approach to calculating quantum arithmetic operations of multiplication and division is performed cyclically in the following way.

A quantum of the extracted substance from the contents of weighing bow l+1 of weighing dispenser 1 through a transportation artery is placed in the quantum ball shown in **Fig. 1.** We call numerical value of the weight of a quantum ball its quantum number. The moment of emergence of quantum of the extracted substance in the quantum ball is the beginning of the cycle and the existence of a quantum ball in it in the process of calculating the arithmetic operations of multiplication/division. The quantum ball is assigned with the sign of the weighing bowl from which the quantum mass of the substance is extracted. In the case under consideration, the quantum ball is assigned with a plus sign, since the extraction of a quantum of matter occurs from weighing bowl 1 + of weighing dispenser 1. Next, as shown in **Fig. 2**, the weight of the quantum ball is multiplied by coefficient K and it displaces along the respective transportation arteries to weighing bowl 2 + weighing dispenser 2.

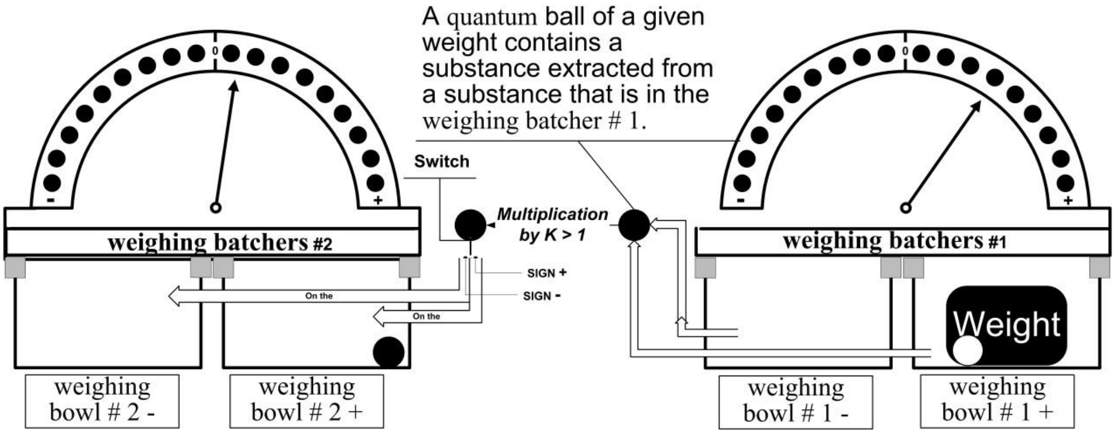

Fig. 2. The first cycle of quantum computation of arithmetic operations of multiplication/division for a physical model shown in **Fig. 1**

In this case, this is explained by the fact that, after multiplying the positive quantum sphere by positive coefficient K, its sign did not change. The second and the last (n-th) cycles of quantum computation of arithmetic operations of multiplication / division are shown in **Fig. 3, 4.**

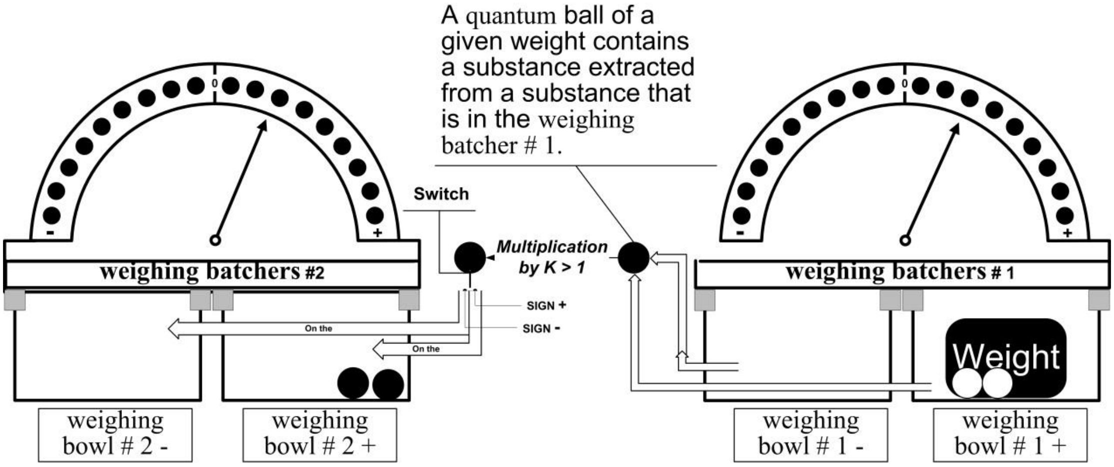

Fig. 3. The second cycle of quantum computation of arithmetic operations of multiplication/division using a physical model shown in **Fig. 1**

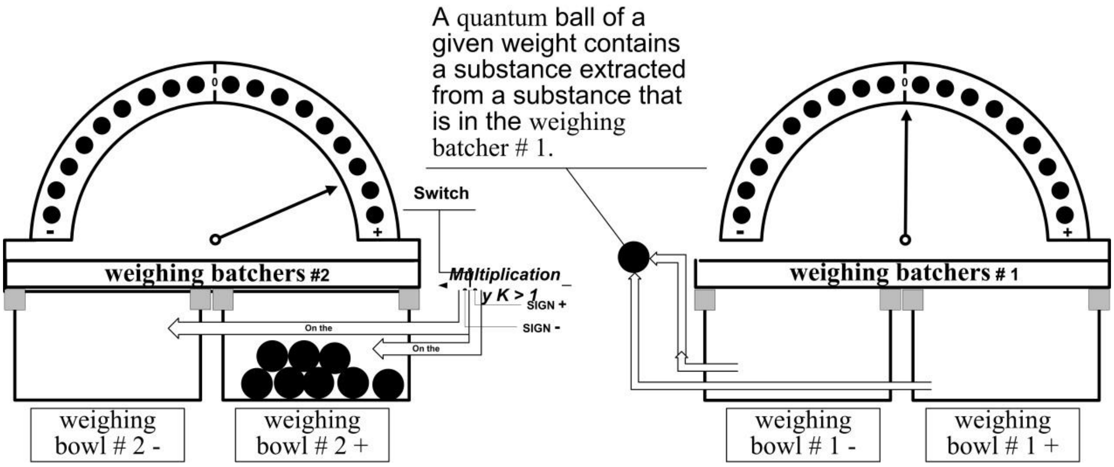

Fig. 4. N-th cycle of quantum computation of arithmetic operations of multiplication/division using a physical model shown in **Fig. 1**

The cycles of quantum computation of arithmetic operations of multiplication/division will end at the point in time when the weight of the contents of weight bowl 1 + weighing dispenser 1 becomes less than the weight of the quantum ball. The accuracy of quantum computation of arithmetic operations of multiplication/division using a mechanical model, shown in **Fig. 1**, is given in **Table 1.**

Table 1
Results of testing the accuracy of calculation by the mechanical model of quantum computation of arithmetic operations of multiplication/division

UN	QNQB	k	EN=UN*k	K=k* QNQB	NQB= =|UN/QNQB|	FN=NQB*K	DBE_FN= =UN–FN
3,275.17	0.2	1	3,275.17	0.2	16,375	3,275	0.17
3,275.17	0.2	3.17	10,382.2889	0.634	16,375	10,381.75	0.5389
3,275.17	0.1	3.17	10,382.2889	0.317	32,751.7	10,382.12089	0.16801
3,275.17	0.01	3.17	10,382.2889	0.0317	32,7517	10,382.2889	0
0.32517	0.1	1	0.032517	0.1	3	0.3	0.0251
0.32517	0.1	0.3	0.097551	0.03	10.839	0.03	0.29517
0.32517	0.01	0.3	0.097551	0.003	108.39	0.32517	0

Table 1, the following designations are used: 1. **N –** Initial number. 2. **QNQB –** Quantum number of the quantum ball. 3. **k –** Coefficient of multiplication. 4. **EN –** Expected number. 5. **K –** Transformed multiplication factor. 6. **NQB –** Number of quantum balls. 7. **FN –** Resulting number obtained as a result of multiplying all quantum balls by the transformed coefficient. 8. **DBE_FN –** Difference between the expected and the resulting numbers.

It is known that any real number represented as a floating-point number can be represented by a certain number of quantum balls whose quantum numbers are assigned from the permissible range for the data type that the real number belongs to. Results given in **Table 1** confirmed the reliability of the proposed mechanical model for quantum computation of arithmetic operations of multiplication / division, which, of course, does not require its direct implementation. Terminological concepts, introduced in the process of creating a quantum mechanical model for performing quantum computations of arithmetic operations of multiplication and division, are given below:

1. **Quantum ball** is a sphere filled with a quantum of the substance, extracted externally, of a certain weight represented by a real number.
2. **Quantum number of the quantum sphere** is the numerical value of the weight that it contains.
3. **Elementary particle of the real number** is a quantum ball whose quantum number is equal to the minimum possible numerical value within a permissible range for the data type to which the type of the numerical value belongs.
4. Quantum ball has two states – the active state, or the passive state.
5. Quantum ball is considered to be active if its quantum number is not zero.
6. Quantum ball is considered to be passive if its quantum number is zero.

The above terminological concepts have been applied, in particular, when constructing a quantum mechanical mathematical model for calculating arithmetic operations of multiplication and division.

2. 2. Quantum Mathematical Model for Calculating Arithmetic Operations of Multiplication and Division

The construction of a quantum mathematical model for calculating arithmetic operations of multiplication and division is based on the structure of Markov random chains and, as a consequence, on the method of number-average dynamics (dynamics of average) [17]. In the proposed quantum-mechanical mathematical model for quantum computations of arithmetic operations of multiplication and division, a mechanism is implied for the interaction of positive and negative elementary computational particles, belonging, respectively, to set $\mathbf{S}_1$ and $\mathbf{S}_2$, between each other. Set $\mathbf{S}_1$ contains $\mathbf{N}_{s1}$ active elementary computational particles. In the process of quantum computation, the arithmetic operations of multiplying and dividing $\mathbf{N}_{s1}$ of active elementary computational particles are multiplied by a factor. As a result of multiplying active elementary computational particles, a new set of active elementary computational particles is created. To identify the particles of a newly formed set, we call each particle of a given set simply an active particle. Set $\mathbf{S}_2$ contains $\mathbf{N}_{s2}$ passive elementary computational particles, called (for the sake of textual compactness) simply passive particles. In the created quantum-mechanical mathematical model for quantum computing of arithmetic operations of multiplication and division, each active particle $\mathbf{S}_1$ of the set creates a Poisson flow for the interaction with passive particle $\mathbf{S}_2$ of the set. The Poisson flow is carried out with certain intensity λ_1, which can be either constant or variable over time. The interaction between active and passive particles is as follows: 1. Each active particle $\mathbf{S}_1$ of the set performs a random search for passive particle $\mathbf{S}_2$ of the set; 2. Each randomly selected active particle $\mathbf{S}_1$ sets the value of its quantum number to the quantum number of the randomly selected passive particle $\mathbf{S}_2$ of the set while the original value of the quantum number is reset. As a result, the selected active particle $\mathbf{S}_1$ becomes passive; 3. A passive particle, which acquired a quantum number, passes from the passive state into the active state. The graphs of states of the particles belonging to sets $\mathbf{S}_1$ and $\mathbf{S}_2$ are represented by two sub graphs $\mathbf{S}_1$ and $\mathbf{S}_2$, which are shown in **Fig. 5.**

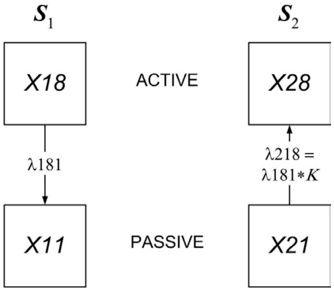

Fig. 5. Graphs of states of the particles belonging to sets S_1 and S_2 are represented by two sub graphs S_1 and S_2

X18 and X28 (**Fig. 5**) is the number of particles $\mathbf{S}_1$, $\mathbf{S}_2$ of the sets that are in the active state. X11, X21 is the number of particles $\mathbf{S}_1$, $\mathbf{S}_2$ of the sets that are in the passive state. $\lambda 181$ is the intensity of transition of particles $\mathbf{S}_1$ of the set from the active state into the passive state. $\lambda 218$ is the intensity of transition of particles $\mathbf{S}_2$ of the set from the passive state into the active state. X18, X11 and X28, X21 is the number of states of the particles belonging to set $\mathbf{S}_1$ and $\mathbf{S}_2$, at time point t; we shall denote them as X18(t), X11(t) and X28 (t), X21(t). X18(t) of the number-average active particles $\mathbf{S}_1$ of the set with intensity λ_1 interacts with the passive particles $\mathbf{S}_2$ of the set.

All together, per time unit, they produce

$$\lambda \cdot \mathrm{K} \cdot \mathrm{X18(t)}, \tag{1}$$

successful interactions on average.

These interactions are evenly distributed among all the passive particles of the $\mathbf{S}_2$ array, so that each of them has

$$\lambda_{218} = \frac{\lambda_1 \cdot \mathrm{K} \cdot \mathrm{X18(t)}}{\mathrm{X21(t)}}, \tag{2}$$

successful interactions on average.

Intensity (2), in accordance with [1], should be multiplied by function

$$R(X21(t))=\begin{cases}X21(t) & \text{at } X21(t)>0,\\ 0 & \text{at } X21(t)<0.\end{cases} \quad (3)$$

Function R(X21(t))=0 if, at time point t, set $\mathbf{S}_2$ contains no particles in the passive state. In this case, active particles $\mathbf{S}_1$ of the set cannot conduct the process of interaction of active and passive particles.

According to [1]

$$\frac{R(X21(t))}{X21(t)}=\rho(X21(t)). \quad (4)$$

It follows from equation (4) that

$$R(X21(t))=X21(t)\cdot\rho(X21(t)). \quad (5)$$

After multiplying equation (2) by function R(X21(t)), which is represented by equation (5), we obtain that each passive particle $\mathbf{S}_2$ of the set has, on average successful interactions:

$$\lambda_{218}=\frac{\lambda_1\cdot K\cdot X18(t)\cdot X21(t)\rho(X21(t))}{X21(t)}=\lambda_1\cdot K\cdot X18(t)\rho(X21(t)), \quad (6)$$

$$\rho(X21(t))=\begin{cases}1 & \text{at } X21(t)>1e-6;\\ 0 & \text{at } X21(t)<1e-6.\end{cases}$$

By definition, λ_{181}=1/K * λ_{218} then:

$$\lambda_{181}=\lambda_1 X18(t)\rho(X21(t)). \quad (7)$$

Let us replace the names of particle numbers X18(t), X11(t), X28(t), X21(t) by the names of the average-numerical states of particles m18(t), m11(t), m28(t), m21(t), respectively. Next, when one knows intensities λ_{181}, λ_{218} shown in **Fig. 5**, by using the principle of quasi-regularity, we can immediately record a system of logical-differential equations for the dynamics of average-numerical states of particles $\underline{S}_1$, $\mathbf{S}_2$ of sets

$$\frac{d(m11(t))}{dt}=\lambda_1 m18(t)\rho(m21(t)),. \quad (8)$$

$$\frac{d(m18(t))}{dt}=-\lambda_1 m18(t)\rho(m21(t)), \quad (9)$$

$$\frac{d(m21(t))}{dt}=-coeff\_mltpl\cdot\lambda_1\cdot m18(t)\rho(m21(t)), \quad (10)$$

$$\frac{d(m28(t))}{dt}=coeff\_mltpl\cdot\lambda_1\cdot m18(t)\rho(m21(t)). \quad (11)$$

$$\rho(m21(t))=\begin{cases}1 & \text{for } m21(t)>1e-6;\\ 0 & \text{for } m21(t)<1e-6.\end{cases}. \quad (12)$$

3. Quantum calculations of arithmetic operations of multiplication and division by an unconventional method

Fig. 6 shows results of the number-average states of particles belonging to sets $\mathbf{S}_1$, $\mathbf{S}_2$, obtained as a result of solving the system of differential equations (8=12), describing the process of quantum computing of arithmetic operation m18*K, for different values of m18, K, intensity constant values $\lambda_1 = 65$, quantization period h=0.03. In **Fig. 6, *a*** – m18=75.3795, K=3.5349e+011. In **Fig. 6**, ***b*** m18=75.3795, K=–3.5349e+011. In **Fig. 6, *c*,** m18=75.3795, K=1./3.5349e+011. In **Fig. 6, *d*** m18=75.3795, K=–1./3.5349e+011.

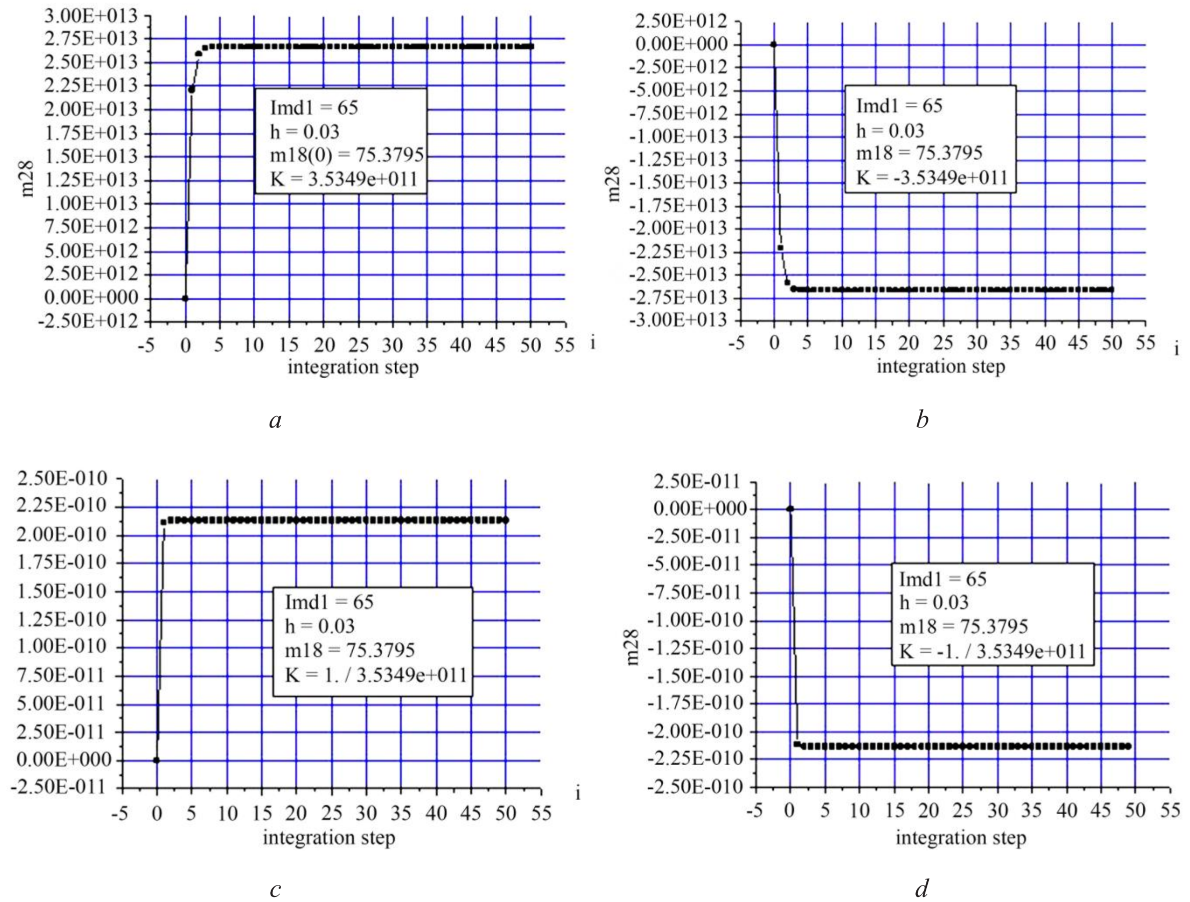

Fig. 6. Dynamics of the number-average states of particles S_1, S_2 of the sets: *a* – m18=75.3795, K=3.5349e+011; *b* – m18=75.3795, K=–3.5349e+011; *c* – m18=75.3795, K=1. / 3.5349e+011; *d* – m18=75.3795, K=–1. / 3.5349e +011

Analysis of the results of calculating the number-average states of the particles of sets $\mathbf{S}_1$, $\mathbf{S}_2$ allowed us to conclude that the non-traditional mathematical model proposed for the quantum computation of mathematical operations of multiplication and division is correct. Direct practical use of the mathematical model of quantum computation of arithmetic operations (in this case, multiplication and division) is complicated by the fixed time of integration of the system of differential equations. By analogy to the problems solved in [13, 14], in our case, there was a problem to complete the process of integrating the system of differential equations (8)–(12) when a numerical value of the integral variable becomes equal to the required accuracy. It is known that each stage of the integration of system of differential equations adds up to the improvement in accuracy of the integral variable. To complete the process of integrating a system of differential equations (8)–(12), we propose the following procedure. The proposed procedure comes down to the following:

1. The accuracy of calculation of the integral variable is specified.

2. At each integration stage, it is necessary to bear in mind absolute values of the integral variables, which are calculated at the preceding and subsequent integration stage.

3. At each stage of integration, it is necessary to know absolute difference between the absolute values of integral variables, which are computed at the preceding and subsequent stages of integration.

4. It is necessary, before each stage of integration, that the absolute difference of absolute values of the integral variables, calculated at the preceding and subsequent integration stage, be compared to the specified accuracy of calculation of the integral variable.

5. If the result of the comparison is positive, the integration process is completed, otherwise, it continues.

The proposed procedure for completing the process of integrating a system of differential equations (8)–(12) made it possible to create a mathematical model of adaptive quantum computation of arithmetic operations of multiplication and division. The mathematical model constructed automatically changes data of the integration algorithm, which leads to the calculated value of the integral variable at reaching the required accuracy, the completion of the integration process and creation of the adaptive-quantum mechanism for calculating arithmetic operations of multiplication and division. **Fig. 7** shows results of the number-average states of particles as a result of quantum computations of arithmetic operation mq18 * K, obtained by the adaptive integration of a system of differential equations (8)–(12), for different values of m18, K, intensity constant values λ_1 =lmd1=65 and quantization period h = 0.03. The adaptive integration of the system of differential equations (8)–(12) made it possible to transform the forms of the number-average states of particles $\mathbf{S}_1$, $\mathbf{S}_2$ of the sets, shown in Fig. 6, into forms shown in **Fig. 7**. As a result, we obtain: 1. **Fig. 6, *a*** was transformed to **Fig. 7, *a***; 2. **Fig. 6, *b*** was transformed to **Fig. 7, *b***; 3. **Fig. 6, *c*** was transformed to **Fig. 7, *c***; 4. **Fig. 6, *d*** was transformed to **Fig. 7, *d*.**

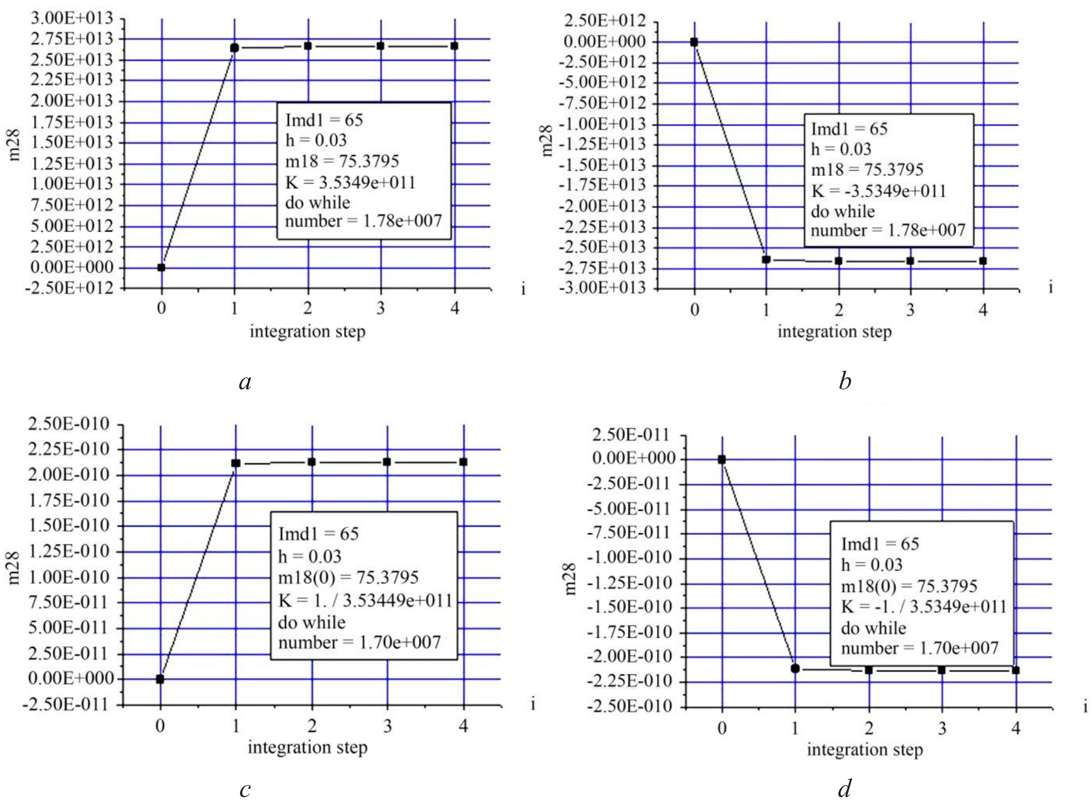

Fig. 7. Dynamics of the number-average states of particles S_1, S_2 of sets, obtained by means of adaptive integration of a system of differential equations (8)–(12): *a* – K=3.5349e+011; *b* – K=–3.5349e+011; *c* – K=1. / 3.5349e+011; *d* – K=–1. / 3.5349e+011

The adaptive execution time of the quantum arithmetic operations of multiplication and division is determined by the user.

4. Conclusions

An approach is proposed for solving a number of problems of quantum computation of elementary arithmetic operations, specifically, multiplication and division, using contemporary computer technologies. The mechanical model of quantum computation of arithmetic operations of multiplication and division that we constructed made it possible to solve a problem of the lacking terminological concepts in the field of quantum computation of elementary arithmetic operations, listed above, in traditional computer technologies. The proposed mathematical model for quantum computation of arithmetic operations of multiplication and division, based on the principle of predetermined random self-organization, suggested by Ashby in 1966, as well as the application of the method of the dynamics of average, allowed us to solve the problem on the implementation of interaction between positive and negative particles from sets S1 and S2. The results of calculating arithmetic operations of multiplication and division, performed using the proposed mathematical model of quantum computation of arithmetic operations, given above, made it possible to conclude that the proposed mathematical model is correct. The problem on completing the process of integrating a system of differential equations (8–12) after the user obtains the calculated value of the integral variable at achieving the required accuracy, is solved by using the constructed mathematical model of adaptive-quantum computation of arithmetic operations of multiplication and division. The proposed mathematical model of adaptive-quantum computation of arithmetic operations of multiplication and division might become a driver for creating a number of mathematical models for adaptive-quantum computation of other arithmetic operations. One of the authors of present paper believes that in the future there will emerge functional nodes to integrate the systems of logical-differential equations for adaptive-quantum computation of arithmetic operations, which could enable the possibility to transfer the calculated numerical values to other functional nodes, as well as obtaining numerical values from other functional nodes. The functional nodes that might be created would form the basis for constructing adaptive-quantum computational networks to implement the algorithms of the problem being solved. The only problem is the need to accurately exchange the calculated numerical values between the functional nodes.

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