Original Research Article: full paper

(2018), «EUREKA: Physics and Engineering» Number 5

INVESTIGATION OF THE NON-EQUILIBRIUM IMPACT ON THE PROPERTIES OF A SUPERCONDUCTOR USING A PLASMA SUPERCONDUCTIVITY MECHANISM

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Abstract

The results of theoretical studies of a semiconductor structure are presented which, upon irradiation with a particle stream or light, transforms into a superconducting phase state.

The work is carried out:

– analysis of ion-acoustic oscillations in non-degenerate and degenerate plasma;

– analysis of the possibility of using the jelly model to describe the weak coupling and strong coupling;

– determination of the value of the order parameter when the equilibrium distribution is replaced by a non-equilibrium distribution function;

– analysis of the dispersion properties of the medium for a non-equilibrium stationary distribution of charged particle;

– determination of the critical temperature of a superconducting transition when using the plasma mechanism for describing superconductivity.

The aim of research: determination of the frequency of oscillations of the "ion sound" type with sufficiently large mobility of the sites of the absence of electrons (holes); determination of the method for calculating the damping of ion-acoustic oscillations, which are determined by the imaginary part of the dielectric constant (decrement damping); numerically solution of the equation for the width of the energy gap; determination of the method of estimating the critical temperature of a superconductor in the plasma description of the process of the onset of superconductivity.

Keywords: superconductor, plasma superconductivity mechanism, critical temperature of a superconductor.

DOI: 10.21303/2461-4262.2018.00720

1. Introduction

Many characteristics of matter (for example, matrix elements of quasi-particle interaction, distribution functions, etc.) on which superconducting properties depend, can be highly modified under the influence of external fields and sources and sinks of particles. Most types of phase transitions are associated with collective effects. Collective effects are very sensitive to the deviation of the system from equilibrium and the distribution of quasi-particles in the energy space. The influence of external sources just leads to a change in the number of quasi-particles (their density) in comparison with the equilibrium and the change in their distribution in the energy space. At the same time, let's emphasize that not only quantitative characteristics can change, but also the form of the distribution function can change qualitatively. New parameters appear that affect the characteristics of the phase transition. Non-equilibrium stationary distributions of charged particles can significantly change the dispersion properties of the medium and, consequently, can lead to a significant change in the temperature of the superconductor phase transition [1]. The magnitude of the order parameter in non-equilibrium conditions can often be expressed by the usual relations, but with the replacement of the equilibrium distribution by a non-equilibrium distribution function. The kinetic equation for the distribution function, as a rule, [2–5], also depends on the order parameter and, therefore, the analysis of the phase transition under non-equilibrium conditions is a complex non-linear problem. Therefore, the analysis of the effect of the non-equilibrium of the system on superconducting properties of semiconductor systems with a possible plasma mechanism of superconductivity is relevant [1, 3].

2. Materials and methods of research

2. 1. Ionic-sound oscillations in a degenerate plasma

In rarefied non-isothermal ion-electron plasma, when $T_e \gg T_i$ (where T_e , T_i –the temperature of electrons and ions, respectively), there exists, as is well known [4], a low-frequency branch of weakly damped ion-acoustic oscillations with a frequency:

$$\omega_q = \frac{qc_s}{\sqrt{1+q^2d_e^2}}, \tag{1}$$

where $c_s = \sqrt{\frac{T_e}{m_i}}$ – the phase velocity of ion sound; m_i – mass of ion; $d_e = \sqrt{\frac{T_e}{4\pi e^2 n}}$ – Debye-Hückel electron screening radius; n – concentration of charged particles, and because of the neutrality of the plasma $n = n_e = Zn_i$ (in what follows let's assume that $Z = 1$). The damping of these oscillations is due, in the main, to their interaction with resonant particles (Landau damping).

An analogous branch of longitudinal collective oscillations can also exist in a dense isothermal plasma $T_e = T_i = T$ with degenerate electrons, where $T \ll E_F$ (where E_F – the Fermi energy, $p_F = (3\pi^2 n)^{1/3}$ – the Fermi momentum of electrons, m_e – their mass; $\hbar = 1$), and, in particular, in the degenerate electron-hole plasma of some semiconductors (semimetals) with significantly different effective masses and sufficiently large mobilities of free carriers. Let's first consider the case of ion-electron plasma.

Let's assume that the ions are non-degenerate and assume that the plasma density satisfies the following conditions: $\hbar = 1$, so that the energy of the Coulomb interaction between the particles is small in comparison with their average energy. The longitudinal dielectric constant of such "almost ideal" plasma has the form [1–3]:

$$\varepsilon(q,\omega) = 1 + \frac{3\Omega_e^2}{q^2 v_{Fe}^2}\{1 + \frac{1}{2x}\left[1 - \frac{1}{4}(x - y/x)^2\right]\times$$
$$\frac{x - y/x + 2}{x - y/x - 2} + \frac{1}{2x}\left[1 - \frac{1}{4}(x + y/x)^2\right]\ln\frac{x + y/x + 2}{x + y/x - 2}\} + \frac{1}{q^2 d^2}\left[1 + i\sqrt{\pi}zW(z)\right], \tag{2}$$

where $\Omega_e = \left(\frac{4\pi e^2 N}{m_e}\right)^{1/2}$ – the electron Langmuir (plasma) frequency; $v_{Fe} = P_F/m_e$ – Fermi velocity of electrons;

$$x = q/P_F,\ y = \frac{\omega}{E_{Fe}};\ z = \frac{\omega}{qv_i};\ v_i = \left(\frac{2T}{m_i}\right)^{1/2},\ d = \left(\frac{T}{4\pi e^2 N}\right)^{1/2},$$

$$W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right). \tag{3}$$

In the region of phase velocities $v_i << \omega/q << v_e$ let's use (3) to obtain the dispersion equation for the longitudinal collective oscillations of the plasma:

$$\mathrm{Re}\,\varepsilon(q,\omega_q) \cong 1 + \frac{3\Omega_e^2}{q^2 v_{Fe}^2} - \frac{\Omega_i^2}{\omega_q^2} = 0, \tag{4}$$

where $\Omega_i = \left(\frac{4\pi e^2 N}{m_i}\right)^{1/2}$ – the ion Langmuir frequency.

Hence, for the oscillation frequency, an expression analogous to (1) follows:

$$\omega_q = \omega_s(q) = \frac{qu_s}{\left(1 + q^2 D_e^2\right)^{1/2}}, \tag{5}$$

where $u_s = v_{Fe}(m_e/3m_i)^{1/2}$ – phase velocity of ion sound, and $D_e = \left(\frac{E_{Fe}}{6\pi e^2 N}\right)^{1/2}$ – the electron Debye screening radius in degenerate plasma. An analogous branch of longitudinal oscillations for a strongly compressed substance was considered in [3].

The damping of ion-acoustic oscillations, determined by the imaginary part of the dielectric constant (3), in the region $x < 2$ is due to electron-hole decays and is a quantum analog of Landau damping. The damping rate

$$\gamma_s(q) = -\mathrm{Im}\,\varepsilon(q,\omega_q)\left(\frac{\partial \mathrm{Re}\,\varepsilon}{\partial \omega}\right)^{-1}$$

is:

$$\gamma_s(q) = -\frac{\pi/4}{1+q^2D_e^2}\begin{cases} \dfrac{\omega_s^2}{qv_{Fe}}, & 0 < \omega_s < E_{Fe}x(2-x); \\ \dfrac{\omega_s}{2x\left[1-\frac{1}{4}(x-y/x)^2\right]}, & E_{Fe}x(2-x) < \omega_s < E_{Fe}x(2+x); \\ 0, & \omega_s \le E_{Fe}x(x-2). \end{cases} \tag{6}$$

In the region $x >> 1$, when $qD_e >> 1$ and $\omega_s \approx \Omega_i$, the Landau damping on resonance ions becomes important, which is particularly large at $qd \ge 1$.

2. 2. The degenerate electron-hole plasma of semiconductors (semimetals)

Let's consider collective low-frequency oscillations of the "ion sound" type in the degenerate electron-hole plasma of certain semiconductors (semimetals), when the role of ions is played by "heavy" holes with an effective mass m_p considerably exceeding the effective mass of the conduction electrons m_n. Let's note that degenerate electron-hole plasma in intrinsic semiconductors can be created by "pumping" carriers with a powerful light source (laser) [5].

Using the equations of motion and continuity

$$\frac{\partial \vec{v}_p}{\partial t} = -\frac{e}{m_p}\vec{\nabla}\varphi;\ \frac{\partial}{\partial t}(\delta N_p) + N_p \mathrm{div}\vec{v}_p = 0, \tag{7}$$

where $\vec{v}_p$ and δN_p – the perturbation of the velocity and density of holes; N_p – their unperturbed concentration ($N_p >> |\delta N_p|$); φ – potential of the longitudinal self-consistent electric field of the oscillations ($\vec{E} = -\vec{\nabla}\varphi$), τ_p – the characteristic time of scattering of holes by phonons and impurities, for the Fourier component let's obtain the expression:

$$\delta N_p(q) = \frac{eN_p q^2 \varphi(q)}{m_p \omega(\omega + 1/\tau_p)}. \tag{8}$$

On the other hand, neglecting the inertia of the light conduction electrons, from the equilibrium condition:

$$N_n e\vec{\nabla}\varphi = \vec{\nabla}\delta P_n, \tag{9}$$

where N_n – unperturbed concentration of electrons, and a δP_n – the perturbation of the pressure of the degenerate electron gas, let's have:

$$\delta N_n(q) = \frac{3}{2}\frac{eN_n}{E_{Fn}}\varphi(q), \tag{10}$$

where $E_{Fn} = P_{Fn}^2/2m_n$ – Fermi energy of the electrons reckoned from the bottom of the conduction band, and $P_{Fn} = (3\pi^2 N_n)^{1/3}$ – their Fermi momentum.

Substituting (8) and (10) into the Poisson equation [4], let's arrive at the following dispersion equation for low-frequency collective oscillations of carriers in a semiconductor (semimetal):

$$\varepsilon(q,\omega) = \varepsilon_0\left[1 + \frac{1}{q^2 D_n^2} - \frac{\Omega_p^2}{\omega(\omega + i/\tau_p)}\right]. \tag{11}$$

Here $\Omega_p = \left(\frac{4\pi e^2 N_p}{m_p \varepsilon_0}\right)^{1/2}$ – the Langmuir frequency of the holes; $D_n \approx (a_n/N_n^{1/3})^{1/2}$ – effective electron screening radius, where $a_n = \varepsilon_0/m_n e^2$ – the Bohr radius of the electron. Since the screening radius can't be less than the mean distance between particles $\sim N_n^{-1/3}$, this expression for D_n is valid, generally speaking, only for sufficiently high concentrations of conduction electrons, when $a_n N_n^{1/3} \geq 1$, so that the electron gas can be considered almost ideal , In the region of low concentrations, when $a_n N_n^{-1/3} < 1$, it is to be supposed that $D_n \sim N_n^{-1/3}$, ε_0 – the longitudinal permittivity of the crystal (far from the natural frequencies of the crystal ε_0=const).

Hence, under the condition $\omega\tau_p >> 1$, i. e. for sufficiently large hole mobility, for the frequency of oscillations of the "ion sound" type, let's obtain the expression:

$$\Omega_s(q) = \frac{qU_s}{(1+q^2D_n^2)^{1/2}} = \Omega_p\left(\frac{q^2}{q^2+\kappa_n^2}\right)^{1/2}, \tag{12}$$

where

$$U_s \equiv D_n\Omega_p \approx v_{Fn}\left(\frac{m_n}{m_p}\frac{N_p}{N_n}\right)^{1/2};\ v_{Fn} = P_{Fn}/m_n;\ \kappa_n = D_n^{-1}. \tag{13}$$

As follows from (11), the damping of the oscillations in this case is determined mainly by the mobility of the holes and is small under the condition $\Omega_p\tau_p >> 1$.

An analogous branch of collective oscillations can also exist in semiconductors with "heavy" and "light" carriers of one type (electrons or holes). Such oscillations arise due to additional minima (valleys) in the conduction band (in the case of n-type semiconductors) or maxima in the valence band (in the case of p-type semiconductors).

2. 3. The «jelly» model. The approximation of weak coupling

One of the simplest models in the theory of superconductivity, along with the BCS models [2–10] and Frohlich [3], is the so-called jelly model proposed by Pines [3–10]. The Pines model allows one to consider electron-phonon and Coulomb interaction in metals in a unified manner on the basis of generalized (dynamic) dielectric permittivity $\varepsilon(\vec{q},\omega)$ [3].

However, this model can't claim a detailed quantitative description of the properties of real superconductors, since it does not take into account the features of the crystal and band structure of the metal, the processes of "transfer", etc. At the same time, the jelly model can serve as a fairly good approximation for the study of superconductivity in such systems with Coulomb interaction. Such systems include: an electron-hole plasma of isotropic degenerate semiconductors and semimetals; two-component plasma s and d-electrons in transition metals and alloys. In this case, the interaction between the conduction electrons in semiconductors (semimetals) in the approximation of an almost ideal Fermi gas is described by the following vertex part

$$\Gamma_{nn}(\vec{q},\omega) = \frac{4\pi e^2}{q^2\varepsilon(\vec{q},\omega)} = \frac{4\pi e^2}{\varepsilon_i\left[q^2+\chi_n^2(q)\right]}\frac{\omega^2}{\omega^2-\Omega_q^2}, \tag{14}$$

where

$$\Omega_q = \left[\frac{4\pi e^2 N_p}{\varepsilon_i m_p} \frac{q^2}{q^2+\kappa_n^2(q)}\right]^{1/2};\ \kappa_n(q) \cong \left[\frac{6\pi e^2 N_n}{\varepsilon_i E_{Fn}}\left(1-\frac{q^2}{8p_{Fn}^2}\right)\right]^{1/2}; \tag{15}$$

N_n and N_p – the concentration of electrons and holes $N_n \neq N_p$), $P_{Fn} = \left(3\pi^2 N_n\right)^{1/2}$ – the Fermi momentum of the conduction electrons, and ε_i – the dielectric constant of the crystal (for simplicity, let's assume $\varepsilon_i \approx \varepsilon_\infty = \text{const}$ in the frequency region $\omega \sim \Omega_p$). In this case, the equation for the gap in the electron spectrum at absolute temperature zero $(T=0)$ in approximation weak coupling has the form [3]:

$$C(\xi) = \int_0^E d\xi' K(\xi,\xi') \frac{C(\xi')}{\sqrt{\xi'^2 + C^2(\xi')}}, \tag{16}$$

where

$$K(\xi,\xi') = 1/2\left[Q(\xi-\xi') + Q(\xi+\xi')\right]; \tag{17}$$

$$Q(\omega) = \frac{\alpha\omega^2}{\Omega_p^2 - \omega^2(1-\alpha/2)},\ \alpha = \frac{\kappa_n^2(0)}{4p_{Fn}^2} \equiv \frac{e^2}{\pi e^i v_{Fn}}; \tag{18}$$

$$\beta(\omega) = \frac{\alpha\omega^2}{\Omega_p^2 - \omega^2(1-\alpha/2)},\ \alpha = \frac{\kappa_n^2(0)}{4p_{Fn}^2} \equiv \frac{e^2}{\pi e^i v_{Fn}}; \tag{19}$$

$v_{Fn} = p_{Fn}/m_n$ – Fermi-velocity of conduction electrons.

The kernel $Q(\omega) \equiv K(\omega,0)$ is shown in **Fig. 1** by a continuous curve.

Let's note that for the jelly model, the high-density condition $p_{Fn}\alpha_n \gg 1$ (where $\alpha_n = \varepsilon_i / e^2 m_n$ – the effective Bohr radius of the conduction electron) is simultaneously a condition for the applicability of the weak-coupling approximation $\alpha \equiv 1/\pi p_{Fn}\alpha_n \ll 1$.

Let's find the approximate (asymptotic) solution of equation (16), taking into account the exponential smallness of the gap. Replacing for simplicity the function $C(\xi)$ under the root on the right-hand side of (16) by its value on the Fermi surface $C(0) \equiv \Delta$, by means of integration by parts let's reduce equation (16) to the form:

$$C(\xi) = -K(\xi,0)\Delta \ln\frac{\Delta}{2E} - \int_0^E \frac{d}{d\xi'}\left[K(\xi,\xi')C(\xi')\right]\ln\frac{\xi'}{E}d\xi'. \tag{20}$$

Equation (20) coincides, in fact, with the asymptotic expansion of the equation (16), obtained in [3], up to terms $\sim \Delta\ln\Delta$ and Δ. Introducing a new function $\varphi(\xi)$ related to $C(\xi)$

$$C(\xi) = 2E\frac{\varphi(\xi)}{\varphi(0)}e^{-1/\varphi(0)} \equiv \Delta\frac{\varphi(\xi)}{\varphi(0)}, \tag{21}$$

let's obtain for it, according to (20), the following linear integral equation:

$$\varphi(\xi) = K(\xi,0) - \int_0^E \frac{d}{d\xi'}\left[K(\xi,\xi')\varphi(\xi')\right]\ln\frac{\xi'}{E}d\xi'. \tag{22}$$

A non-trivial solution of equation (16), smoothly branching from the trivial solution $C(\xi) \equiv 0$ when the interaction $K(\xi,\xi')$ on and corresponding to the appearance of superconductivity, exists only under the condition $\varphi(0) > 0$ (the latter, generally speaking, can be satisfied even for $K(0,0) < 0$). However, the method for solving Eq. (22) is unsuitable in the case of the jelly model, since here, according to (19)–(17), a static interaction on the Fermi surface $K(0,0) = 0$.

Nevertheless, in this case it is also easy to obtain an approximate solution of Eq. (22) (and, consequently, and (16)) if take into account that the kernel $K(\xi,0)=K(0,\xi)=Q(\xi)$ has a logarithmic singularity for $\beta(\xi)=1$, i. e., in a point $\xi=\Omega\equiv\Omega_p(1+\alpha/2)^{-1/2}$. The same feature, according to (22), must also have a function $\varphi(\xi)$, so that the derivative $d\left[K(\xi,\xi')\varphi(\xi)\right]/d\xi'$ suffers a break $(\mp\infty)$ at a point $\xi'=\Omega$ for any value ξ. Taking the slowly varying function $\ln(\xi'/E)$ from under the sign of the integral at the point of discontinuity, let's obtain an approximate functional equation for $\varphi(\xi)$:

$$\varphi(\xi)=K(\xi,0)-\ln\frac{\Omega}{E}\left[K(\xi,E)\varphi(E)-K(\xi,0)\varphi(0)\right], \tag{23}$$

from which simple recurrence relations follow:

$$\begin{cases}\varphi(0)\left[1-K(0,0)\ln\dfrac{\Omega}{E}\right]=K(0,0)-\varphi(E)K(0,E)\ln\dfrac{\Omega}{E};\\ \varphi(E)\left[1-K(E,E)\ln\dfrac{\Omega}{E}\right]=K(E,0)\left[1+\varphi(0)\ln\dfrac{\Omega}{E}\right].\end{cases} \tag{24}$$

Solving the system of equations (24) with respect to, let's obtain

$$\frac{1}{\varphi(0)}=\frac{1-K(E,E)\ln(E/\Omega)}{K(0,0)\left[1-K(E,E)\ln(E/\Omega)\right]+K(0,E)K(E,0)\ln(E/\Omega)}-\ln\frac{\Omega}{E}. \tag{25}$$

Let's note that the results of BCS 9 and Bogoliubov-Tolmachev [3, 5] for the gap follow immediately from (25) with allowance for (21)

$$\Delta\equiv 2E\exp\left\{-1/\varphi(0)\right\}. \tag{26}$$

In the case of the jelly model, according to (17)–(19):

$$K(0,0)\equiv Q(0)=0,\, K(0,E)\equiv K(E,0)=Q(E),$$

$$K(E,E)=1/2Q(2E).$$

Under the condition $E\gg\Omega$, the kernel $Q(\omega)$ in the energy region $\omega\geq E$ is practically independent of ω and equal to its asymptotic value when $\omega\to\infty$:

$$Q(\infty)\equiv-\rho(\alpha)=-\frac{\alpha}{2-\alpha}\ln\left(\frac{2+\alpha}{2\alpha}\right), \tag{27}$$

which corresponds to the screened Coulomb repulsion.

As a result, taking into account (25)–(27), let's obtain the following approximate formula for the gap:

$$\Delta=2\Omega\exp\left\{-\frac{2+\rho(\alpha)\ln(E/\Omega)}{2\rho^2(\alpha)\ln(E/\Omega)}\right\}. \tag{28}$$

It follows, in particular, that in the framework of the jelly model, there always exists a non-trivial solution of equation (16) corresponding to the superconducting state. In other words, on the basis of this model, a criterion for the superconductivity of metals can't be obtained [2–4]. Formally, in the limit $E\to\infty$ and $\alpha\to 0$ from (28) let's obtain a simpler expression:

$$\Delta=2\Omega_p\exp\left\{-\frac{1}{\alpha\ln(1/\alpha)}\right\}, \tag{29}$$

$$Q_n / E_{Fn} \equiv (16\alpha/3)^{1/2} < 1,$$

which qualitatively agrees with Abrikosov's general conclusion about an almost exponential decrease in the gap with increasing density.

However, because of the strong decay of elementary excitations, the Coulomb interaction in the energy region $\omega \geq \Omega_n$ is cut off (where $\Omega_n = (4\pi e^2 N_n / \varepsilon_i m_n)^{1/2}$ – the electron plasma frequency), so that it is natural to choose the upper limit $E = \min\{E_{Fn}, Q_n\}$ in (28)/ (Relation at $\alpha < 0.2$

$$Q_n / E_{Fn} \equiv (16\alpha/3)^{1/2} < 1.$$

It is interesting to note that from (21) and (25), as a special case, follows the formula for the gap obtained [3]. Indeed, setting in (26) $K(E,E) = Q(E) \cong -\rho(\alpha)$, with $E = E_F, \Omega = \omega_i$ and $\alpha \ll 1$, let's obtain:

$$\Delta = 2\omega_i e^{-1/g};\ g = \frac{[(\alpha/2)\ln(1/\alpha)]^2 \ln(E_F/\omega_i)}{1 + (\alpha/2)\ln(1/\alpha)\ln(E_F/\omega_i)}. \tag{30}$$

It is easy to show that expression (30) corresponds to an approximate solution of equation (16) with a kernel $K(\xi,\xi')$ having the form of a "rectangular well"

$$K(\xi,\xi') = \begin{cases} -(\alpha/2)\ln\left(\dfrac{1}{\alpha}\right); & \omega_i \leq |\xi|, |\xi'| \leq E_F; \\ 0; & |\xi|,|\xi'| < \omega_{i,}\ |\xi|,|\xi'| > E_F. \end{cases} \tag{31}$$

As we'll see below, the formulas (28) and (30), which are the most interesting from the practical point of view, give very low values of the gap, compared with the numerical solution (which, by the way, depends weakly on the upper limit E).

2. 4. The method of numerical solution of the equation for a gap

In this section let's briefly outline the main idea of the method used in the paper for the numerical solution of the non-linear integral Fredholm equation of the first kind, which is the equation for the gap (16):

$$y(x) = \int_0^L dx' K(x,x') \frac{y(x')}{\sqrt{x'^2 + y^2(x')}}, \tag{32}$$

where dimensionless quantities are introduced $x = \xi / E_{Fn}, y(x) = C(\xi)/E_{Fn}$ and $L = E/E_{Fn}$.

The mathematical difficulties arising in the numerical solution of an equation of the form (32) are mainly due to the fact that it is, firstly, homogeneous (and therefore additional complications arise with the allocation of a non-trivial solution) and, secondly, essentially non-linear. Zubarev and Garland [3] independently proposed a method of "quasi-linearization" of Eq. (32), which simultaneously allows to bypass both these difficulties and ensure a sufficiently rapid convergence. With the help of the appropriate normalization of the kernel $I(x,x') = K(x,x') / K(0,0)$ and the gap $\Phi(x) = y(x)/y(0)$, and taking into account the boundary conditions on the Fermi surface $I(0,0) = \Phi(0,0) = 1$, Eq. (32) reduces to an inhomogeneous integral equation (Fredholm of the second kind) with a weak non-linearity, the solution of which can be easily obtained by iteration.

However, in the case of the jelly model, the Zubarev-Garland method is inapplicable, since here $K(0,0) = 0$. Therefore, we partition the domain of integration $[0,L]$ into (32) into intervals (x_i, x_{i+1}) with a constant step δ (usually, $L = 1$ and $\delta = 0.05$ were chosen). Approximating $y(x)$ by a step function, let's obtain instead of the integral equation (32) a system of homogeneous non-linear algebraic equations of the n-th order:

$$y_i = \sum_{j=1}^{n} K_{ij} \frac{y_j}{\sqrt{y_j^2 + x_j^2}}; \; i = 1,...,n, \tag{33}$$

where

$$K_{ij} = \int_{x_j}^{x_{j+1}} dx' K(x_i, x'),\; y_i \equiv y(x_i).$$

Since the kernel $K(x, x')$ has logarithmic singularities (27), (28), in order to improve the accuracy of calculating the coefficients, an additional partition of the interval (x_j, x_{j+1}) into small intervals with a step $h \ll \delta$ was carried out.

Along with (33) let's consider the system of inhomogeneous equations

$$f_i = y_i - \sum_{j=1}^{n} K_{ij} y_j \left(y_j^2 + x_j^2\right)^{-1/2}, \tag{34}$$

which is equivalent to the system (33) for $f_i \equiv 0\ (i = 1,...,n)$. Thus, if considered f_i as a function of a vector $Y = [y_1,...,y_n]$ in an n-dimensional Euclidean space, then the search for a solution of the homogeneous system of equations (32) reduces to the selection of such set of values y_i^* for which all $f_i(Y^*)$ in (34) would tend to zero. For this purpose let's introduce a function

$$F(Y) = \sum_{i=1}^{n} f_i^2(Y) \Big/ \sum_{i=1}^{n} y_i^2 \equiv \sum_{i=1}^{n} \left\{ y_i - \sum_{j=1}^{n} \frac{K_{ij} y_j}{\sqrt{y_j^2 + x_j^2}} \right\}^2 \Big/ \sum_{i=1}^{n} y_i^2 . \tag{35}$$

The denominator here is introduced in order to get rid of the trivial solution $y_i \equiv 0$ of system (33). In the case when there exists at least one non-trivial solution $y_i^* \neq 0$ of system (33), the corresponding value $F(Y^*) \equiv 0$. As a result, let's arrive at the problem of minimizing a non-negative function $F(Y)$ in the space of vectors Y.

As it is known, the main difficulty of any method of minimization is choosing the step. In this respect, methods are convenient in which the step is not an external parameter of the task algorithm, but is selected automatically during the computation process. Such methods include, for example, the method of steepest descent. However, the rate of convergence of this method essentially depends on the relief of the function being minimized, and, in particular, for functions of the "ravine" type can be very small. The analysis shows that near the minimum $F(Y)$ the function has strongly elongated level lines resembling the "ravine" in form. This leads to the fact that the angle between the direction of the maximum gradient and the direction of the bottom (channel) of the "ravine" is close to and the method of steepest descent becomes ineffective. Therefore, in order to increase the rate of convergence of the process of minimizing the function $F(Y)$ (by the method of steepest descent), the space Y was "stretched" in the direction of maximum gradients in order to reduce the local "ravine" coefficients (i. e. local curvature of the level line), which led to a significant reduce the counting time ($\min\{F(Y)\} - 10^{-15} \div 10^{-16}$).

It should be noted that the original equation (32) is true when using the weak-coupling mechanism. Similarly, a solution of the gap equation in the tight-binding approximation can be obtained, and the additional non-linearity of the kernel, which can be taken into account by the method of successive approximations, leads only to insignificant corrections for a sufficiently small $|C(\xi)|$.

2. 5. Critical temperature

To determine the critical temperature of a superconducting transition T_c, it is necessary to generalize the equation for the gap (28) to the case of finite temperatures $T \neq 0$. To this end, in accordance with the method presented in [3, 10], let's perform the following procedure: the denominators of the form $\left[\omega' \mp \omega \mp \Omega_q\right]^{-1}$ in the kernel of equation (28) are multiplied by the sum of the distribution functions of the electrons

$$f(\mp\omega')=\left[\exp\{\mp\omega'/T\}+1\right]^{-1}$$

and plasmons

$$N(\pm\Omega_q)=\left[\exp\{\pm\Omega_q/T\}-1\right]^{-1},$$

and the term corresponding to the Coulomb repulsion is $\left[1-2/(\omega')\right]$. As a result, the expression in curly braces is given in the form:

$$\begin{aligned}&\left\{\frac{\Omega_q}{2}[\left(N(\Omega_q)+f(-\omega')\right)\left(\frac{1}{\omega'-\omega+\Omega_q}+\frac{1}{\omega'+\omega+\Omega_q}\right)-\right.\\&-\left(N(\Omega_q)+f(\omega')\right)\left(\frac{1}{-\omega'-\omega+\Omega_q}+\frac{1}{-\omega'+\omega+\Omega_q}\right)]-\\&-\frac{\Omega_q}{2}[\left(N(-\Omega_q)+f(-\omega')\right)\left(\frac{1}{\omega'-\omega-\Omega_q}+\frac{1}{\omega'+\omega-\Omega_q}\right)-\\&\left.-\left(N(-\Omega_q)+f(\omega')\right)\left(\frac{1}{-\omega'-\omega-\Omega_q}+\frac{1}{-\omega'+\omega-\Omega_q}\right)]-2\left[1-2f(\omega')\right]\right\}.\end{aligned}\tag{36}$$

Similarly, it is possible to transform the kernel of the equation for the function $f_0(\omega)$. However, as was shown above, up to terms $\sim\rho^2(\alpha)$ the renormalization of the interaction can be neglected. And so in the sequel let's omit the terms containing $f_0(\omega)$. At the same time, to determine the damping of the excitations, it is necessary to take the imaginary part $f_1(\omega)\equiv\mathrm{Im}\{f_n(\omega)\}$ into account. Thus, the equation for the gap at $T\neq 0$ takes the form:

$$\begin{aligned}&C(\omega,T)=\frac{\alpha}{2}\int_0^{2p_{Fn}}\frac{qdq}{q^2+\kappa_n^2(q)}\int_{\Delta(T)}^{\infty}d\omega'\mathrm{Re}\left\{\frac{C(\omega',T)}{\sqrt{\omega'^2-C^2(\omega',T)}}\right\}\bullet\\&\bullet\left\{\frac{\Omega_q}{2}[\left(\mathrm{th}\frac{\omega'}{2T}+\mathrm{cth}\frac{\Omega_q}{2T}\right)\left(\frac{1}{\omega'-\omega+\Omega_q}+\frac{1}{\omega'+\omega+\Omega_q}\right)-\right.\\&\left.-\left(\mathrm{th}\frac{\omega'}{2T}-\mathrm{cth}\frac{\Omega_q}{2T}\right)\left(\frac{1}{\omega'-\omega-\Omega_q}+\frac{1}{\omega'+\omega-\Omega_q}\right)]-2\mathrm{th}\frac{\omega'}{2T}\right\}.\end{aligned}\tag{37}$$

Let's note that (37) differs from the gap equation in the case of the electron-phonon interaction obtained in [3] by the method of temperature Green's functions, only by changing ω_q to Ω_q and a_q^2 to $\frac{2\pi e^2\Omega_q}{\varepsilon_i(q^2+\chi^2)}$, and also taking into account the Coulomb repulsion. Up to terms $\sim\left(T_c/\Omega_p\right)^2$, equation (37) at $T\to T_c$ can be reduced to the form:

$$C(\omega,T_c)=\int_0^{\infty}\frac{d\omega'}{\omega'}K^+(\omega,\omega')C(\omega',T_c)\mathrm{th}\frac{\omega'}{2T_c}.\tag{38}$$

From the comparison of (37) and (38) it follows that even with the "plasmon" mechanism of superconductivity, the usual BCS ratio [10] between the gap $\Delta(0)$ at $T=0$ and the critical temperature T_c is preserved with great accuracy.

An approximate solution of the equation for the critical temperature takes the form [9]:

$$T_c=1.14\Omega\exp\left\{-\frac{1-\rho_0(\alpha)}{\rho(\alpha)}\right\}.\tag{39}$$

Let's note that expression (39) gives the correct order of magnitude for superconducting metals with electron-phonon interaction, when, $\mu \equiv M_i / z^* m_0 \sim 10^4 \div 10^5$, $\alpha \approx 0.3 \div 0.4$ and $T_c \leq 0.1\omega_D \leq 10^{-3} E_F$ (z^* – the effective valence, that is, the number of free electrons per atom. So, for example, for Al let's obtain an estimate $T_c = 2.4$. In the case of the "plasmon" mechanism of superconductivity in degenerate semiconductors (semimetals), approximate formulas lead to values that are underestimated by almost two orders of magnitude in comparison with the exact solution.

3. Discussion of research results

The advantage of the published studies is the obtaining of an expression for determining the critical temperature of a superconductor, which, with a 6 % error, corresponds to the values obtained in the experiment for metals and semiconductors. In addition, to calculate the width of the energy gap of a superconductor, the proposed numerical solution of the equation is applicable when using mechanisms, both with weak and strong coupling

The disadvantage of the studies is the fact that the proposed mechanisms work only partially for high-temperature superconductors and additional theoretical and experimental studies are required to verify the statement. The proposed model does not take into account the contacts [4], which must be introduced into the superconductor to match it with the common wire systems, including during the experimental electromagnetic measurements of superconducting properties.

The above studies will make it possible to determine what conditions must be created for the transition of metals and semiconductors to the superconducting state, and to estimate their critical temperature already as superconductors.

The direction of further research is the improvement of models of the superconductivity process for high-temperature [6, 7, 12] samples and thin films based on them.

4. Conclusions

From the most general propositions of the theory of many-particle systems (the Goldstone theorem), it follows that in the degenerate ion-electron plasma (or in a two-component plasma) of semiconductors, a collective branch of natural oscillations of the "ion sound" type with a frequency $\omega_q \to 0$ of $q \to 0$. And these oscillations are a consequence of the symmetry breaking of the Coulomb Hamiltonian due to the difference in charge masses. On the other hand, since the branch of normal oscillations with $\lim_{q\to 0} \omega_q = 0$ can exist only in systems with a finite radius of interaction between particles, an "ion sound" in degenerate plasma appears as a result of the Debye screening of the Coulomb interaction by light carriers (electrons).

The paper proves that the superconductivity criterion for metals can't be obtained within the framework of the jelly model, however, for high-temperature superconductors its application is possible.

To calculate the width of the energy gap of a superconductor, the proposed numerical solution of the equation is applicable when using mechanisms for both weak and strong coupling.

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