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Validity of Finite Element Method: Analysis of Laminated Composite Decks Plates Subjected to in Plane Loading

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Abstract

To verify the accuracy of the present technique, buckling loads are evaluated and validated with other works available in the literature. Further comparisons were carried out and compared with the results obtained by the ANSYS package and experimental results. The good agreement with available data demonstrates the reliability of the finite element method used.

***Keywords**: validity, plane loads, finite element, fortran program, laminated decks plates.*

Introduction

Several numerical methods could be used in this study, but the main ones are finite difference method (FDM), dynamic relaxation coupled with finite difference method (DR), and finite element method (FEM).

In the finite difference method, the solution domain is divided into a grid of discrete points or nodes. The partial differential equation is then written for each node and its derivatives are replaced by finite divided differences. Although such pointwise approximation is conceptually easy to understand, it becomes difficult to apply for a system with irregular geometry, unusual boundary conditions, and heterogeneous composition.

The DR method was first proposed in 1960th; see Rushton (1968); Cassel & Hobbs (1966); and Day (1965). In this method, the equations of equilibrium are converted to dynamic equations by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations. The optimum damping coefficient and the time increment used to stabilize the solution depend on the stiffness matrix of the structure, the applied load, the boundary conditions and the size of mesh used (Suleiman, 2017; Arauz *et al*., 2017).

In the present work, a numerical method known as the finite element method (FEM) is used. It is a numerical procedure for obtaining solutions to many of the problems encountered in engineering analysis. It has two primary subdivisions. The first utilizes discrete elements to obtain the joint displacements and member forces of a structural framework. The second uses the continuum elements to obtain approximate solutions to heat transfer, fluid mechanics, and solid mechanics problem. The formulation using the discrete element is referred to as matrix analysis of structures and yields results identical with the classical analysis of structural frameworks. The second approach is the true finite element method. It yields approximate values of the desired parameters at specific points called nodes. A general finite element computers program, however, is capable of solving both types of problems and the name" finite element method" is often used to denote both the discrete element and the continuum element formulations.

The finite element method combines several mathematical concepts to produce a system of linear and non – linear equations. The number of equations is usually very large, anywhere from 20 to 20,000 or more and requires the computational power of the digital computer.

It is impossible to document the exact origin of the finite element method because the basic concepts have evolved over a period of 150 or more years. The method as we know it today is an outgrowth of several papers published in the 1950th that extended the matrix analysis of structures to continuum bodies. The space exploration of the 1960th provided money for basic research, which placed the method of a firm mathematical foundation and stimulated the development of multi-purpose computer programs that implemented the method. The design of airplanes, unmanned drones, missiles, space capsules, and the like, provided application areas.

The finite element method (FEM) is a powerful numerical method, which is used as a computational technique for the solution of differential equations that arise in various fields of engineering and applied sciences. The finite element method is based on the concept that one can replace any continuum by an assemblage of simply shaped elements, called finite elements with well-defined force, displacement, and material relationships. While one may not be able to derive a closed – form solution for the continuum, one can derive approximate solutions for the element assemblage that replaces it. The approximate solutions or approximation functions are often constructed using ideas from interpolation theory, and hence they are also called interpolation functions. For more details refer to References (Constance *et al.,* 1966; Whitney & Pagano, 1970; Seloodeh & Karami, 2004).

In a comparison between the finite element method (FEM) and dynamic relaxation method (DR), Aalami (1972), found that the computer time required for the finite element method is eight times greater than for DR analysis, whereas the storage capacity for FEM is ten times or more than that for DR analysis. This fact is supported by Putcher & Reddy (1986), Turvey & Osman (1990, 1989, 1991) and Osama Mohammed Elmardi Suleiman (2014, 2015a, 2015b, 2015c, 2016).

who noted that some of the finite element formulations require large storage capacity and computer time. Hence due to the large computations involved in the present study, the finite element method (FEM) is considered more efficient than the DR method. In another comparison, Vernon (1972), found that the difference in accuracy between one version of FEM and DR may reach a value of more than 15 % in favor of FEM. Therefore, the FEM can be considered of acceptable accuracy. The apparent limitation of the DR method is that it can only be applied to limited geometries, whereas the FEM can be applied to different intricate geometries and shapes (Gamez *et al.,* 2017; González *et al.,* 2017).

Verification of the Computer Program

Convergence Study: The optimum number of plate elements in the x any y directions (i.e. mesh size or discretization), to be used in order to compute the buckling loads with reasonable accuracy can be obtained by a convergence study. The suitable number of finite elements is determined by a number of factors which include material properties, plate dimensions, lamination scheme, boundary conditions and the storage capacity of the computer ram.

It can be observed that, as the number of modes increases, the number of finite elements required increases. Therefore, it is expected that the higher modes need number of elements.

All of the analyses described in the present thesis have been undertaken to assume the plate to be subjected to identical and/ or different support conditions on the four edges of the plate. The three sets of the edge conditions used here are designated as clamped-clamped (CC), simply – simply supported (SS), clamped – simply supported (CS), are shown in Table 1 below.

Table 1
Boundary conditions

Boundary Conditions	Plate dimensions in y – coordinate $x = 0\,, x = a$	Plate dimensions in x – coordinate $y = 0\,, y = b$
CC	$w = \phi = \psi = 0$	$w = \phi = \psi = 0$
SS	$w = \psi = 0$	$w = \phi = 0$
CS	$w = \phi = \psi = 0$	$w = \phi = 0$

Table 2 shows the convergence study of non – dimensional buckling load of simply supported SS square isotropic plate with length to thickness ratio (a/h=20) having the following material properties: material 1:
$E_y/E_x = 1.0\,, G_{xy}/E_x = G_{yz}/E_x = G_{xz}/E_x = 0.4\,, v_{xy} = 0.25$

It could be observed from Table 2 that the values of the buckling parameter $\bar{P} = Pb^2/E_2h^3$ converge as the number of elements in the mesh are increased (i.e. as the mesh size is progressively reduced). These results suggest that a 6×6 mesh over the plate is adequate for the present work (i.e. less than 1.32% difference compared to the finest mesh result available). Therefore, a mesh size of 6×6 is found to be sufficient to predict the first seven modes of buckthe ling load. In practice only the first three modes of buckling are sufficient.

Table 2
Convergence study of non – dimensional modes of buckling $\bar{P} = Pa^2/E_1h^3$ of simply supported (SS) isotropic square plate with a/h=20. (material 1)

Mesh Size	Mode Sequence Number						
	1	2	3	4	5	6	7
2×2	30.69	76.89	83.18	83.49	94.71	94.95	101.78
3×3	32.64	79.12	79.18	117.58	179.04	189.78	191.05
4×4	33.60	82.38	82.44	123.22	165.70	166.35	192.53
5×5	34.10	84.08	84.14	127.71	168.69	168.92	202.10
6×6	34.39	85.10	85.15	130.85	170.41	170.52	208.35
7×7	34.58	85.75	85.79	133.03	171.55	171.61	212.50
8×8	34.70	86.19	86.23	134.57	172.34	172.39	215.79
9×9	34.78	86.50	86.53	135.68	172.92	172.97	218.07
10×10	34.84	86.72	86.75	136.52	173.35	173.40	219.78

Validation of the finite element program: In order to check the validity, applicability, and accuracy of the present FE method, many comparisons were performed. The comparisons include theoretical, ANSYS simulation and experimental results.

Comparisons with Theoretical Results

In Table 3 the non – dimensional critical buckling load is presented in order to compare with References Yu & Wang (2008); Mohammadi *et al.,* (2009); Mohammadi *et al.,* (2009), for an isotropic plate of material 1 with different aspect ratios. As the Table shows, the present results have a good agreement with References Yu & Wang (2008); Mohammadi *et al.,* (2009); Mohammadi *et al.,* (2009).

Table 3
Comparison of the non – dimensional critical buckling load $\bar{\mathrm{P}} = \mathrm{Pa}^2/\mathrm{D}$ for an isotropic plate (material 1)

Aspect Ratio a/b	References			
	Ref. [162]	Ref. [163]	Ref. [164]	Present Study
0.5	12.33	12.3370	12.3370	12.3
1.0	19.74	19.7392	19.7392	19.7

Table 4 below shows the effect of plate aspect ratio and modulus ratio on non – dimensional critical loads $\bar{P} = P(b^2\pi^2/D_{22})$ of rectangular laminates under biaxial compression. The following material properties were used: material 2: $E_1/E_2 = 5,\ 10,\ 20, 25$ and 40; $G_{12} = G_{13} = G_{23} = 0.5\, E_2$; $\nu_{12} = 0.25$ and $a/h = 20$. It is observed that the non – dimensional buckling load increases for symmetric laminates as the modular ratio increases. The present results were compared with Mahmoud Yassin Osman & Osama Mohammed Elmardi Suleiman (2017) and Reddy (2004). The verification process showed good agreement especially as the aspect ratio increases and the modulus ratio decreases.

Table 4
Buckling load for (0/ 90/ 90/ 0) simply supported (SS) plate for different aspect and moduli ratios under biaxial compression (material 2)

Aspect Ratio a/b	Modular Ratio	Biaxial Compression				
	E_1/E_2	5	10	20	25	40
0.5	Present	10.864	12.122	13.215	13.726	14.000
	Ref. [165]	-	12.307	-	13.689	-
	Ref. [166]	11.120	12.694	13.922	14.248	14.766
1.0	Present	2.790	3.130	3.430	3.510	3.645
	Ref. [165]	-	3.137	-	3.502	-
	Ref. [166]	2.825	3.174	3.481	3.562	3.702

1.5	Present	1.591	1.602	1.611	1.613	1.617
	Ref. [165]	-	1.605	-	1.606	-
	Ref. [166]	1.610	1.624	1.634	1.636	1.641

Table 5 shows the effect of plate aspect ratio and modulus ratio on non – dimensional critical buckling loads $\bar{P} = P(b^2/\pi^2 D_{22})$ of simply supported (SS) antisymmetric cross – ply rectangular laminates under biaxial compression. The properties of material 2 were used. It is observed that the non – dimensional buckling load decreases for antisymmetric laminates as the modulus ratio increases. The present results were compared with Reddy Reddy (2004). The validation process showed good agreement especially as the aspect ratio increases and the modulus ratio decreases.

Table 5
Buckling load for (0/ 90/ 90/ 0) simply supported (SS) plate for different aspect and moduli ratios under biaxial compression (material 2)

Aspect Ratio a/b	Modular Ratio E_1/E_2	Biaxial Compression				
		5	10	20	25	40
0.5	Present	4.000	3.706	3.535	3.498	3.442
	Ref. [166]	3.764	3.325	3.062	3.005	2.917
1.0	Present	1.395	1.209	1.102	1.079	1.045
	Ref. [166]	1.322	1.095	0.962	0.933	0.889
1.5	Present	1.069	0.954	0.889	0.875	0.853
	Ref. [166]	1.000	0.860	0.773	0.754	0.725

Table 6 below shows the effect of plate aspect ratio, and modulus ratio on non – dimensional critical buckling loads of simply supported (SS) antisymmetric angle-ply rectangular laminates under biaxial compression.

The properties of material 2 were used. It is observed from Table 6 that the prediction of the buckling loads by the present study is closer to that of Mahmoud Yassin Osman & Osama Mohammed Elmardi Suleiman (2017) and Reddy Reddy (2004).

Table 6
Buckling load for antisymmetric angle-ply $(45/-45)_4$ plate with different moduli and aspect ratios under biaxial compression (material 2)

Aspect Ratio a/b	Modular Ratio	Biaxial Compression			
	E_1/E_2	10	20	25	40
	Present	19.376	36.056	44.400	69.440
0.5	Ref. [165]	19.480	-	44.630	-
	Ref. [166]	18.999	35.076	43.110	67.222
	Present	9.028	17.186	21.265	33.512
1.0	Ref. [165]	9.062	-	21.345	-
	Ref. [166]	8.813	16.660	20.578	32.343
	Present	6.144	11.596	14.322	22.013
1.5	Ref. [165]	6.170	-	14.383	-
	Ref. [166]	6.001	11.251	13.877	21.743

In Tables 7 and 8, the buckling loads for symmetrically laminated composite plates of layer orientation (0/ 90/ 90/ 0) have been determined for three different aspect ratios ranging from 0.5 to 1.5 and two modulus ratios 40 and 5 of material 2. It is observed that the buckling load increases with increasing aspect ratio for biaxial compression loading. The buckling load is maximum for clamped-clamped (CC), and clamped – simply supported (CS) boundary conditions, while the minimum for simply – simply supported (SS) boundary conditions. It is seen from Tables 7 and 8 that the values of buckling loads by the present study is much closer to the of Mahmoud Yassin Osman & Osama Mohammed Elmardi Suleiman (2017).

Table 7
Buckling load for (0/ 90/ 90/ 0) plate with different boundary conditions and aspect ratios under biaxial compression ($\overline{P} = Pa^2/E_1h^3$) (material 2) $E_1/E_2 = 40$; $G_{12} = G_{13} = G_{23} = 0.5\ E_2$; and $\nu_{12} = 0.25$

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.0742	0.4143	0.9679
	Ref. [165]	1.0827	0.4213	1.0022
1.0	Present	1.3795	0.4409	1.0723
	Ref. [165]	1.3795	0.4411	1.0741
1.5	Present	1.6402	0.4400	1.2543
	Ref. [165]	1.6367	0.4391	1.2466

Table 8
Buckling load for (0/ 90/ 90/ 0) plate with different boundary conditions and aspect ratios ($\overline{P} = Pa^2/E_1h^3$) (material 2) $E_1/E_2 = 5$; $G_{12} = G_{13} = G_{23} = 0.5\ E_2$; and $\nu_{12} = 0.25$

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.7786	0.6787	1.6325
	Ref. [165]	1.8172	0.6877	1.6838
1.0	Present	2.1994	0.6972	1.8225
	Ref. [165]	2.2064	0.6985	1.8328
1.5	Present	2.7961	0.8943	1.7643
	Ref. [165]	2.8059	0.8962	1.7618

The same behavior of buckling load applies to symmetrically laminated composite plates (0/ 90/ 0) as shown in Tables 9 and 10.

Table 9
Buckling load for (0/ 90/ 0) plate with different boundary conditions and aspect ratios ($\overline{P} = Pa^2/E_1h^3$) (material 2) $E_1/E_2 = 40$; $G_{12} = G_{13} = G_{23} = 0.5\ E_2$; and $\nu_{12} = 0.25$

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.7471	0.3238	0.6870
	Ref. [165]	0.7529	0.3325	0.7201
1.0	Present	0.9523	0.3485	0.7925
	Ref. [165]	0.9511	0.3489	0.7932
1.5	Present	1.1811	0.3530	0.8190
	Ref. [165]	1.1763	0.3514	0.8099

Table 10
Buckling load for (0/ 90/ 0) plate with different boundary conditions and aspect ratios ($\overline{P} = Pa^2/E_1h^3$) (material 2) $E_1/E_2 = 5$; $G_{12} = G_{13} = G_{23} = 0.5\ E_2$; and $\nu_{12} = 0.25$

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.6947	0.6772	1.5842
	Ref. [165]	1.7380	0.6871	1.6337
1.0	Present	2.1669	0.6970	1.7009
	Ref. [165]	2.1744	0.6984	1.7113
1.5	Present	2.5008	0.8224	1.7658
	Ref. [165]	2.5075	0.8235	1.7622

Comparisons with the Results of Ansys Package

ANSYS is a general-purpose finite element modeling package for numerically solving a wide variety of mechanical problems. These problems include static/ dynamic structural analysis (both linear and non – linear), heat transfer and fluid problems, as well as acoustic and electromagnetic problems. The problem of buckling in ANSYS is considered a static analysis. To validate the present results with ANSYS, the present results were converted from its non – dimensional form to the dimensional form by using the formula $\bar{P} = Pa^2/E_1h^3$. The E – glass/ Epoxy material is selected to obtain the numerical results for the comparisons. The mechanical properties of this material (material 3) is given in Table 11 below.

Table 11
Mechanical Properties of the E – glass/ Epoxy material (material 3)

Property	Value
E_1 or E_x	38.6 GN/m^2
E_2 or E_y	8.27 GN/m^2
G_{12} or G_{xy}	4.14 GN/m^2
G_{13} or G_{xz}	4.14 GN/m^2
G_{23} or G_{yz}	3.4 GN/m^2
ν_{12} or ν_{xy}	0.28

Tables 12 to 15 shows comparisons between the results of the present study and that simulated by ANSYS technique. Table 12 shows the effect of boundary conditions on dimensional buckling loads of symmetric angle-ply (30/ -30/ -30/ 30) of square thin laminates ($a/h = 20$) under biaxial compression. The properties of material 3 in Table 11 were used. Small differences were shown between the results of the two techniques. The difference ranges between 0.6% to less than 2%. It is observed that as the mode serial number increases, the difference increases. The same behavior of buckling load of both techniques applies to symmetrically laminated composite plates of the order (45/ -45/ -45/ 45), (60/ -60/ -60/ 60) and (0/ 90/ 90/ 0) shown in Tables 13, 14 and 15.

Table 12
Dimensional buckling load of symmetric angle–ply (30/ -30/ -30/ 30) square thin laminates with different boundary conditions (a/h=20) (material 3)

Boundary Conditions	Method	Mode Serial Number		
		1	2	3
SS	Present	109.5 N	193.4 N	322.8 N
	ANSYS	109.4 N	206.5 N	315.8 N
CS	Present	234.7 N	257.2 N	371.41 N
	ANSYS	233.21 N	255.6 N	378.7 N

Table 13
Dimensional buckling load of symmetric angle-ply (45/-45/-45/45) square thin laminates with different boundary conditions (a/h=20) (material 3)

Boundary Conditions	Method	Mode Serial Number		
		1	2	3
SS	Present	115.24 N	219.5 N	305.4 N
	ANSYS	116.3 N	225.5 N	312.7 N
CS	Present	196.33 N	282.8 N	439.53 N
	ANSYS	194.7 N	287.6 N	444.51 N

Table 14
Dimensional buckling load of symmetric angle-ply (60/-60/-60/60) square thin laminates with different boundary conditions (a/h=20) (material 3)

Boundary Conditions	Method	Mode Serial Number		
		1	2	3
SS	Present	109.39 N	193.213 N	322.19 N
	ANSYS	109.6 N	191.13 N	325.37 N
CS	Present	161.4 N	279.1 N	370.5 N
	ANSYS	160.6 N	280.4 N	377.7 N

Table 15
Dimensional buckling load of symmetric cross–ply (0/ 90/ 90/ 0) square thin laminates with different boundary conditions (a/h=20) (material 3)

Boundary Conditions	Method	Mode Serial Number		
		1	2	3
SS	Present	93.4 N	170.4 N	329 N
	ANSYS	94.4 N	181.4 N	315 N
CS	Present	244.5 N	263.7 N	366.23 N
	ANSYS	244.4 N	265.8 N	369.6 N

Comparisons with Experimental Results

Many numerical and mathematical models exist which can be used to describe the behavior of a laminate under the action of different forces. When it comes to buckling, a mathematical model can be developed which is used to model the phenomenon of buckling. But numerical methods become complicated as the number of assumptions and variables increase. Also, once the model is formed, it takes a lot of time to solve the partial differential equations and then arrive at the final result. This process becomes very cumbersome and time-consuming. In view of the above-mentioned limitations, experimental methods are followed. The experimental process needs less time and less computational work. Also, the results obtained in experiments are fairly close to that which is obtained theoretically.

The composites have two components. The first is the matrix which acts as the skeleton of the composite and the second is the hardener which acts as the binder for the matrix. The reinforcement that was used for the present study was woven glass fibers. Glass fibers are materials which consist of numerous extremely fine fibers of glass. The hardener that utilized was epoxy which functions as a solid cement to keep fiber layers together.

To manufacture the composites the following steps were taken:

1) The weight of the fiber was noted down, then the approximately 1/3rd mass of epoxy was prepared for further use.
2) A clean plastic sheet was taken and the mold releasing spray was sprayed on it. After that, a generous coating of the hardener mixture was coated on the sheet. A woven fiber sheet was taken and placed on top of the coating. A second coating was done again, and the second layer of fiber was placed, and the process continued until the required thickness was obtained. The fiber was pressed with the help of rollers.
3) Another plastic sheet was taken and the mold releasing spray was sprayed on it. The plastic sheet was placed on top of the fiber with hardener coating.
4) The plate obtained was placed under weights for a period of 24 hours.
5) After that, the plastic sheets were removed and the plates separated.

The buckling test rig for biaxial compression was developed in Tehran University of Science and Technology, College of Engineering, Iran. The frame was built using rectangular shaped mild steel channels. The channels were welded to one another and then the frame was prepared. Two-Ton hydraulic jacks were assembled into the frame to provide the necessary hydraulic forces for biaxial compression of the plates. The setup can be easily assembled and disassembled. Thus, the setup offers flexibility over the traditional buckling setups.

It is proposed to undertake some study cases and obtain experimental results of non – dimensional buckling of rectangular laminated plates subjected to in-plane biaxial Compressive loads. The plates are assumed to be either simply supported on all edges (SS), or a combined case of clamped and simply supported (CS), or clamped on all edges (CC).

The effects of various parameters like material anisotropy, fiber orientation, aspect ratio, and edge conditions on the buckling load of laminated plates are to be investigated and compared with the present finite element results. The plates are made of graphite – epoxy material (material 3) and generally, square with side $a = b = 250mm$ and length to thickness ratio (a/h) =20. The required experiments are explained below:

Experiment (1): effect of material anisotropy (E_1/E_2)

Cross-ply symmetric laminates with length to thickness ratio of $(a/h = 20)$ are to be tested. The ratio of longitudinal to transverse modulus (E_1/E_2) is to be increased from 10 to 50. The required number of plies is 8. The plate is simply – supported (SS) on all edges. The experimental values of bucthe kling load were compared with the present theoretical results as shown in Table 16.

Table 16
Effect of material anisotropy on buckling load, $a/h = 20$

E_1/E_2	Method	Buckling loads
10	Present	0.5537
	Experimental	0.4985
20	Present	0.4789
	Experimental	0.4310
30	Present	0.4536
	Experimental	0.4082
40	Present	0.4418
	Experimental	0.3976
50	Present	0.4343
	Experimental	0.3908

It is observed that the buckling load decreases with the increase in material anisotropy (E_1/E_2). The present theoretical results were about 10% higher than the experimental values which is considered to be acceptable.

Experiment (2): effect of fiber orientation (θ)

Symmetric and antisymmetric cross-ply laminated plates (0/ 90/ 90/ 0) and (0/ 90/ 0/ 90) with length to thickness ratio (a/h) are to be tested. The required number of plies is 8. The plate is simply supported (SS) on four edges. As shown in Table 17 below, the theoretical buckling load was found to be 10% above the experimental value.

Table 17
Effect of fiber orientation on buckling load, $E_1/E_2 = 40$, $a/h = 20$

Orientation	Method	Buckling loads
Symmetric	Present	0.4418
	Experimental	0.3976
Anti – Symmetric	Present	0.4417
	Experimental	0.3975

Experiment (3): effect of aspect ratio (a/b)

The effect of aspect ratio (a/b) on the buckling load is studied by testing cross – ply symmetric (0/ 90/ 90/ 0) laminates with length to thickness ratio $(a/h = 20)$. The aspect ratios 0.5, 1, 1.5 and 2.0 are to be tested. The required number of plies is 8. The plate is simply supported on four edges and the modulus ratio is taken to be $(E_1/E_2 = 40)$. As shown in Table 18 below, the difference between the theoretical and experimental buckling was found to be about 10%.

Table 18
Effect of aspect ratio on buckling load, $E_1/E_2 = 40$, $a/h = 20$

Aspect Ratio (a/b)	Method	Buckling loads
0.5	Present	0.4192
	Experimental	0.3773
1.0	Present	0.4418
	Experimental	0.3976
1.5	Present	0.7187
	Experimental	0.6468
2.0	Present	1.2324
	Experimental	1.1092

Experiment (4): Effect of Boundary Conditions

Cross-ply symmetric laminates (0/ 90/ 90/ 0) can be used to study the effect of the boundary conditions on the buckling load. The length to thickness ratio is taken to be $(a/h = 20)$. The boundary conditions used are SS, CS and CC. The required number of plies is 8 and the modulus ratio (E_1/E_2) is selected to be 40. As shown in Table 19 below, the same difference between the theoretical and experimental results was observed.

Table 19
Effect of boundary conditions on buckling load, $E_1/E_2 = 40$, $a/h = 20$

Boundary Conditions	Method	Buckling loads
SS	Present	0.4418
	Experimental	0.3976
CS	Present	1.2882
	Experimental	1.1594
CC	Present	1.3812
	Experimental	1.2431

Conclusions

The finite element model has been formulated to compute the buckling loads of laminated plates with rectangular cross-section. To verify the accuracy of the present technique, buckling loads are evaluated and validated with other works available in the literature. Further comparisons were carried out and compared with the results obtained by the ANSYS package and experimental results. The good agreement with available data demonstrates the reliability of the finite element method used.

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