Robust Control of a Quadcopter Flying via Sliding Mode

Rudi Uswarman[1], Swadexi Istiqphara[1], Rajif Agung Yunmar [2], Arkham Zahri Rakhman [2]

[1] Electrical Engineering Department, Institut Teknologi Sumatera, Lampung Selatan, Lampung
[2] Informatics Engineering Department, Institut Teknologi Sumatera, Lampung Selatan, Lampung

Abstract. Sliding Mode Control (SMC) used to control the stability of a quadcopter from disturbances and uncertainties. This technique has two main advantages: the nonlinear dynamics and modelling errors of the quadcopter can be eliminated by switching function and the uncertainty problem can be overcome with a closed-loop response. The controller of the sliding mode technique consists of two components. The first is design equivalent control law to maintain the system state trajectory on the sliding surface. The second is design switching control law to reach the sliding surface. The Lyapunov theorem is used to ensure the stability of the system. Simulation results verify the robustness of the controller.

I. Introduction

Quadcopter has been applied in many fields such as business [1], agriculture [2], surveillance, and security [4]. Nowadays many methods have been developed to control quadcopter including linear and non-linear controls. Some previous studies were simulation's research of quadcopter controller using PD [5], PID [6] [7] [8] and sliding mode control (SMC) [9]. The PD controller can be reduced the uncertainties of a quadcopter by manually select filter parameters and the results show that the output can follow the reference but there is a steady state error when added disturbance [5]. The PID controller can follow the desired path but level of precision in moving need to be improved [6] [7] [8]. The simulation results of illustrate that proposed SMC controller provides significant reduction of disturbances influence on Quadcopter tracking performances but the gain of switching control could cause high chattering phenomenon if added external disturbance [9]. Based on some literature the SMC control has a robust response, although there is weakness such as high chattering. This weakness can be overcome by adjusting the gain value in the switching control.

SMC with automatic tuning of gain value on switching control will be investigated on Quadcopter flying. This technique can reduce the effect of the uncertainties and disturbances then minimize the chattering.

II. Dynamics of a Quadcopter

The mathematical model of the quadcopter uses 12 states, namely x, y, z as position of quadcopter, $\dot{x}, \dot{y}, \dot{z}$ as velocity of x, y, z, then ϕ, θ, ψ as roll, pitch, yaw angle, and $\dot{\phi}, \dot{\theta}, \dot{\psi}$ as angular velocity. The dynamic of system will be represent in state space model.

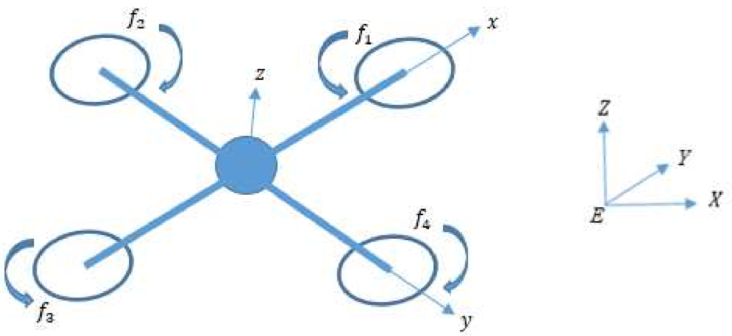

Fig. 1. Model of quadcopter

The Quadopter model is obtained from the Newton-Euler equation. In the first stage, use Newton second law to get the equation of translational motion [10].

$$\boldsymbol{F} + \boldsymbol{d_F} = \boldsymbol{m\dot{v}} + \boldsymbol{\omega} \times \boldsymbol{mv}, \quad (1)$$

$$\boldsymbol{F_g} - \boldsymbol{F_{thrust}} + \boldsymbol{d_F} = \boldsymbol{m\dot{v}} + \boldsymbol{\omega} \times \boldsymbol{mv}, \quad (2)$$

where $\boldsymbol{F_g}$ is gravity, $\boldsymbol{F_{thrust}}$ are rotation matrix of vertical thrust $[0\ 0\ \boldsymbol{U_1}]^T$, $\boldsymbol{d_F} = [\boldsymbol{d_x d_y}\ \boldsymbol{d_z}]^T$ are external disturbances, m is mass of quadcopter, $\boldsymbol{v} = [\dot{x}\ \ \dot{y}\ \ \dot{z}]^{\boldsymbol{T}}$ are velocity, $\boldsymbol{\omega} = [\dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi}]^{\boldsymbol{T}}$ are angular velocity.

$$\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{mg} \end{bmatrix} - \left(\boldsymbol{R_z(\psi)} \cdot \boldsymbol{R_y(\theta)} \cdot \boldsymbol{R_x(\phi)}\right) \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{U_1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{d_x} \\ \boldsymbol{d_y} \\ \boldsymbol{d_z} \end{bmatrix} = \boldsymbol{m} \left[\begin{bmatrix} \ddot{\boldsymbol{x}} \\ \ddot{\boldsymbol{y}} \\ \ddot{\boldsymbol{z}} \end{bmatrix} + \begin{bmatrix} \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix} \times \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{y}} \\ \dot{\boldsymbol{z}} \end{bmatrix} \right]. \quad (3)$$

The rotation matrix of roll, pitch, and yaw denoted as :

$$\boldsymbol{R_z(\psi)\cdot R_y(\theta)\cdot R_x(\phi)} = \begin{bmatrix} c_\psi & s_\psi & 0 \\ -s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{bmatrix}$$

$$= \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & -s_\phi c_\psi + c_\phi s_\theta s_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}. \tag{4}$$

Substituting equation 4 into equation 3

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m}\left[\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} - \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & -s_\phi c_\psi + c_\phi s_\theta s_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ U_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}\right] - \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}.$$

[5]

Assume that $\boldsymbol{v} = [\dot{x}\ \ \dot{y}\ \ \dot{z}]^{\boldsymbol{T}}$ and $\boldsymbol{\omega} = [\dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi}]^{\boldsymbol{T}}$ are equal with 0, we can find $\dot{\boldsymbol{v}}$

$$\ddot{x} = -\frac{1}{m}(s_\phi s_\psi + c_\phi s_\theta c_\psi)U_1 + \frac{1}{m}d_x\ , \tag{6}$$

$$\ddot{y} = -\frac{1}{m}(-s_\phi c_\psi + c_\phi s_\theta s_\psi)U_1 + \frac{1}{m}d_y\ , \tag{7}$$

$$\ddot{z} = g - \frac{1}{m}(c_\phi c_\theta)U_1 + \frac{1}{m}d_z\ . \tag{8}$$

Rotational Euler equation can be written as :

$$\boldsymbol{M_b} = \boldsymbol{\omega} \times (\boldsymbol{I}\cdot\boldsymbol{\omega}) + \boldsymbol{I}\cdot\dot{\boldsymbol{\omega}} = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix}, \tag{9}$$

where $[\boldsymbol{U_2}\ \boldsymbol{U_3}\boldsymbol{U_4}]^T$ are the control torques of roll, pitch, and yaw,

$$\begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} I_{xx}\dot{\phi} \\ I_{yy}\dot{\theta} \\ I_{zz}\dot{\psi} \end{bmatrix} + \begin{bmatrix} I_{xx}\ddot{\phi} \\ I_{yy}\ddot{\theta} \\ I_{zz}\ddot{\psi} \end{bmatrix}. \tag{10}$$

Based on equation 10, we can get :

$$U_2 = (I_{zz} - I_{yy})\dot{\theta}\dot{\psi} + I_{xx}\ddot{\phi}\ , \tag{11}$$

$$U_3 = (I_{xx} - I_{zz})\dot{\phi}\dot{\psi} + I_{yy}\ddot{\theta}\ , \tag{12}$$

$$U_4 = (I_{yy} - I_{xx})\dot{\phi}\dot{\theta} + I_{zz}\ddot{\psi}\ . \tag{13}$$

Finally, the derivatives of the angular velocity can be written as :

$$\ddot{\phi} = \frac{U_2 + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi}}{I_{xx}}\ , \tag{14}$$

$$\ddot{\theta} = \frac{U_3 + (I_{zz} - I_{xx})\dot{\phi}\dot{\psi}}{I_{yy}}\ , \tag{15}$$

$$\ddot{\psi} = \frac{U_4 + (I_{xx} - I_{yy})\dot{\phi}\dot{\theta}}{I_{zz}}\ . \tag{16}$$

The state space of the quadcopter has the following form

$$X = [\, x_1\; x_2\; x_3\; x_4\; x_5\; x_6\; x_7\; x_8\; x_9\; x_{10}\; x_{11}\; x_{12}]^T, \quad (17)$$

$$X = [\, x\; y\; z\; \dot{x}\; \dot{y}\; \dot{z}\; \phi\; \theta\; \psi\; \dot{\phi}\; \dot{\theta}\; \dot{\psi}]^T\;. \quad (18)$$

Then, we get

$$\dot{x}_1 = \dot{x} = x_4\;, \quad (19)$$

$$\dot{x}_2 = \dot{y} = x_5\;, \quad (20)$$

$$\dot{x}_3 = \dot{z} = x_6\;, \quad (21)$$

$$\dot{x}_4 = \ddot{x} = -\frac{1}{m}(s_\phi s_\psi + c_\phi s_\theta c_\psi)U_1 + \frac{1}{m}d_x, \quad (22)$$

$$\dot{x}_5 = \ddot{y} = \ddot{y} = -\frac{1}{m}\left(-s_\phi c_\psi + c_\phi s_\theta s_\psi\right)U_1 + \frac{1}{m}d_y\;, \quad (23)$$

$$\dot{x}_6 = \ddot{z} = g - \frac{1}{m}\left(c_\phi c_\theta\right)U_1 + \frac{1}{m}d_z\;, \quad (24)$$

$$\dot{x}_7 = \dot{\phi} = x_{10}\;, \quad (25)$$

$$\dot{x}_8 = \dot{\theta} = x_{11}\;, \quad (26)$$

$$\dot{x}_9 = \dot{\psi} = x_{12}\;, \quad (27)$$

$$\dot{x}_{10} = \ddot{\phi} = \frac{U_2 + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi}}{I_{xx}}\;, \quad (28)$$

$$\dot{x}_{11} = \ddot{\theta} = \frac{U_3 + (I_{zz} - I_{xx})\dot{\phi}\dot{\psi}}{I_{yy}}\;, \quad (29)$$

$$\dot{x}_{12} = \ddot{\psi} = \frac{U_4 + (I_{yy} - I_{xx})\dot{\phi}\dot{\theta}}{I_{zz}}\;, \quad (30)$$

and input control of u_x and u_y can be represented as

$$u_x = s_\phi s_\psi + c_\phi s_\theta c_\psi\;, \quad (31)$$

$$u_y = -s_\phi c_\psi + c_\phi s_\theta s_\psi\;. \quad (32)$$

The total thrust/force U_1, roll torque U_2, pitch torque U_3, and yaw torque U_4 can be expressed as follows

$$U_1 = F = f_1 + f_2 + f_3 + f_4\;, \quad (33)$$

$$U_2 = f_2 - f_4\;, \quad (34)$$

$$U_3 = f_1 - f_3\;, \quad (35)$$

$$U_4 = f_1 + f_3 - f_2 - f_4\;. \quad (36)$$

Quadcopter parameters used can be seen in table 1.

Table 1. Parameter of a quadcopter.

Parameter	Value
Distance rotor from centre (l)	0.3 m
Mass (m)	2.5 kg
Inertial moment on x axis (J_{xx})	0.04 $kg \cdot m^2$
Inertial moment on y axis (J_{yy})	0.04 $kg \cdot m^2$
Inertial moment on z axis (J_{zz})	0.045 $kg \cdot m^2$
Gain (K)	150 rad/s

Actuator Bandwidth ($\boldsymbol{\omega}$)	18 $\boldsymbol{N}$

III. Controller Design

The SMC control consists of equivalent control and switching control. In the equivalent control for the Z axis, the first step is designing a sliding surface. The sliding surface is defined as:

$$\boldsymbol{s_5} = \dot{\tilde{\boldsymbol{n}}}_5 + \boldsymbol{\lambda_5}\tilde{\boldsymbol{n}}_5 \ , \tag{37}$$

$$\boldsymbol{s_5} = (\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\boldsymbol{x_5} - \boldsymbol{x_{5d}}) \ , \tag{38}$$

where $\boldsymbol{\lambda_5} > 0$, then derivative of $\boldsymbol{s_5}$

$$\dot{\boldsymbol{s}}_5 = \ddot{\tilde{\boldsymbol{n}}}_5 + \boldsymbol{\lambda_5}\dot{\tilde{\boldsymbol{n}}}_5 = \boldsymbol{0} \ , \tag{40}$$

$$\boldsymbol{0} = (\ddot{\boldsymbol{x}}_5 - \ddot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) \ , \tag{41}$$

$$\boldsymbol{0} = (\dot{\boldsymbol{x}}_6 - \ddot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) \ , \tag{42}$$

$\dot{\boldsymbol{x}}_6$ can be defined as :

$$\dot{\boldsymbol{x}}_6 = \ddot{\boldsymbol{z}} = \boldsymbol{g} - \frac{\boldsymbol{1}}{\boldsymbol{m}}(\boldsymbol{c_\phi c_\theta})\boldsymbol{U_1} + \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{d_z} \ . \tag{43}$$

Substituting equation (43) to equation (42)

$$\boldsymbol{0} = \left(\boldsymbol{g} - \frac{\boldsymbol{1}}{\boldsymbol{m}}(\boldsymbol{c_\phi c_\theta})\boldsymbol{U_1} + \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{d_z} - \ddot{\boldsymbol{x}}_{5d}\right) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) \ , \tag{44}$$

and we get equivalent control:

$$\boldsymbol{U}_{\mathbf{eq}} = \frac{\boldsymbol{m}}{\boldsymbol{cos\ x_7 \cdot cos\ x_8}}\left(\boldsymbol{g} + \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{d_z} - \ddot{\boldsymbol{x}}_{5d}\right) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) \ . \tag{45}$$

The switching control for Z axis can be written as:

$$\boldsymbol{u_s} = -\boldsymbol{K_5}\ \boldsymbol{sign}\ (\boldsymbol{s_5}) \ . \tag{46}$$

Finally, SMC controller can be obtained by adding equation (45) and (46)

$$\boldsymbol{U_1} = \frac{\boldsymbol{m}}{\boldsymbol{cos\ x_7 \cdot cos\ x_8}}\left(\boldsymbol{g} + \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{d_z} - \ddot{\boldsymbol{x}}_{5d}\right) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) - \boldsymbol{K_5}\ \boldsymbol{sign}\ (\boldsymbol{s_5}) \ . \tag{47}$$

The stability of the system can be proven using the Lyapunov stability method
Lyapunov function is defined as:

$$V = \frac{1}{2}S^2 \ > 0, \tag{48}$$

and

$$\dot{V} = s\dot{s} < 0$$

$$= \left((\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\boldsymbol{x_5} - \boldsymbol{x_{5d}})\right)\left((\ddot{\boldsymbol{x}}_5 - \ddot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d})\right) . \tag{49}$$

Subtituting equation (43) into equation (49) :

$$\dot{V} = \left((\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d}) + \boldsymbol{\lambda_5}(\boldsymbol{x_5} - \boldsymbol{x_{5d}})\right)\left(\left(\boldsymbol{g} - \frac{\boldsymbol{1}}{\boldsymbol{m}}(\boldsymbol{c_\phi c_\theta})\boldsymbol{U_1} + \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{d_z} - \ddot{\boldsymbol{x}}_{5d}\right) + \boldsymbol{\lambda_5}(\dot{\boldsymbol{x}}_5 - \dot{\boldsymbol{x}}_{5d})\right).$$

(50)

Then, substituting equation (47) into equation (50):

$$\dot{V} = \left((\dot{x}_5 - \dot{x}_{5d}) + \lambda_5(x_5 - x_{5d})\right)\left(\left(g - \frac{1}{m}(c_\phi c_\theta)\left(\frac{m}{\cos x_7 \cdot \cos x_8}\left(g + \frac{1}{m}d_z - \ddot{x}_{5d}\right) + \lambda_5(\dot{x}_5 - \dot{x}_{5d}) - K_5\, sign\,(s_5)\right) + \frac{1}{m}d_z - \ddot{x}_{5d}\right) + \lambda_5(\dot{x}_5 - \dot{x}_{5d})\right). \quad (51)$$

We can eliminated some equation in equation (51) and choose K to be minus large enough to get stable in the sense of Lyapunov. K gain can change when the system gets disturbances. The equation can be seen in (52)

$$\hat{K}_5 = -\mu_5\, m\, Sign\,(s_5)(-\varepsilon_5)\,, \quad (52)$$

where ε_5 is defined as:

$$\varepsilon_5 = m\left(\dot{x}_6 + g - K_5\, sign\,(s_5)\right) - U_1\,, \quad (53)$$

Then, control for Z axis can be written as:

$$U_1 = \frac{m}{\cos x_7 \cdot \cos x_8}\,(g + \frac{1}{m}d_z - \ddot{x}_{5d}) + \lambda_5(\dot{x}_5 - \dot{x}_{5d}) - \hat{K}_5\ sign\,(s_5)\,, \quad (56)$$

with same technique we get input control

$$u_x = \left(\frac{d_x}{m} + \ddot{x}_{3d} - \left(\lambda_3(\dot{x}_3 - \dot{x}_{3d})\right)\right)\frac{m}{U_1} - \hat{K}_3 sign(s_3)\,, \quad (57)$$

$$u_y = \left(\frac{d_y}{m} + \ddot{x}_{4d} - \lambda_4(\dot{x}_4 - \dot{x}_{4d})\right)\frac{m}{U_1} - \hat{K}_4 sign(s_4)\,, \quad (58)$$

$$U_2 = \left(I_{zz} - I_{yy}\right)x_{11}x_{12} + \left(\ddot{x}_{9d} - \lambda_9(\dot{x}_9 - \dot{x}_{9d})\right)I_{xx} - K_9\, sign(s_9)\,, \quad (60)$$

$$U_3 = (I_{xx} - I_{zz})x_{10}x_{12} + \left(\ddot{x}_{10d} - \lambda_{10}(\dot{x}_{10} - \dot{x}_{10d})\right)I_{yy} - K_{10} sign\,(s_{10})\,, \quad (61)$$

$$U_4 = \left(I_{yy} - I_{xx}\right)x_{10}x_{11} + I_{zz}\left(\ddot{x}_{11d} - \lambda_{11}(\dot{x}_{11} - \dot{x}_{11d})\right) - K_{11} sign\,(s_{11})\,, \quad (62)$$

IV. Simulation Results

In Figure 2 shows the quadcopter can follow the set point without steady state error and requires a rise time of 8 seconds to reach the x-axis change as far as 5 meters. The output response in Figures 3 and 4 shows good performance despite being given payload as disturbance (Figure 8). Then in Figures 5 and 6 shows the response of the Roll angle and Pitch angle to the reference signal has fast rise time. Figure 7 shows that the gain in switching controls can change automatically adjusting changes in dynamics of the system due to disturbance and uncertainty. When given a load as a disturbance (Figure 8), the gain value at Kz increases so that the quadcopter can maintain its position on the Z axis.

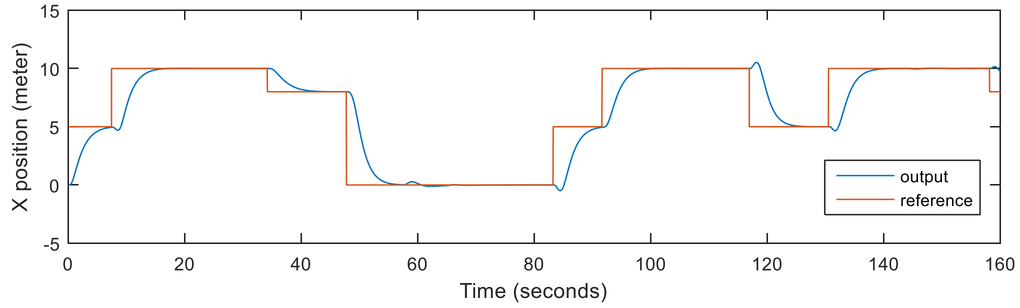

Figure 2 Position on X axis

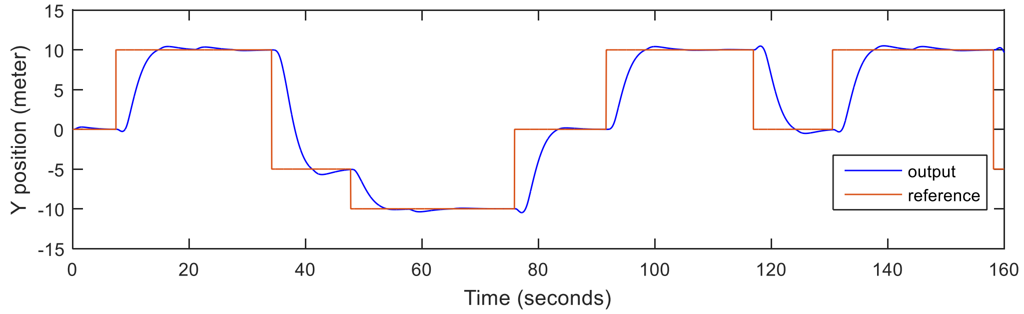

Figure 3 Position on Y axis

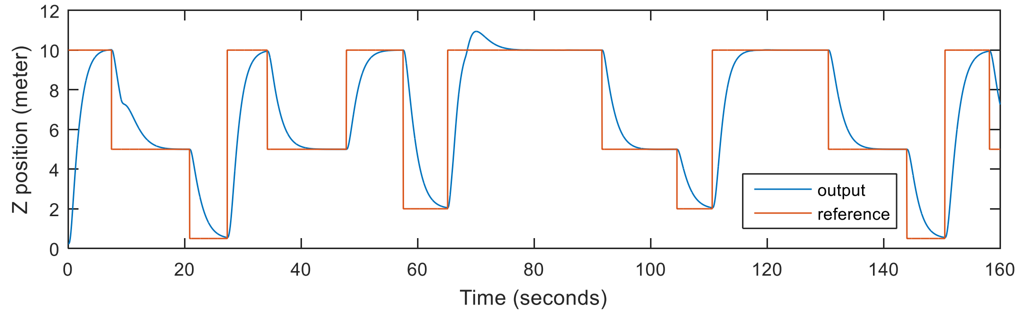

Figure 4 Position on Z axis

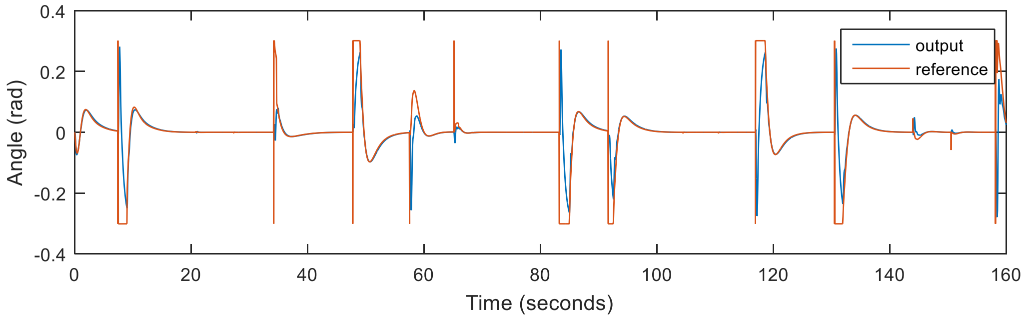

Figure 5 Roll angle

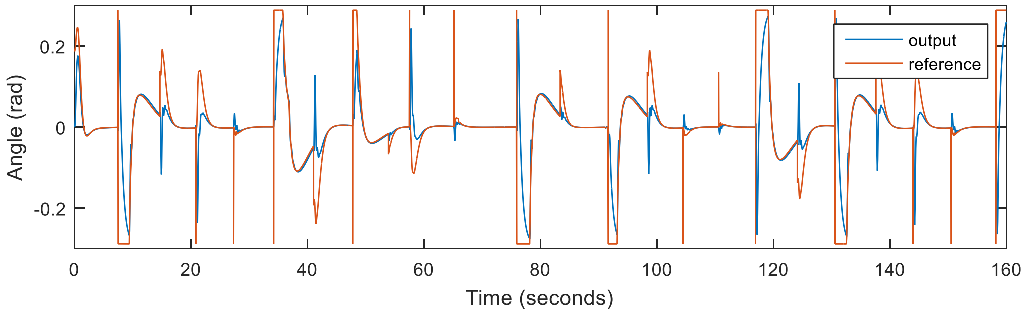

Figure 6 Pitch angle

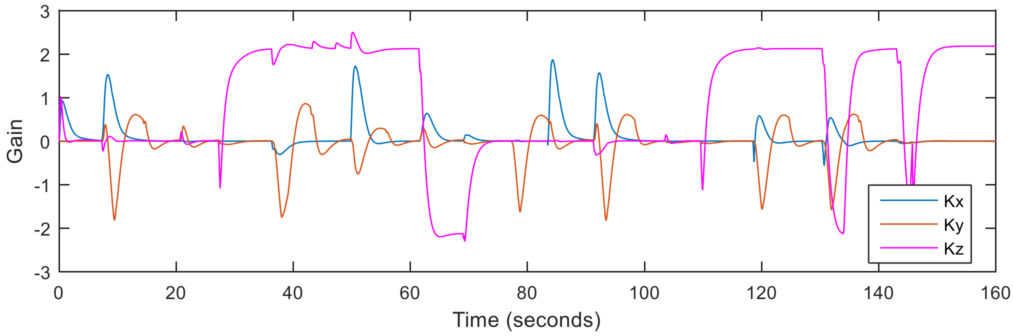

Figure 7 Gain change in simulation

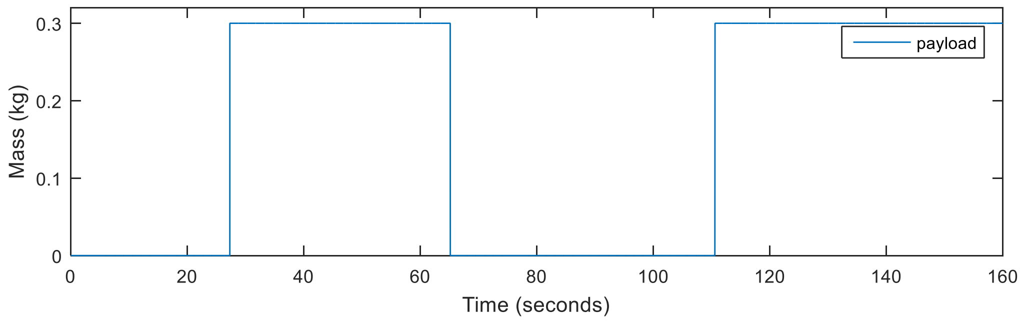

Figure 8 Payload as disturbance

V. Conclusion

The simulation results show that the SMC can control the quadcopter following the reference of position without causing a chattering phenomenon, then the switching control can change automatically when there are changes in parameters and disturbances.

References

[1] A. M. Lawson, "Consumer Behaviors of Drone Delivery," Bachelor Theses, Appalachian State University, 2017.

[2] L. P. Hafsal, "Precision Agriculture with Unmaned Aerial Vehicles for SMC estimation – Towards a more Sustainable Agriculture," Master Thesis, Hedmark University of Applied Sciences, 2016.

[3] N. H. Motlagh, M. Bagaa, and T. Taleb, "UAV-Based IoT Platform: A Crowd Surveillance use Case," IEEE Communications Magazine, vol. 55, pp. 128-134, February 2017.

[4] D. He, S. Chan, and M. Guizani, “Drone-Assisted Public Safety Networks: The Security Aspect,” IEEE Communications Maagazine, vol. 55, pp. 218-233, April 2017.

[5] G. Shincy and E. Varghese, “Design and Simulation of Robust Filter for Tracking Control of Quadcopter System,” IEEE International Conference on circuits Power and Computing Technologies, Apr. 2017.

[6] R. A. Garcia, F. R. Rubio, and M. G. Ortega,”Robust PID Control of the Quadrotor Helicopter,” IFAC Conference, Mar. 2012.

[7] S. Abdelmoeti and R. Carloni, “Robust Control of an Unmanned Aerial Vehicle Using the Parameter Space Approach,” Project on Robotics and Mechatronics at University of Twente.

[8] B. J. Emran, J. Dias, L. Seneviratne, and G. Cai, “Robust Adaptive Design for Quadcopter Payload Add and Drop Applications,” Chinese control conference, pp. 3252-3257, July 2015.

[9] Stevanovic, Kasac, and Stepanic,”Robust Tracking Control of a Quadrotor Helicopter without Velocity Measurement,” International DAAAM Symponsium, vol. 23, pp. 595-600, 2012.

[10] P. Pounds, R. Mahony, and P. Corke, “Modelling and control of a quad-rotor robot,” in Australasian Conference on Robotics and Automation 2006. Auckland, New Zealand: Australian Robotics and Automation Association Inc., 2006.