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# Comparison among Original AHP, Ideal AHP and Moderate AHP Models

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**Abstract**

Decisions always involve getting the best solution, selecting the suitable experiments, most appropriate judgments, taking the quality results etc., using some techniques. Every decision making can be considered as the choice from the set of alternatives based on a set of criteria. The analytic hierarchy process (AHP) is a multi-criteria decision making and is dealing with decision-making problems through pairwise comparisons. This paper is concerned with the moderate AHP decision model is always the same as the original AHP decision model. It does not violate the rule itself.



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## 1. Introduction

Aragonés-Beltrán *et al.,* (2014), Cheng *et al.,* (1999), to achieve the evaluation for the problems in this project, the (Analytic Hierarchy Process) AHP has been preferred among the other methods of MADM the AHP is a structured technique for dealing with a complex decision. Rather than prescribing a ‘correct’ decision, the AHP helps the decision makers find the one that best suits their needs and their understanding of the problem.

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## 2. Materials and Methods

### *2.1 Analytic Hierarchy Process (AHP)*

The main essence of the AHP method is analyzing complex problems into a hierarchy with an aim at the top of the hierarchy, criterions at levels of the hierarchy and decision alternative at the bottom of the hierarchy. Elements at given hierarchy level are compared in pairwise to calculate their relative importance with respect to each of the elements at the next higher level. Dyer & Forman (1992), Ishizaka & Labib (2011), Lee & Kozar (2006), the AHP method calculates and totalizes their eigenvectors until the composite last vector of weight comparisons for alternatives is achieved. The entries of last weight comparisons vector reflect the importance value of each alternative with respect to the aim stated at the top of hierarchy [12].

### *2.2 Procedure for AHP*

The first step in the AHP procedure is the decomposition of a complex issue into a structure (hierarchy) with the aim criteria (Boroushaki & Malczewski, 2008) at the top of the structure. The criteria and sub-criteria allocated at level and sub-levels of the structure, and decision alternatives or comparisons at the bottom of the structure, as depicted in figure 1.

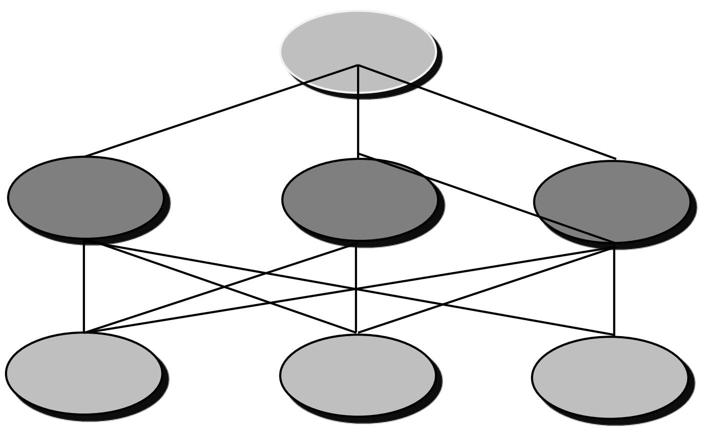


Figure 1. AHP Structure (Hierarchy) of decision issue

## 3. Results and Discussions

### *3.1 Pair –wise comparison*

Saaty (1987), (1990), (1994),(2012), pair –wise comparison can be explained as the procedure of comparing units in pairs to find out which one is selected. In other words, for each unit or entity of the hierarchy, the entire entities which are associated in the low hierarchy are compared in pair-wise. We can observe from figure 1 that the number of comparisons or alternatives in a combination of the number of entities or elements based on that, the number of comparisons in figure 1 is three which is shown in table 1.

Table 1
Number of Alternatives

| No. of elements | 1 | 2 | 3 | 4 | 5 | 6 | 7 | n |
|---|---|---|---|---|---|---|---|---|
| No. of comparisons | 0 | 1 | 3 | 4 | 10 | 15 | 21 | $\frac{n(n-1)}{2}$ |

The main aim of calculation technique is to make a reciprocal matrix comparison expressing the relative values of a set of attributes. The comparisons are used to structure a matrix of pair –wise comparisons called the judgment matrix or square matrix A (Thirumalaivasan *et al.,* (2003). For instance, let consider n elements to be compared $D_1$, $D_2$, ……$D_n$ are indicated to the relative or priority weight of $D_i$ with respect to $D_j$ by $a_{ij}$ and form a square matrix A = $(a_{ij})$ of order n with the constraints that $a_{ij} = 1/a_{ij}$, for i≠j, and $a_{ii} = 1$, all i, such a matrix is said to be a reciprocal matrix. In other words if $a_{ij}$ is the element of row i column j, the lower diagonal is filled by employing this formula be $a_{ji} = 1/a_{ij}$ the weight n elements. For instance of $a_{ij} = 3$ it implies and i is moderately important than j or i 3 times important than j this is called crisp evaluation (Ramik & Korviny, 2010). The structure of the matrix illustrated as follows.

$$A = (a_{ij}) = \begin{array}{c} \\ D_1 \\ D_2 \\ \\ D_n \end{array} \begin{array}{c} D_1 \quad D_2 \quad \ldots\ldots \quad D_n \\ \left( \begin{array}{cccc} 1 & a_{12} & \ldots\ldots & a_{1n} \\ 1/a_{12} & 1 & \ldots\ldots & a_{2n} \\ . & . & \ldots\ldots & \\ 1/a_{n} & 1/a_{2n} & \ldots\ldots & 1 \end{array} \right) \end{array} \qquad (1)$$

Where A = comparison pair –wise reciprocal matrix,

To find out the relative selection for n elements of the hierarchy matrix, the Saaty's fundamental scale of value from 1-9 is used to consider the intensity priority between two elements and, using the verbal scale associated with the 1-9 scale as shown in table 2.

Table 2
Saaty's scales for pair–wise comparison (Saaty, 2008)

| Saaty's Scale value | Priorities represented in linguistic variables |
|---|---|
| 1 | Equal important |
| 2 | Slight or Weak |
| 3 | Moderately important with one over another |
| 4 | Moderately plus |
| 5 | Strongly important |
| 6 | Strongly plus |
| 7 | Very strongly important |
| 8 | Too Strong |
| 9 | Extremely important |

The linguistic variables and ratio scale values are used for weighting tangible and intangible elements. The values of 2, 4, 6, and 8 are selected to specify compromise values of importance. To calculate relative weights of elements in each pair –wise comparison matrix, the Eigen value method can be employed. To compute Eigenvector or priority vector i.e. if we have a matrix three by three. We totalize each column of the matrix, then we divide each elements of the matrix with the total of its column, then we have to normalize relative weight (or) using the software of evaluation for AHP by CGI, to compute eigen vector or priority vector [12].

To normalize Eigen vector, row elements will be summed then divided by a number of elements in the same row, in other words taking the average value. The Eigen vector demonstrates relative weights amongst the objects that we compare. In this comparison method, some inconsistencies may accrue and are usual. For instance, when A contains inconsistencies, the estimated priorities can be achieved by employing the A matrix as the input. The relative weight (W) of matrix A is obtained from the following equation:

$$(A - \lambda_{max} I)\, q = 0 \qquad (2)$$

Where M is the reciprocal matrix?
$\lambda_{max}$ is the biggest Eigen value of matrix,
q is its correct Eigen value, and
I am the unit matrix of size n.

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The Eigen value ($\lambda_{max}$) can be obtained by summing of products between each elements of Eigen vector multiplied by the total of columns of the reciprocal matrix. Every Eigen value is scaled to total up to one to get the priorities. In other words the sum of all elements in Eigen value (priority value) is one. Inconsistency may occur when $\lambda_{max}$ moved away from n this is because of the inconsistency responses in pair –wise comparisons. Saaty (1977) proved that the biggest Eigen value is equal to the number of comparisons ($\lambda_{max}$ = n). Therefore, the matrix A should be examined for consistency by using consistency index CI as given in equation (3).

$$CI = \frac{(\lambda_{max} - n)}{(n-1)} \qquad (3)$$

One of the critical steps of AHP method is to create the comparison matrixes. However, when the number of alternatives increases, more comparisons between alternatives required. This might easily cause the excess of the consistency of the model. Therefore, a consistency check is required for the pair –wise comparison matrix (Saaty, 1992).

The consistency index is used in order to check whether the judgment of decision makers is consistent with respect to a comparison matrix. In other words, this index is important for the decision maker to assure him that his/ her judgments were consistent and that the final decision is made well. Well. While CI depends on n, then should calculate consistency ratio CR as shown in equation (4).

$$CR = \frac{CI}{RI} \qquad (4)$$

Salty proposed that CI used to compare with the appropriate consistency index which is called Random consistency index (RI). In other words, he randomly generated a reciprocal matrix in order to find random consistency index to observe if it is about 0.1 or 10% or less. The random CI is illustrated in table 3.

Table 3
Random consistency index (Saaty & Forman, 1993)

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| RI | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

The matrix will be consistency and acceptable if the consistency ratio is less than 0.1 or (CR < 0.1), if not we have to revise the subjective judgment.
In order to obtain the overall rating for the alternatives as depicted in equation (5).

$$W_j^s = \sum^{j=k} a_{ij}^s \; c_j^e \quad i = 1, \ldots.., n \qquad (5)$$

Where $A_i^s$ = total weight of site i,

$a_{ij}^s$ = weight of alternative (site) i associated to criterion map j.

$a_j^e$ = weight of criteria j,

k = number of criteria,

i = number of alternative

*3.2 Ideal AHP*

Also, we can get ideal AHP matrix (model), by dividing their entries in the column of the original AHP matrix for the corresponding criterion with the largest entry in that selected column. Multiply these values of the alternatives with corresponding the resulting criterion weights (Global priorities). Sum these values to get the final priority vector for the respective alternative. In such a way we find the final priority vectors for the remaining alternatives. After normalization of the final priority vector to have the values with ranking. For rare cases (one or two), it violates the Ideal AHP rule against original AHP.

*3.3 Moderate AHP*

It can be extended to find the final alternative priority vectors for all alternatives from the original AHP decision matrix. It can be obtained from the following formula.

$$MA_j = \sum_{j=1}^{n} W_j (W_j + A_{ij}^1)$$

Where $W_j$ is the weight vector for corresponding resulting criteria and $A^1_{ij}$ is the weight vector (scores) of the $i^{th}$ alternative and $j^{th}$ resulting criteria of the original AHP decision matrix we get moderate AHP decision matrix [7].

For all cases, moderate AHP model is same as original AHP model so it is a better to model. From the suitable sequence of alternatives and criteria of the pairwise comparisons models, we get the following tables.

Table 4
Original AHP decision matrix

| Alternative/ Criteria | S | W | O | T | Final Priority Vector | Ranking |
|---|---|---|---|---|---|---|
| | 0.6336 | 0.1823 | 0.0446 | 0.1395 | | |
| $A_1$ | 0.2902 | 0.4296 | 0.3622 | 0.2122 | 0.3080 | 1 |
| $A_2$ | 0.2143 | 02593 | 0.3189 | 0.2252 | 0.2287 | 4 |
| $A_3$ | 0.2355 | 0.1126 | 0.1139 | 0.4273 | 0.2344 | 2 |
| $A_4$ | 0.2598 | 0.1985 | 0.2050 | 0.1353 | 0.2288 | 3 |

Table 5
Ideal AHP decision matrix

| Alternative/ Criteria | S | W | O | T | Final Priority Vector | Normalization | Ranking |
|---|---|---|---|---|---|---|---|
| | 0.6336 | 0.1823 | 0.0446 | 0.1395 | | | |
| $A_1$ | 1.0000 | 1.0000 | 1.0000 | 0.4966 | 0.9298 | 0.3042 | 1 |
| $A_2$ | 0.7385 | 0.6036 | 0.8805 | 0.5270 | 0.6907 | 0.2260 | 4 |
| $A_3$ | 0.8115 | 0.2621 | 0.3145 | 1.0000 | 0.7155 | 0.2341 | 3 |
| $A_4$ | 0.8952 | 0.4621 | 0.5660 | 0.3166 | 0.7208 | 0.2358 | 2 |

Table 6
Moderate AHP decision matrix

| Alternative/ Criteria | S | W | O | T | Final Priority Vector | Normalization | Ranking |
|---|---|---|---|---|---|---|---|
| | 0.6336 | 0.1823 | 0.0446 | 0.1395 | | | |
| $A_1$ | 0.2902 | 0.4296 | 0.3622 | 0.2122 | 0.7641 | 0.2705 | 1 |
| $A_2$ | 0.2143 | 0.2593 | 0.3189 | 0.2250 | 0.6848 | 0.2425 | 4 |
| $A_3$ | 0.2355 | 0.1126 | 0.1139 | 0.4273 | 0.6906 | 0.2445 | 2 |
| $A_4$ | 0.2598 | 0.1985 | 0.2050 | 0.1353 | 0.6849 | 0.2425 | 3 |

## 4. Conclusion

In original AHP decision model, $A_2 = 0.22874575$ and $A_4 = 0.22884577$ with eight decimal places after the decimal point clear the ranking of the alternatives In Ideal AHP decision model, the ranks of $A_3$ and $A_4$ have changed as 3 and 2 respective instead of 2 and 3 in the original AHP.

It violates the ideal AHP rule. In moderate AHP decision model, the $A_2 = 0.24245858$ and $A_4 = 0.24249398$ with eight decimal places after the decimal point clear the ranking of the alternative as in original AHP decision model. Therefore the moderate AHP is the best and accuracy. Therefore the Moderate AHP rule does not violate itself.

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