PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' UNDERSTANDING OF ABSOLUTE VALUE

Tian Abdul Aziz*, Supiat, and Yohanes Soenarto
Universitas Muhammadiyah Prof. DR. HAMKA
*e-mail: tian_aziz@uhamka.ac.id

Abstract: This study aims to give a comprehensive account of pre-service secondary mathematics teachers' understanding of absolute value. Thirty two-item absolute value understanding test was developed and administered to thirty-eight students attending mathematics education department at one private university in Jakarta City, Indonesia. Five of them were selected purposively and interviewed to gain deep information and confirm their written responses in the test. We find that most participants struggled with the absolute value task. There are inconsistencies of the definition of absolute value expressed by them. Besides, typical mistakes made are: (a) removal of absolute value bars; (b) focus heavily on rules; (c) conversion of absolute value bars to parentheses; (d) exclusion of number inside absolute value bars; (e) poor algebraic manipulation; and (f) inability to draw absolute value graph. Based on the findings, the most common cause of mistakes made by the participants is didactical contract in mathematics teaching and learning. Limitation and implications of the study are presented.

Keywords: ***absolute value, pre-service teachers, understanding, mistakes, secondary mathematics***

PEMAHAMAN MAHASISWA CALON GURU MATEMATIKA SEKOLAH MENENGAH TERHADAP NILAI MUTLAK

Abstrak: Penelitian ini dilakukan untuk memberikan gambaran yang komprehensif tentang pemahaman calon guru matematika terhadap konsep nilai mutlak. Instrumen tes yang terdiri dari 32 soal dikembangkan dan diberikan kepada 38 mahasiswa program studi pendidikan matematika di salah satu universitas swasta di Jakarta, Indonesia. Berdasarkan hasil tes tersebut, lima mahasiswa dipilih secara *purposive* dan diwawancarai dengan tujuan untuk mendapatkan informasi yang mendalam dan mengkonfirmasi jawaban yang mereka berikan dalam tes. Kami menemukan bahwa sebagian besar partisipan mengalami kesulitan dalam mengerjakan soal tes. Mereka mengungkapkan definisi nilai mutlak secara tidak konsisten. Kesalahan umum yang mereka buat adalah: (a) menghapus tanda nilai mutlak; (b) memfokuskan hanya pada rumus; (c) mengubah tanda nilai mutlak menjadi tanda kurung; (d) mengeluarkan bilangan di dalam tanda nilai mutlak; (e) memanipulasi aljabar secara tidak tepat; dan (f) menggambar grafik fungsi nilai mutlak secara tidak tepat. Berdasarkan hasil temuan, kesalahan yang dilakukan umumnya disebabkan oleh *didactical contract* dalam proses pembelajaran matematika. Batasan dan implikasi dari hasil penelitian akan disajikan.

Kata Kunci: ***nilai mutlak, mahasiswa calon guru, pemahaman, kesalahan, matematika menengah***

INTRODUCTION

Pre-service mathematics teachers are required to develop and strengthen their knowledge of subject matter so that it might improve their teaching performance. Also it might help students to comprehend various mathematical concepts in a proper manner and cope with various puzzlements.

One of the mathematical concepts in which most students have puzzlement is the concept of absolute value (Çiltaş & Tatar, 2011). The most widely accepted explanation of absolute value in mathematics classrooms is that it is construed as a distance between two real numbers in number lines (Ellis & Bryson, 2011). However, several algebra students tend to believe that absolute value is defined as positive real numbers (Ponce, 2008; Taylor & Mittag, 2015) or a number without a sign (Gagatsis & Panaoura, 2014). Understanding absolute value becomes critical for advanced mathematics topics in calculus such as limit, continuity, and multivariate (Almog & Ilany, 2012). Therefore, mathematics teachers are responsible for assisting their students in comprehending it well. School mathematics curriculum in most countries introduced the concept of absolute value at the secondary level.

In reference to the 2013 Indonesian mathematics curriculum, the absolute value is introduced in the 11th grade whose basic competencies required are: (1) describing and analysing absolute value; (2) applying absolute value to solve real-world problems; and (3) generating mathematical model involving absolute value from real-world context. In most textbooks applying the 2013 curriculum, the absolute value is defined as the distance between a number and zero on the real number line. In order for students gain a better insight into the concept, they are provided with instances considering a step forward and a step backward. There is no discussion that absolute value is a number without sign or process of removing the absolute sign.

There are abundant techniques developed to help students gain better and meaningful insight of absolute value. Teachers might utilize line numbers to provide an opportunity for students integrating procedural and conceptual knowledge at the same time (Ellis & Bryson, 2011). Besides, by considering the importance of visualization in mathematics learning, teachers could take the use of graphical representation into account (Konyalioglu, Aksu, & Senel, 2012; Stupel & Ben-Chaim, 2014). In addition, a meaningful task such as contextual task could be presented to guide students to comprehend absolute value (Wade, 2012).

The literature contains many studies on disclosing students' difficulties with absolute value and its performance when dealing with mathematical problems (Almog & Ilany, 2012; Elia, Özel, Gagatsis, Panaoura, & Özel, 2016; Gagatsis & Panaoura, 2014). The possible explanations concerning this case are students' overgeneralization (Elia et al., 2016). Students' knowledge is likely to be affected by their previous knowledge. Previous knowledge is likely to be considered as the absolute truth which could be applied in various contexts. Generally speaking in learning algebra, overgeneralization is the main source of students' misconception (Aziz, Pramudiani, & Purnomo, 2017). This explanation also appears to be supported by Schneider (2014) who refers to this students' overgeneralization as epistemological obstacles. This might occur due to their inaccurate understanding as a condition of certain rules is not presented comprehensively by teachers or students focus solely on memorizing and applying formulas to answer questions. This obstacle also is ubiquitous when students attempt to grasp the notion of absolute value. For example, misuse of distributive multiplication leads students to make mistakes when solving problems of absolute value as they regard absolute sign is similar to parentheses.

Another plausible explanation for this case is a problem in a didactic process which is mostly known as the didactic obstacle. It refers to distortion between taught knowledge and scholarly knowledge. As a result, students' knowledge of absolute value is likely to differ considerably from scholarly knowledge. Teachers who take part in transitive work are responsible for presenting this scholarly knowledge in educational institution. Teachers' misconception or mistakes could influence students' understanding of the notion of absolute value. Didactic obstacle might be due to a didactic contract (Brousseau, Sarrazy, & Novotná, 2014). It refers to implicit rules developed and negotiated in the learning process between teachers and students in an attempt to comprehend the concept or solve problems. In the context of absolute value teaching process, didactic contract appears when removing absolute sign is common procedure emphasized by teachers. It might lead students to disengage in making sense of absolute value concept.

A number of studies focusing on absolute value have been conducted intensively in recent years (Elia et al., 2016; Gagatsis & Panaoura, 2014). The focus of their study was to understand high school students' conception of absolute value as well as its obstacles. Nevertheless, a study investigating pre-service secondary mathematics teachers understanding of the notion of absolute value is still not investigated. Therefore, the present study is unique to research literature. Generally speaking, students' inaccurate understanding of absolute value at high school level might be carried over to higher education level. Mathematics teacher education program at university is responsible for refining pre-service teachers' existing misconceptions of absolute value in mathematics teaching and learning process. Otherwise, when they are becoming in-service teachers, their misconception might lead to problems encountered by their students. Therefore, the study becomes critical as identification of pre-service secondary mathematics teachers is a cornerstone of an effort to refine their knowledge about absolute value so that possible misconceptions would be avoided when teaching. Teacher educators might take the finding of this study into consideration as the study might assist them in designing effective instruction concerning absolute value concept.

Building on the aforementioned explanation, the researchers attempt to address the two central questions to be examined in this study: (1) what are pre-service secondary mathematics teachers' understanding of absolute value; and (3) what are typical mistakes made by them when accomplishing absolute value task.

METHODS

As the objectives of this study concerned describing pre-service teachers' understanding of absolute value, a case study was chosen. To address the research questions, paper-and-pencil assessments and one-on-one interviews were carried out for collecting data. In the paper-and-pencil assessments, thirty-eight pre-service secondary mathematics teachers involved. They were pursuing a bachelor degree at the Mathematics Education department of a private university in East Jakarta, Indonesia. Their mean age was 20.8 years, ranging from 20 to 23 years. Nine of the participants were male, and 29 were female. They consisted of twenty second-semester, ten fourth-semester, two sixth-semester, and six eighth-semester students. All of them have completed the course of differential calculus in which the concept of absolute value is explained in detail. In that university, the course is available in semester one.

To gain a better insight into participants' written responses in the test, we drew upon individual in-depth interviews. Interviews were conducted with nine participants involved in the test. The process of selection of the participants was as follows. Firstly, based on their achieved score in the test, participants were categorized into low, middle, and high-achieving groups. Then, five participants were selected from middle (two participants) and low achieving participants (three participants). The reason for recruiting them was that they were more likely to make mistake when coping the task. During the interview, an interviewer employed exploratory questions asking participants to describe their thinking and strategy by providing verbal answers and written solutions if necessary. For item asked, the interviewer took into account the following four-step protocol: (1) ask how the participants deal with the problem; (2) ask a more detail question; (3) ask the reason why he/she uses selected strategy; and (4) ask from whom they learn the strategy. All the responses to the interview questions were audio recorded. Each interview lasted between 30–40 minutes.

Their performance in solving absolute value questions, with a focus on their understanding and mistake, were investigated by means of test administration. The instruments used in this study included paper-and-pencil assessments and in-depth interviews. In consultation with two experts in mathematics education and mathematics as well, a 32-item absolute value understanding test was capitalized on to investigate pre-service teachers' understanding and the mistake made.

Twelve items requested participants to provide a response on statements given. The statements assess participants' knowledge of absolute value concepts. Participants indicated their responses on each statement on four possible choices, "I don't know", "I disagree", "I doubt", and "I agree".

The other ten items asked participants to tackle problems associated with absolute value. Several of them are designed by the researcher and the rest are taken from a study conducted by Gagatsis & Panaoura (2014). As several researchers have suggested that students who encounter difficulty in the notion absolute value have problems with the meaning of absolute value, hence one item requested participants to write their definition of absolute value. Three items assessed the abilities of participants to deal with regular or typical problem of absolute value. The items asked them to determine the value of x in $|x+2|=2$, $|x|>2$, and $|x-3|<0$. Four items requested participants to address questions drawing upon their discursive reasoning rather than algorithmic processes such as determining the value of x in $-x$, $|x-4|<-2$, $|x+2|+|x-2|=0$, and $\left||x-2|-12\right|=-5$.

One item requested participants to make a representation of absolute value in terms of graph in Cartesian coordinate. Finally, one item asked participants to provide one instance representing the notion of absolute value in a real-world context. The participants worked on the test individually and they were provided 100 minutes to complete all questions given. Participants' written responses for each one of the given problems were assessed for identifying their conception and mistakes.

The data taken from the participants' written responses in the test and from the interviews were analyzed for the purpose of description. All interviews were transcribed. Moreover, the explanations provided for the problems were coded to categorize the facets of conception and mistakes. As the present study concatenate quantitative and qualitative data, it allowed both results to notify each other and confirmed triangulation of evidence so that it could increase the reliability, validity, and credibility of the data.

RESULTS AND DISCUSSION

Results

The findings of this study are based on data compelled from 38 pre-service secondary mathematics teachers, who provided information about their understanding of the notion of absolute value through completing a 32-item test and in-depth interviews. Details of this information are discussed with regard to their conception, mistakes, and difficulties.

The conception of absolute value

Students' conception of the meaning of absolute values and its related concepts such as the concept of zero, real numbers line, and intervals. To examine this, twelve statements which requested participants to respond it and one open-ended question were presented to them. Table 1presents the participants' responses to the statements.

Based on Table 1, participants seem to have an uncommon agreement with respect to the statement asking whether zero is a positive number. Only 18 participants out of 38 who disagree with the statement which is the correct response. It inevitably becomes fundament knowledge required for students to have a good grip of the definition of absolute value notion. Definition of absolute value of a real number is asked via three different items i.e. item 2, item 3, and item 4. Participants are relatively inconsistent in their responses to these items. Table 1 indicates that most participants considered that statement in item 2 as correct. As a matter of fact, it is an incorrect statement. Subsequently, item 3 and item 4 are similar statements. However, participants respond to it differently. Participants revealed that when they were in high school level, they grasp the meaning of absolute value as a positive number. In addition, the notion that zero is not a positive number is not articulated properly by their instructors.

Item 5 and 7 examine participants' recognition of several absolute value properties. Most of them didn't seem to have puzzlement with respect to these items as they are able to provide intended responses. Besides, the rest items, i.e. item 6, 8, 9, 10, 11, and 12 challenge participants' comprehension in connecting the concept of absolute value to real number lines. Inconsistencies among participants' responses are identified. Most participants tend to have an inaccurate understanding of the representation of absolute value in the real number line. Based on the interview conducted, several of them stated that they are not informed by their instructor concerning the connection of absolute value and the real number line.

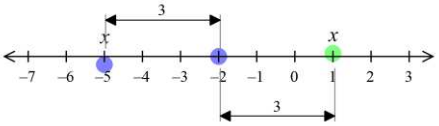
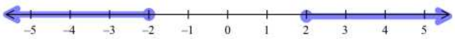
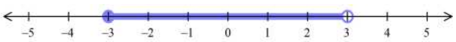
In the subsequent item, we also had participants describe their own arguments concerning the meaning of absolute value of a real number. Table 2 summarizes their responses. It seems that inconsistency exists among participants' responses. Nevertheless, the typical argument given by most participants is that the absolute value of a real number is always positive. In addition, it seems that participants have puzzlement in distinguishing positive and non-negative numbers. Besides, there are only few who are able to provide the meaning of absolute value of a real number properly by referring to the formal definition.

Participants' Mistakes

In this section, participants' mistake when dealing with the rest items of the test. Based on participants' written responses, there are four observed mistakes identified which are explicated in detail in the following subsections.

*Removing absolute value bars.*The most common misconception identified is that participants attempted to solve the problems by removing absolute bars. Even though majority of students were able to deal with the regular questions, several participants showed their inability to solve it. Omitting or ignoring this absolute value bars might be influenced by their conception of which absolute value is always positive. They omitted the absolute value bars and then treated it as a single linear equation. In interviews, respondents who provided such answer stated that $|a|$ equals to a. It is more salient when they encountered advance problems, i.e. item 4, item 5, and item 6.

Table 1. Participants Given Responses on Each Statements

No	Statements	DK	AG	DO	DA	NA
1	Zero is a positive number.	0	17	2	18	1
2	The absolute value of a real number is always positive.	0	36	1	0	1
3	The absolute value of a real number is non negative number.	1	33	2	1	1
4	The absolute value of a real number is positive real number and zero.	0	17	5	14	2
5	$\|a\| = \|-a\|$	0	30	1	6	1
6	$\|a-b\|$ could be interpreted as a distance between point a and point b at real number line.	9	16	5	5	3
7	$\|x-4\| = 9$ could be written as $x-4=9$ or $x-4=-9$	0	27	1	9	1
8	$\|x+2\| = 3$ could be represented in real number line as follow 	1	16	10	10	1
9	$\|x-2\| = 4$ refers to the condition in which the distance between x and point -2 at real number line is 4.	1	21	7	6	3
10	$\|x\|$ refers to the condition in which the distance between x and zero at real number line is zero	1	24	6	4	3
11	$\|x\| \geq 2$ could be represented in real number line as follow 	1	29	1	5	2
12	$\|x\| < 3$ could be represented in real number line as follow 	0	20	2	14	2

Note: DK = I don't know, AG = I agree, DO= I doubt, DA: I disagree, NA: No Answer

Table 2. Participants Perceived Meaning of the Absolute Value of a Real Number

No	Participants perceived meanings	Frequency
1	Distance on real number line	5
2	Non-negative number	1
3	Using absolute sign	1
4	Positive number	16
5	Positive number and zero	3
6	Positive number or non-negative number	2
7	The number without differing positive or negative	5
8	Formal definition	3
9	No responses given	2

Focusing heavily on rules or strategies. Albeit they argue that the absolute value of a real numbers is always positive, it doesn't satisfy with their response to several items, i.e. item 4, item 6, and item 8. For item 4, several participants tried to solve it by taking square of both sides. Taking square of both sides is common strategies to solving absolute value equation. For absolute value inequalities, most participants are likely to take common strategies for granted. The common strategies used are if $|x| > a$ then $x < -a$ or $x > a$ and if $|x| < a$ then $-a < x < a$. However, the participants tend to focus heavily on those strategies in lieu of expression on the right side. Therefore, in coming to grips with the problems such $|x-3| < 0$ and $|x-4| < -2$, the participants took those strategies into granted and the result obtained didn't make sense. In the interview, several participants stated that they did not focus on the right side, thus they executed the rule to solve the problem even though they do not reach the intended answers.

*Converting the absolute value bars to parentheses.*In an attempt to solve presented absolute value questions, several participants changed the absolute value bars into parentheses or brackets. Parentheses or brackets are definitely not the same as absolute value bars in which distinct rules apply for operating them. This confusion might be caused by their incomplete comprehension of procedures to solve absolute value equation. Converting the absolute value bars to parentheses could be done in the wake of implementing the formal definition. Focusing only on a certain condition (positive or zero) and ignoring another (negative) might be the reason why participants treat the absolute value bars as brackets. One participant claimed that when dealing with absolute value questions, he always changes the absolute value bars to parentheses, and his teachers do not provide correction of his work.

Taking out the number inside absolute value bars. Another interesting fact found in participants written responses is that there are some participants took number inside out. This strategy is carried out consistently across item questions given. For example in the case of $|x+2|$, the participants took the number two out and then equated such expression as $|x| + 2$. They might consider that the absolute value bars solely influence the variables such as x instead of numbers inside. In addition, the participant carried out similar strategy when dealing with $|x-4|$ in which they equated it as $|x| + 4$. Taking the numbers inside out and converting it into positive numbers might be performed and as a consequence of her conception held concerning the meaning of absolute value as positive numbers. Also, her glance at the word "numbers" in the definition might lead her to make such mistakes.

Poor performance in algebraic manipulation. In most cases, it seems that one of the critical factors for getting to grips with absolute value task successfully is the ability to manipulate algebraic expression. A lack of proficiency in algebraic manipulation might lead to incorrect solution albeit with a strong understanding of absolute value concept. Therefore, the combination of both proficiencies is necessary. In this study, several participants showed their inability to manipulate algebraic expression. It is obviously seen when they tried to solve the task. For instance, the item of $|x+2| = 2$ was tried to be solved by several participants with taking the square of only on side of the equation. They tend to focus exclusively on expression inside absolute value bars as they might consider that their first strategy was removing the absolute value bars. As a consequence, they disregard to take the square of another side of the equation.

*A lack proficiency in drawing absolute value function.*At the task, one item requested participants to make the graph of $y = |x-1|$. Majority of participants could make the graph of $y = |x-1|$ properly. However, several of them made mistakes. The following are typical mistakes made by participants.

There is a participant who considers

solely positive value of x and disregards the negative value of x. Therefore, the graph drawn is only on the top right of the Cartesian coordinate system. The reason could be due to his conception of the meaning of absolute value which is always positive. This conception leads him to take positive numbers per se into account to construct the graph.

To draw the graph of $y = |x - 1|$, at the beginning several participants took the square of the expression only on the right side to omit the absolute value bars. Therefore, quadratics equation was obtained from this strategy performed. Subsequently, this incorrect strategy and algebraic manipulation lead them to construct the graph of quadratics function. The participants might hold that the most important effort when working with absolute value task is omitting absolute value bars by taking the square.

Several participants attempted to use formal definition of absolute value as their effort to omit the absolute value bars. They defined $y = |x - 1|$ as $x = y + 1$ and $x = 1 - y$. Therefore, this incorrect use of formal definition leads them to construct two different lines on the Cartesian coordinate.

DISCUSSION

The purpose of this study was to explore pre-service secondary mathematics teachers' understanding of absolute value concept by investigating their written responses on absolute value understanding test. The tasks used for this study consisted of items requested participants to deal with the meaning of absolute value and its related concepts, the visual representation of absolute value, and various questions of absolute value. The results show that participants have considerable puzzlements in getting to grips with the task. Several mistakes that students make when dealing with absolute value task have been revealed and similar to those found in the prior studies. In addition, several possible sources of mistakes have been identified.

For real numbers, the absolute value is either zero or positive, that is to say, it is nonnegative. However, based on participants' responses, they tend to perceive that nonnegative means positive. The main reason for that misconception lay in their ignorance related to zero. They claimed that zero is a positive number. In fact, zero is a not positive number. Students' ignorance of it has been disclosed by Ponce (2008) and Taylor & Mittag (2015). In most classrooms, mathematics teachers are likely to disregard this knowledge. It is inevitably important to introduce at the beginning of mathematics lesson as it might contribute to better understanding of absolute value concept.

Subsequently, we get a better insight into participants' conception of the absolute value of a real number. A discrepancy exists among participants' responses on it. The meaning of absolute value as positive number dominates participants' responses. Only a few of them are able to define absolute value by providing in the formal definition. A possible interpretation for this finding is that in Indonesia, it is prevalent that the absolute value of a real number tends to be perceived as a number taken positively by most mathematics teachers as well as students. In addition, their ignorance of that zero is not positive numbers might contribute to this misconception. Literature indicated the discrepancy in the concept of absolute value held by students in various countries. For instances, for Turkish students, the concept of absolute value as a distance from zero was the most considerably used definition (Elia et al., 2016). In addition, a study conducted by Gagatsis & Panaoura (2014) revealed that majority of Cyprus students held the conception of absolute value as numbers without a sign. The didactic process seems to have a significant influence on students' conception held concerning the absolute value. Mathematics teachers' inaccuracy and incomplete understanding of absolute value might contribute to students' misconception of absolute value as well as their mistakes made when dealing with absolute value tasks.

The concept of absolute value could be represented externally and visually using real number line (Ellis & Bryson, 2011; Wagster, 1986) or graphics (Horak, 1994; Wade, 2012). Using these external representations, students could learn the concept meaningfully. Therefore, it might be easier for students to grasp the concept of absolute value by visualizing it. From participants' responses, it seems that there is a disconnection between students' understanding of absolute value concepts and their external representations. The reason might be the lack of students' sense-making of external representations as well as their flexibility. The finding of this study showed that most participants were able to make a graphical representation of the absolute value function of $y = |x - 1|$, yet they get confused when dealing with symbolic representation or describing the meaning of absolute value. In other words, ability in drawing graphical representation of absolute value function might not lead students to have accurate meaning of absolute value concept. Therefore, it is important for mathematics teachers to have their students make sense of various external representations of absolute value by integrating it and its formal definition or meaning. Aziz & Kurniasih (2019) also emphasized the importance of flexibility of external representation in an effort to grasp the mathematical concept.

When attempting to solve the absolute value task, typical mistakes made by participants are removal of absolute value bars, focus on rules of strategies, conversion of the absolute value bars to parentheses, taking out the number inside absolute value bars, poor performance in algebraic manipulation, and a lack proficiency in drawing absolute value function. Several mistakes found in this study are compatible with mistakes found in previous studies. Previous studies have revealed students' misconceptions and mistakes when dealing with absolute value tasks. Almog & Ilany (2012) found students' mistakes when solving absolute value tasks, such as logical mistakes, removing the absolute value symbol, integer only, over-generalization, failure to distinguish, and other mistakes. Gagatsis & Panaoura (2014) revealed that typical mistakes made by students are their tendency to get rid of the absolute value bars. Exclusion of absolute value bars seemed to be a common mistake made by most students in various countries. They are impelled by such mechanical application without taking the meaning into consideration as a product of didactic contract (Gagatsis & Panaoura, 2014). In Indonesian mathematics classrooms, the most prevalent didactic contract is that an absolute value is perceived as a number taken positively. This conception developed and embedded in students' cognition. It might be basic misconception which leads to various mistakes made in solving the absolute vale task such as the exclusion of absolute value bars, conversion of the absolute value bars to parentheses, focus on rules or strategies, taking out the number inside absolute value bars, and drawing absolute value function.

Conversion of the absolute value bars to parentheses stem from participants' over-generalization. Aziz, Pramudiani, & Purnomo (2018) stated that over-generalization made by students as a result of direct interpretation of certain idea without considering other mathematical ideas. In this case, students interpret absolute value bars as parentheses directly devoid of regarding the definition of absolute value. Besides, ability in manipulating algebraic manipulation also is a necessity for successful problem-solving. In this study, understanding the concept of absolute value is not likely to be in line with successful problem solving, and vice versa. Therefore, before presenting the topics of absolute value, mathematics teachers should ensure that students master algebraic manipulation. The combination of a comprehensive understanding of the absolute value and strong ability in algebraic manipulation contributes to this success.

CONCLUSION

It was the main purpose of the study to draw attention to investigate pre-service secondary mathematics teachers understanding of the absolute value and their mistake when coming to grips with the task. From the study that has been carried out, it is possible to conclude that most participants seemed to hold a conception that entailed beliefs that absolute value is always a positive number. These findings give ample evidence that instructors are necessary to revisit notions about zero number prior to a presentation regarding absolute value. In addition, it appears that most participants seem to: (a) focus on removing absolute value bars; (b) focus heavily on rules or strategies; (c) converting the absolute value bars to parentheses; (d) taking out the number inside absolute value bars; (d) have poor performance in algebraic manipulation; and (e) have lack of proficiency in drawing absolute value function.

The insights uncovered in this study suggest at least several implications for the further development of pre-service secondary mathematics teachers' absolute value understanding. The findings of this study also have implications for teaching the concept of absolute value especially as teachers or instructors look for ways to improve students' comprehension. At least, teacher educators need to ensure that pre-service teachers: (1) understand the concept of the zero number; (2) present definition of absolute value precisely; (3) perform algebraic manipulation correctly; (4) convert one representation to other representation of absolute value function. Such implications could be useful to consider when designing absolute value activities in a teacher preparation program.

The findings and conclusions of this study should be regarded in the perspective of the limitation that the data resulted from investigating pre-service secondary mathematics teachers in one private university. It is also acknowledged that this is a small-scale investigation that included 38 pre-service secondary mathematics teachers and a 32-item instrument.

Further study of the issue is still required. Even though most pre-service teachers lack comprehension of absolute value, what is still not well known is how teacher educators are addressing the absolute value topic in the teaching process. Pre-service teachers' puzzlement on the topics might be caused by methods that teacher educators implement. Therefore, observation of teacher educators' teaching process on absolute value may deliver appealing data that might be a benefit for other teacher educators.

In addition, observation of high school mathematics teachers could be carried out to gain more comprehensive information regarding this topic. Similarities and differences in the methods for teaching absolute value in university and high school level could be investigated.

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