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Emphasis on Mathematical Modeling: The Problems of Contour Values in Calculating the Deflection of a Beam

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***Abstract*—** *Mathematical Modeling has played a fundamental role in the process of teaching and learning mathematics at the various levels of education. The great challenge of today's education is to create means to minimize the distance between the relations of mathematical theory and practical models of everyday life. This work aimed to treat the study of Ordinary Differential Equations through a very usual model of civil construction. Often, students of Mathematics Degree develop skills to understand the abstractions of the vast theory associated with the areas of mathematics, but without being able to exemplify and / or correlate with everyday models.*
Keywords: Mathematics Teaching and Learning. Contour problems. Deflection and Beams.

I. INTRODUCTION

In the present day a great amount of publications in the area of Mathematical Modeling has been a prominent factor in the Brazilian scientific production in Mathematical Education. The evolution of knowledge and new technologies have transformed the teaching and learning process of mathematics into modern and motivating educational practices, and therefore mathematical modeling has played an extremely important role in this transformation.[1] Thinking Mathematical Modeling as one of the possible paths of a new way of establishing, in school spaces, the insertion of the way of thinking the relations of mathematical knowledge and the most participatory and democratic society."

Contemporary education, more precisely, the way in which educators construct their knowledge for their apprentices, there is a gap between the traditionalist way of teaching and the evolution of the modern world. The scientific and technological transformations that have occurred in the last decades have ignited a warning signal for a large number of theoretical educators and resistant to change in the way of teaching mathematics.

For [2], knowing how to differentiate what is a problem from an exercise is one of the most common and most difficult practices for the teacher. A real problem must be a real challenge in which students, by means of sequences of actions, will seek to obtain the results. In this way, Mathematical Modeling has a crucial role of minimizing the distances between contents without immediate applications and the usual models of daily life.

Over time, Modeling has been developed under various contours in the classroom and these different forms of conception reflect the experiences lived by its followers. Thus, the experience and level of teaching in which the teacher works, whether in Higher Education or in Basic Education, offer different characteristics, perceptions and directions to work with Modeling. It is important to make it clear that we understand that there is not one conception of Modeling that is truer than another, or one that is more correct than the other, but what is here is the need for clarity about the conceptions of "Man", "Education "," Teaching and Learning "and the object of study, which implies in epistemological questions that guide our educational practice within the different levels and modalities of teaching [3].

In view of the above and the concerns about the best way of conducting the concepts of Equations Ordinary Differences, through Mathematical Modeling, for students of the degree course in Mathematics, came the idea of using a civil construction application.

Many natural phenomena, from physics to biology, can be described by means of differential equations. For example, suppose two working men are driving a large piece of board over their shoulders, each at one end of the board. It is easy to see that the middle of the board will curve downwards. This effect, we say that the board has suffered a deflection.

Solving a differential equation means finding an adequate family of curves. In engineering, there are a lot of problems in which we resort to these equations to find a solution. One of these problems is to determine the static deflection of an elastic beam caused by its weight or by

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an external load. These beams, under load, undergo a deformation, flexing. The loads that produce this deformation can be of two types: they can be concentrated in one more points of the beam, or they can be evenly distributed by the beam. This article aims to find solutions of the equations of the curves in which the beams deform under a load uniformly distributed by the beam

II. THE DIFFERENTIAL EQUATION OF THE DEFLECTION CURVE

[4] [5] [6] [7] Ordinary Differential Equation of order n is every equation, $F(x, y, \frac{dY}{dx}, \frac{d^2Y}{dx^2}, \ldots, \frac{d^nY}{dx^n}) = 0$, which has an unknown function Y and its derivatives. The transverse deflection of a beam $Y(x)$, at the point x, satisfies a fourth order no homogeneous linear differential equation of the type: $\frac{d^4Y}{dx^4} = \frac{W(x)}{EI}$. Where, $W(x)$ it is a vertical load per unit length, which acts transversely to the beam. The constant EI is called beam stiffness - E it is the modulus of elasticity of Yang of the material that is made to the beam and I is the moment of inertia of a cross section of the beam. The contour conditions (Table I) associated with the equation depend on the manner in which the beam is supported.

Table I: Types of beams under contour conditions

Types	Conditions
Crimped Beam	$Y = Y' = 0$
Rotary Beam	$Y = Y'' = 0$
Fixed End Beam	$Y''' = Y'' = 0$

Source: Elaboration of the author, 2018.

Theorem: The solution of the fourth order non-homogeneous linear differential equation $\frac{d^4Y}{dx^4} = \frac{W(x)}{EI}$ is given by

$$Y(x) = C_1 + C_2x + C_3x^2 + C_4x^3 + \frac{W_0}{24EI}x^4.$$

Where, $W(x) = W_0$ it is a constant charge evenly distributed along its length.

Proof: Let $\frac{d^4Y}{dx^4} = \frac{W_0}{EI}$ (1) be the equation we wish to determine the general solution.

Hence, integrating successively in (1), we have:

$$Y'''(x) = \frac{d^3Y}{dx^3} = 6C_4 + \frac{W_0}{EI}x \quad (2),$$

$$Y''(x) = \frac{d^2Y}{dx^2} = 2C_3 + 6C_4x + \frac{W_0}{2EI}x^2 \quad (3),$$

$$Y'(x) = \frac{dY}{dx} = C_2 + 2C_3x + 3C_4x^2 + \frac{W_0}{6EI}x^3 \quad (4).$$

Therefore,

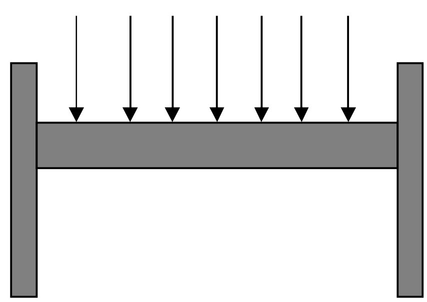
$$Y = C_1 + C_2x + C_3x^2 + C_4x^3 + \frac{W_0}{24EI}x^4 \quad (5). \bullet$$

The arbitrary constants C_1, C_2, C_3 y C_4 are determined from the initial boundary conditions. That is, for each type of beam used, the constants have different values.

III. PROBLEMS OF CONTOUR VALUES AND MAXIMUM DEFLECTION OF A BEAM

A Contour Value Problem is characterized by a differential equation and extra conditions supplied at more than one point. Suppose a beam of length L is homogeneous and has a uniform cross-section along its length. Also, suppose a load is evenly distributed over the beam over its entire length. Under the presented conditions, we will show the solution of the contour problem for the calculation of the deflection of a beam, in the following cases:

Case 1: Beam Set at both ends: $x = 0$ and $x = L$.



Contour Conditions: $\begin{cases} Y(0) = Y(L) = 0 \\ Y'(0) = Y'(L) = 0 \end{cases}$

Hence, since $Y(0) = 0$ and $Y'(0) = 0$, by (5) and (4), respectively, we have $C_1 = 0$ and $C_2 = 0$.

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Under the conditions $Y(L) = 0$ and $Y'(L) = 0$, respectively, they give us:

$$C_3 L^2 + C_4 L^3 + \frac{W_0}{24EI} L^4 = 0 \quad \text{and}$$

$$2C_3 L + 3C_4 L^2 + \frac{W_0}{6EI} L^3 = 0.$$

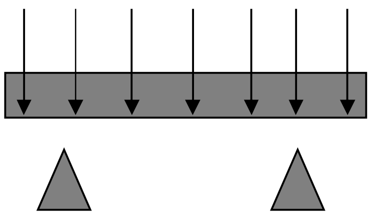
Solving the system, we will have:

$$C_3 = \frac{W_0 L^2}{24EI} \quad \text{and} \quad C_4 = -\frac{W_0 L}{12EI}$$

Therefore, the deflection, at the point *x*, is:

$$Y(x) = \frac{W_0 L^2}{24EI} x^2 - \frac{W_0 L}{12EI} x^3 + \frac{W_0}{24EI} x^4$$

Case 2: Beam at both ends: $x = 0$ *and* $x = L$.



Contour Conditions: $\begin{cases} Y(0) = Y(L) = 0 \\ Y''(0) = Y''(L) = 0 \end{cases}$

Hence, since $Y(0) = 0$ and $Y''(0) = 0$, by (5) and (3), respectively, we have $C_1 = 0$ and $C_3 = 0$.
Under the conditions $Y(L) = 0$ and $Y''(L) = 0$, respectively, they give us:

$$C_2 L + C_4 L^3 + \frac{W_0}{24EI} L^4 = 0 \quad \text{and}$$

$$6C_4 L + \frac{W_0}{2EI} L^2 = 0.$$

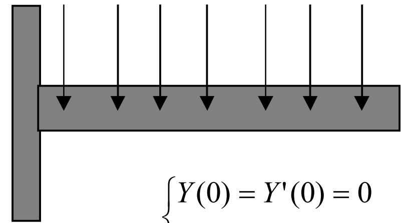
Solving the system, we will have:

$$C_2 = \frac{W_0 L^3}{24EI} \quad \text{and} \quad C_4 = -\frac{W_0 L}{12EI}$$

Therefore, the deflection, at the point *x*, is:

$$Y(x) = \frac{W_0 L^3}{24EI} x - \frac{W_0 L}{12EI} x^3 + \frac{W_0}{24EI} x^4$$

Case 3: Beam Set in $x = 0$ and free end in $x = L$.



Contour Conditions: $\begin{cases} Y(0) = Y'(0) = 0 \\ Y''(L) = Y'''(L) = 0 \end{cases}$

Hence, since $Y(0) = 0$ and $Y'(0) = 0$, by (5) and (4), respectively, we have $C_1 = 0$ and $C_2 = 0$.
Under the conditions $Y''(L) = 0$ and $Y'''(L) = 0$, respectively, they give us:

$$2C_3 + 6C_4 L + \frac{W_0}{2EI} L^2 = 0 \quad \text{and}$$

$$6C_4 + \frac{W_0 L}{EI} = 0.$$

Solving the system, we will have:

$$C_3 = \frac{W_0 L^2}{4EI} \quad \text{and} \quad C_4 = -\frac{W_0 L}{6EI}$$

Therefore, the deflection, at the point *x*, is:

$$Y(x) = \frac{W_0 L^2}{4EI} x^2 - \frac{W_0 L}{6EI} x^3 + \frac{W_0}{24EI} x^4$$

When a load W_0 is uniformly distributed over a beam of length L, maximum deflection occurs in the middle of the beam. This deflection represents the maximum limit at which the beam can flex before breaking. As in our work, we take length beams, their maximum deflection occurs at point $x = \frac{L}{2}$. The table II, shows the maximum deflection reached by beam analyzed in the previous section.

Table II: Maximum Deflection of a Beam

Types of Beam	Maximum Deflection
Case 1	$Y_{max} = \frac{W_0 L^4}{384EI}$
Case 2	$Y_{max} = \frac{5W_0 L^4}{384EI}$
Case 3	$Y_{max} = \frac{17W_0 L^4}{384EI}$

Source: Elaboration of the author, 2018.

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IV. PROPOSAL OF THE USE OF MATHEMATICAL MODELING IN CLASSROOM

The major concern of most mathematics teachers in higher education is to try to make the teaching and learning process of mathematics efficient. The use of Mathematical Modeling - Deflection Calculation of a beam - in the Ordinary Differential Equations is a good example for a motivational educational practice, because it leads the student to realize that mathematical theory has well defined, practical, everyday results. It is necessary to observe that this passage from the theoretical abstract model to the concrete practical model must happen following the demands and rigor in the treatment of mathematical concepts.

The development of the modeling activity involves procedures such as the search for information, the identification and selection of variables, the elaboration of hypotheses, the simplification, the obtaining of a mathematical model, the resolution of the problem by means of adequate procedures and the analysis of the identifying its acceptability (or not) [8].

The purpose of this article was to present a classroom practice that would lead the student to engage in mathematical relationships with the modern world. According to [9] we can say that students are able to produce three types of discussion in the Modeling environment: **mathematics** - refer to mathematical ideas, concepts, and algorithms; **technical** - refer to the representation of the situation - problem in mathematical terms; **reflective** - refer to the relationship between the criteria used in the construction of a mathematical model and its results.

Thus, our pedagogical proposal, for undergraduate mathematics students, for a good understanding of the Equations Ordinary Differences is the use of Mathematical Modeling, following rigorously pre-established steps and tasks.

Step I: Define Ordinary Differential Equation.

Step II: Show the differential equation of the transverse deflection of a beam.

Step III: Show the types of beams under contour conditions.

Step IV: Find the solution of the differential equation of the transverse deflection of a beam,

Step V: Calculate the deflection of a beam, in the cases: Beam, Fixed Beam and Fixed End Beam.

Step VI: Classroom Activity: Determine the Maximum Deflection of a Beam, in each case: Beamed Beam and Fixed End Beam. Present the one that suffers the greatest deflection.

It should be noted that the construction of knowledge must follow very well defined steps and tasks that facilitate the teaching and learning process, respectively, of the teachers and students involved. This pedagogical proposal follows goals, without loss of generality, for the construction of a real, comprehensible and modern mathematical model.

V. CONCLUSION

This article was developed from a proposal to use Mathematical Modeling in Ordinary Differential Equations, using the calculation of Deflection of a Beam. Another fundamental point, highlighted in this article, refers to the use of Mathematical Modeling in higher education, often of high lack of consideration by traditional teachers and resistant to change.

This is a special time for college students to realize the many ways mathematics can be used on a day-to-day basis. The current profile of undergraduate courses in Mathematics in Brazil allows to create mechanisms in pedagogical practice to reduce the gap between what is presented as a theory and what is used as a practice. It can be affirmed that there are several ways to present Mathematical Modeling, at whatever level of education, it is enough only that the stages and tasks of the defined pedagogical proposal be completed. In view of the above, it is expected that this work may stimulate further research in mathematical modeling in order to minimize the lags between mathematical theory and its practice.

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