https://dx.doi.org/10.22161/ijaems.4.10.4 ISSN: 2454-1311

Modelling the Increasing Differential Effects of the First Inter-Competition Coefficient on the Biodiversity Value; Competition between two Phytoplankton Species

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Abstract— *One of the intrinsic factors that affects the growth of two phytoplankton species is called the inter-competition coefficient. When this parameter value is decreased, the first phytoplankton specie benefit from biodiversity gain whereas the second phytoplankton specie is vulnerable to biodiversity loss. In contrast, when the same parameter value is increased from the value of 0.0525 to 0.099 the first phytoplankton specie dominantly suffers from a biodiversity loss whereas the second phytoplankton specie benefits from a biodiversity gain. The novel results that we have obtained have not been seen elsewhere but compliments our current contribution to knowledge in this challenging interdisciplinary research; these full results are presented and discussed quantitatively.*

Keywords*— *Differential effect, inter-competition coefficient, phytoplankton specie, biodiversity richness, continuous differential equation.

I. INTRODUCTION

Heterogeneity of species is a very important factor in biodiversity richness. Phytoplankton play a very crucial role in ocean ecology [Saha and Bandyopadhyay 2009].

While some species of phytoplankton are known to produce toxins which can contaminate seafood, others are high producers of biomass which in high concentrations can cause mortalities of marine life. However, the toxin producing phytoplankton are known to play very significant roles in the growth of zooplankton [Bandyopadhyay et al 2008]. Atsu & Ekaka-a (2017) have shown that as fractional order dimension increases from 0.1 to0.75, there is a reduction in specie depletion. The implication is that an increased fractional order dimension results in a biodiversity gain.

II. MATERIALS AND METHODS

We have considered the following Lotka–Volterra model equations of competition indexed by a system of continuous non-linear first order ordinary differential equations:

$$\frac{dN_1(t)}{dt}=N_1(t)[\alpha_1 - \beta_1 N_1(t) - \gamma_1 N_2(t)] \quad (1)$$

$$\frac{dN_2(t)}{dt}=N_2(t)[\alpha_2 - \beta_2 N_2(t) - \gamma_2 N_1(t)] \quad (2)$$

Here, the initial conditions are defined by $N_1(0)=N_{10} \geq 0$ and $N_2(0)=N_{20} \geq 0$, whereas $N_1(t)$ and $N_2(t)$ specify the densities of the two phytoplankton species (measured as the number of cells per litter). For the purpose of this formulation, $\alpha_1$and α_2 specify the cell proliferation rate per day;β_1 and β_2 specify the rate of intra-specific competition terms for the first and second species;γ_1 and γ_2 specify the rate of inter-specific competition. The units of α_1,α_2,β_1, β_2,$\gamma_1$and γ_2 are per day per cell and day is the unit of time.

Following the parameter values as proposed by Bandyopadhyaya, et al ,2008, where $\alpha_1 = 2$, $\alpha_2 = 1$, ,$\beta_1 = 0.07$, $\beta_2 = 0.08$, $\gamma_1 = 0.05$, $\gamma_2 = 0.015$.

The method is hereby stated step by step as follows:

Step 1: Consider a scenario where N_1(old) is the predicted biomass [N_1(old) is the population density of the first phytoplankton species otherwise called the biomass of the first phytoplankton species when all other parameter values are fixed at time t.

Step 2: Replace N_1(old) with N_1(new) due to a variation of the inter-competition coefficient γ_1.

https://dx.doi.org/10.22161/ijaems.4.10.4 ISSN: 2454-1311

Step 3: If N_1(new) is strictly less than N_1(old), it indicates that the variation of $\gamma_1$has predicted a depletion which mimics biodiversity loss. In this scenario, the appropriate mathematically formula for the quantification of biodiversity loss is defined as follows:

$$BL(\%)= 100[\frac{N_1(old) - N_1(new)}{N_1(old)}]$$

Step 4: If N_1(new) is strictly greater than N_1(old), due to the variation of γ_1,then a biodiversity gain has occurred which can be similarly defined as follows:

$$BG(\%)=100\ [1- \frac{N_1(new)}{N_1(old)}]$$

III. RESULTS

On the application of the above mentioned methods, we have obtained the following empirical results that we have not seen elsewhere.

Table.1: *Evaluating the extent of biodiversity for* $\gamma_1 = 0.049$ *with experimental time of 10 years using ODE 45*

N_1	N_{1m}	BG(%)	N_2	N_{2m}	BL(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	10.7674	0.80	10.6094	10.6048	0.04
17.3033	17.4700	0.96	10.1236	10.1074	0.16
20.5151	20.6862	0.83	9.4452	9.4213	0.25
21.7063	21.8641	0.73	8.9593	8.9328	0.30
22.1823	22.3306	0.67	8.6637	8.6368	0.31
22.4060	22.5491	0.64	8.4908	8.4641	0.31
22.5238	22.6639	0.62	8.3900	8.3637	0.31
22.5896	22.7279	0.61	8.3312	8.3051	0.31
22.6273	22.7645	0.61	8.2967	8.2708	0.31

Table.2: Evaluating the extent of biodiversity for $\gamma_1 = 0.0525$ *with experimental time of 10 years using ODE 45 numerical*

N_1	N_{1m}	$BL(\%)$	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	10.4703	1.98	10.6094	10.6206	0.11
17.3033	16.8882	2.40	10.1236	10.1638	0.40
20.5151	20.0849	2.10	9.4452	9.5052	0.64
21.7063	21.3077	1.84	8.9593	9.0259	0.74
22.1823	21.8070	1.69	8.6637	8.7316	0.78
22.4060	22.0442	1.62	8.4908	8.5582	0.79
22.5238	22.1696	1.57	8.3900	8.4567	0.79
22.5896	22.2399	1.55	8.3312	8.3971	0.79
22.6273	22.2803	1.53	8.2967	8.3621	0.79

Table.3: *Evaluating the extent of biodiversity for* $\gamma_1 = 0.055$ *with experimental time of 10 years using ODE 45*

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	10.2618	3.93	10.6094	10.6317	0.21
17.3033	16.4754	4.79	10.1236	10.2036	0.79
20.5151	19.6507	4.21	9.4452	9.5653	1.27
21.7063	20.9031	3.70	8.9593	9.0933	1.50
22.1823	21.4256	3.41	8.6637	8.8004	1.58
22.4060	21.6763	3.26	8.4908	8.6266	1.60
22.5238	21.8095	3.17	8.3900	8.5244	1.60
22.5896	21.8844	3.12	8.3312	8.4642	1.60
22.6273	21.9277	3.09	8.2967	8.4286	1.59

Table.4: Evaluating the extent of biodiversity for $\gamma_1 = 0.0575$ *with experimental time of 10 years using ODE 45*

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	10.0566	5.85	10.6094	10.6427	0.31
17.3033	16.0652	7.16	10.1236	10.2432	1.18
20.5151	19.2130	6.35	9.4452	9.6254	1.91
21.7063	20.4927	5.59	8.9593	9.1613	2.26
22.1823	21.0378	5.16	8.6637	8.8701	2.38
22.4060	21.3021	4.93	8.4908	8.6961	2.41
22.5238	21.4434	4.80	8.3900	8.5932	2.42
22.5896	21.5231	4.72	8.3312	8.5323	2.41
22.6273	21.5693	4.68	8.2967	8.4962	2.40

Table.5: Evaluating the extent of biodiversity for $\gamma_1 = 0.06$ *with experimental time of 10 years using ODE 45*

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	9.8545	7.75	10.6094	10.6536	0.42
17.3033	15.6578	9.51	10.1236	10.2825	1.57
20.5151	18.7722	8.50	9.4452	9.6857	2.55
21.7063	20.0764	7.51	8.9593	9.2299	3.02
22.1823	20.6436	6.94	8.6637	8.9408	3.20
22.4060	20.9216	6.63	8.4908	8.7666	3.25
22.5238	21.0711	6.45	8.3900	8.6631	3.25
22.5896	21.1558	6.35	8.3312	8.6015	3.25
22.6273	21.2050	6.29	8.2967	8.5649	3.23

Table.6: Evaluating the extent of biodiversity for $\gamma_1 = 0.095$ *with experimental time of 10 years using ODE 45*

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	7.3483	31.21	10.6094	10.7915	1.72

https://dx.doi.org/10.22161/ijaems.4.10.4 ISSN: 2454-1311

17.3033	10.3835	39.99	10.1236	10.7873	6.56
20.5151	12.4577	39.28	9.4452	10.5173	11.35
21.7063	13.7002	36.88	8.9593	10.2380	14.27
22.1823	14.4254	34.97	8.6637	10.0218	15.68
22.4060	14.8564	33.70	8.4908	9.8706	16.25
22.5238	15.1193	32.87	8.3900	9.7695	16.44
22.5896	15.2828	32.35	8.3312	9.7030	16.47
22.6273	15.3860	32.00	8.2967	9.6598	16.43

Table.7: Evaluating the extent of biodiversity for $\gamma_1 = 0.0975$ with experimental time of 10 years using ODE 45

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	7.1914	32.68	10.6094	10.8004	1.80
17.3033	10.0462	41.94	10.1236	10.8196	6.88
20.5151	12.0143	41.44	9.4452	10.5739	11.95
21.7063	13.2174	39.11	8.9593	10.3112	15.09
22.1823	13.9344	37.18	8.6637	10.1041	16.63
22.4060	14.3678	35.88	8.4908	9.9574	17.27
22.5238	14.6356	35.02	8.3900	9.8580	17.50
22.5896	14.8041	34.47	8.3312	9.7920	17.54
22.6273	14.9113	34.10	8.2967	9.7487	17.50

Table.8: Evaluating the extent of biodiversity for $\gamma_1 = 0.099$ with experimental time of 10 years using ODE 45

N_1	N_{1m}	BL(%)	N_2	N_{2m}	BG(%)
4.0000	4.0000	0	10.0000	10.0000	0
10.6819	7.0987	33.55	10.6094	10.8057	1.85
17.3033	9.8468	43.09	10.1236	10.8388	7.07
20.5151	11.7501	42.73	9.4452	10.6075	12.31
21.7063	12.9272	40.45	8.9593	10.3550	15.58
22.1823	13.6374	38.52	8.6637	10.1537	17.20
22.4060	14.0713	37.20	8.4908	10.0099	17.89
22.5238	14.3415	36.33	8.3900	9.9117	18.14
22.5896	14.5127	35.76	8.3312	9.8461	18.18
22.6273	14.6223	35.38	8.2967	9.8028	18.15

IV. DISCUSSION OF RESULTS.

With an inter-competition coefficient value of $\gamma_1 = 0.049$ and an experimental time of ten (10) years, the biodiversity gain percentage value has maintained a maximum value of 0.96 and biodiversity loss maximum percentage value of 0.31 from the sixth month. As γ_1 increases to 0.0525, the maximum biodiversity gain percentage value is 2.40occurs at the second month while the maximum biodiversity loss percentage value of 0.79 starts occurring from the seventh month. At an γ_1 value of 0.055, the maximum biodiversity gain percentage value is 4.79 and biodiversity loss percentage value is 1.60. When the γ_1 value is 0.0575, the maximum biodiversity gain percentage value is 7.16 while the maximum biodiversity loss percentage value is 2.42. An γ_1 value of 0.06 results in a maximum biodiversity gain percentage value of 9.51 at the second month while the maximum biodiversity loss percentage value is 3.25 occurring at the seventh and eighth months. When γ_1 is 0.095, the maximum biodiversity gain percentage value is 39.99 and maximum biodiversity loss percentage value is 16.47. At γ_1 = 0.0975, the maximum biodiversity gain value is 41.94 while maximum biodiversity loss value is 17.54 and when γ_1=0.099, maximum biodiversity gain percentage value is 43.09 and maximum biodiversity loss percentage value is maintained at 18.18. Predominantly a biodiversity gain is predicted at the second month while a biodiversity loss is predicted from the seventh month and higher.

V. CONCLUSION

The MATLAB ODE45 numerical scheme has been used to predict biodiversity gain and biodiversity loss resulting from inter-competition between two phytoplankton species at increasing inter-competition coefficient levels. A relatively higher inter-competition coefficient would result in a relatively lower biodiversity gain.

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