

# Search and optimization of the periodic gait of leg locomotion systems

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**Abstract**— *The task of reproducing artificial legged locomotion presents lots of challenges, most of them related to the system dynamics, which is non-linear, hybrid and under actuated. Recently the scope of legged locomotion changed from static stability to dynamical stability, in which the dynamics of the system plays an important role in the development of control algorithms that have as a main goal the construction of a stable limit cycle. This article projects a periodic control for trot without the air phase for a quadruped through the multiple shooting method with Hermite-Simpson direct collocation. The algorithm was first tested in a monopod where the control for hopping and forward hopping was searched, such that the optimization occurred after finding the periodic gait and for each case the stability of the control was determined through the Floquet Multipliers from Poincare's map. The quadruped system could not be wholly analyzed because the search for the periodic gait failed for the initial estimation, leaving this analysis for future researches.*

**Keywords**— *Legged locomotion, Monopod, Optimization, Periodic gait, Quadruped, Stability.*

## I. INTRODUCTION

The need to explore unhealthy and high risk environments for humans has brought the advent of autonomous robotics. Robots may be used to perform tasks that are too dangerous or difficult for humans to implement directly (e.g. nuclear waste cleanup) or may be used to automate repetitive tasks [1]. If there is one technological advancement that makes human living easier, robot would be the answer [2].

Robots that use the wheel as a means of locomotion have difficulties in dealing with rough terrain, crossing obstacles and moving on non-rigid or soft / penetrating terrain. In order to overcome these adversities, the alternative method of locomotion by legs is used.

Although many systems using leg locomotion have been proposed and constructed, none of them seem to be able

to reproduce the elegance, energy efficiency, and apparent ease with which land animals move through various types of terrain and the various gears that they use.

Seeking to imitate these inherent abilities of animals, this article studies the search and optimization of the periodic gait of a quadruped robot locomotion system by legs.

## II. METHODS

### 2.1 System Simulation

Due to the nonlinearity of the system and the complexity of the functions, the model developed does not have a trivial analytic solution. Therefore, the numerical integration was necessary to obtain the system solution, using the MATLAB "ode45" function.

In addition, the "ode45" function allows to define an auxiliary function that determines the occurrence of the pre-established events, interrupting the integration. The events used were: beginning and end of the contact of the feet with the ground. When one of the events occurs, the value at which the simulation stops is used to perform the discrete dynamics according to the established event and then the integration is resumed until the end of the March period or the next event.

### 2.2 Optimization

The goal of simulation optimization is minimizing the assets spent while amplifying the information acquired in a simulation experiment [3].

This work uses the multiple shooting direct optimization method. The multiple shooting [4] have been gaining much attention in the implementation of optimal control due to its success in many fields of robotics and trajectory optimization [5], [6] e [7].

The nonlinear problem (NLP) was solved using the "fmincon" function of MATLAB. As in [8], the "interior-point" method was chosen for the resolution of gait NLP. There is no defined methodology for determining the initial conditions of an optimization problem. However, it is considered good practice to start with initial conditions in which equality constraints are met and there is no

violation of inequality constraints. In this work there are only equality restrictions, so the search for the periodic gait occurred before the optimization process, using the "fsolve" function of MATLAB.

### 2.3 Monopod model

To test the elaborated algorithm, a monopod model was used (Figure 1), which consists of three bodies: the hip, the leg and the foot.

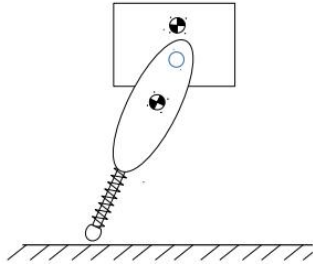


Fig. 1: Monopod scheme

The degrees of freedom that determine the complete dynamics of this system are in Table 1.

Table 1: Monopod generalized coordinates

| Variable                | Symbol         |
|-------------------------|----------------|
| Hip horizontal position | q <sub>1</sub> |
| Hip vertical position   | q <sub>2</sub> |
| Hip angular orientation | q <sub>3</sub> |
| Leg angular orientation | q <sub>4</sub> |
| Leg spring deformation  | q <sub>5</sub> |

The algorithm sought the solution for the monopod in two situations: jump without advance and jump with advance.

### 2.4 Quadruped model

For the quadrupedal model, the configuration of the prismatic joint presented in [9] was used, because it presents a greater correspondence with the dynamics of locomotion observed in animals. Although the leg of an animal has several segments, this configuration can capture the main dynamics involved in walking, simplifying the system [10]. The configuration of the model is shown in Figure 2.

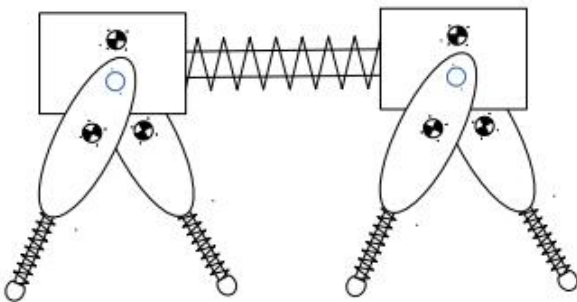


Fig. 2: Quadruped scheme

This model uses the degrees of freedom in Table 2.

Table 2: Quadruped generalized coordinates

| Variable                                  | Symbol          |
|---|-----------------|
| Front quarter horizontal position         | q <sub>1</sub>  |
| Front quarter vertical position           | q <sub>2</sub>  |
| Angled position of the forequarters       | q <sub>3</sub>  |
| Angle position of left front leg          | q <sub>4</sub>  |
| Angle position of right front leg         | q <sub>5</sub>  |
| Deformation of left front leg spring      | q <sub>6</sub>  |
| Spring deformation of the right front leg | q <sub>7</sub>  |
| Distension of the springs                 | q <sub>8</sub>  |
| Angle position of left rear leg           | q <sub>9</sub>  |
| Angle position of right rear leg          | q <sub>10</sub> |
| Deformation of the left rear leg spring   | q <sub>11</sub> |
| Rear right leg spring deformation         | q <sub>12</sub> |

For the quadruped, the gait is limited. The legs on the right side are in antiphase with those on the left side. Another point is that the trot can occur with or without aerial phase.

## III. RESULTS AND DISCUSSION

### 3.1 Monopod results:

For the jump in the same position, only one Fourier series term was used for the initial estimate of the control. She was found by trial and error.

The search process for the periodic condition obtained the solution with the algorithm returning to the parameters of Table 3. The algorithm ended when the stop criterion related to the size of the step standard was obeyed.

Table 3: Periodic search results for Multiple Shooting

| Iteration<br>s | Evaluate<br>d<br>functions | Residua<br>l | Optimalit<br>y | Lambd<br>a | Step<br>rule |
|----------------|----------------------------|--------------|----------------|------------|--------------|
| 134            | 200                        | 3.097e-26    | 6.33e-11       | 1e-09      | 3.296e-11    |

The periodic solutions returned by the algorithm are in Figures 3 and 4. As in the initial estimation, the other degrees of freedom and their first derivatives remained in the initial conditions throughout the period. The markers represent the value of the states in the final grid of the algorithm, the curve is the result of the integration of the system by the "ode34" function and straight marking the initial condition of the system.

The analysis of the stability of the periodic orbit is in Figure 5. The multipliers that are inside it are stable, those that are on it indicate neutral stability and the multipliers outside are unstable.

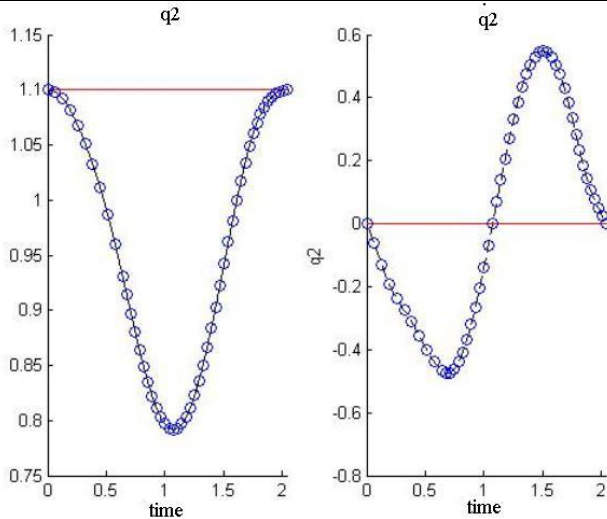
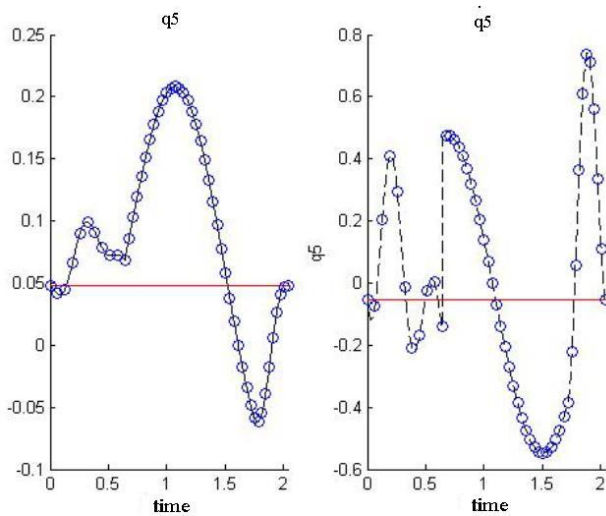
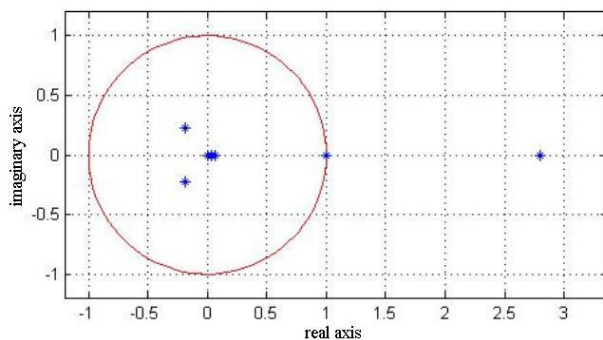

Fig. 3: Periodic solution for  $q_2$ 

Fig. 4: Periodic solution for  $q_5$ 


Fig. 5: Flock float multipliers with no feed

The other condition in which the search for the periodic march was used was the monopod jumping with a horizontal speed of one unit. For the optimization process, the initial conditions of the variables  $q_1$ ,  $q_2$  and its first derivative were kept fixed and the others were released for modification. The periodic constraint encompasses all variables, except for the horizontal position, which grows monotonically at each period.

The optimization was performed for the 10 terms of the Fourier series. The final values obtained are in Table 4.

Table 4: Optimization Results

|                              | Not optimized | Optimized |
|------------------------------|---------------|-----------|
| Transportation costs         | 1.6812        | 1.0964    |
| Travelled distance           | 0.6618        | 0.5291    |
| Positive work                | 1.1126        | 0.5801    |
| Frequency of excitation (Hz) | 0.4695        | 0.3892    |

Floquet analysis can be seen in Figure 6. All eigenvalues are stable except for one in 4.6, indicating that although the solution obtained is periodic, it is unstable for more than one cycle.

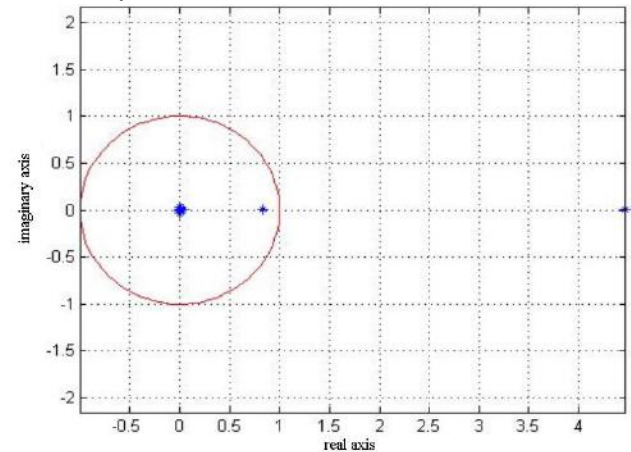


Fig. 6: Stability analysis for forward jump

### 3.2 Quadruped results:

The initial estimate was determined in order to ensure that the feet in the support phase did not penetrate the ground and that the normal force of the support phases were greater than 0. To this end, the trial and error first term of the Fourier series of the excitation function, until an acceptable initial condition was obtained.

The forces of normal reaction of the legs in the support phase and the vertical position of the legs of the legs in the balance phase for half the gait period are represented in Figures 7 and 8, respectively.

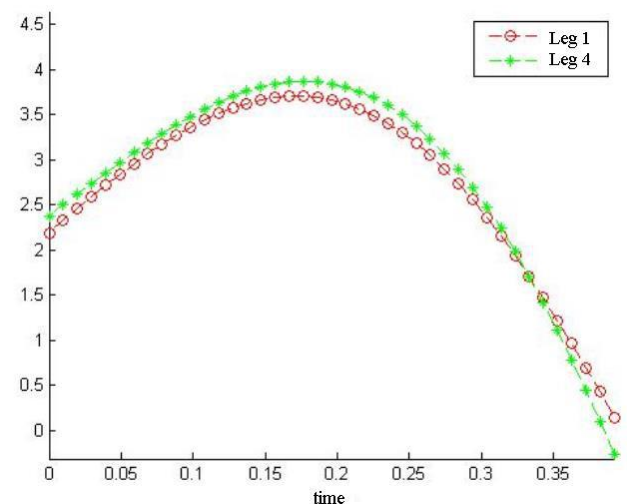


Fig. 7: Soil reaction forces on legs 1 and 4

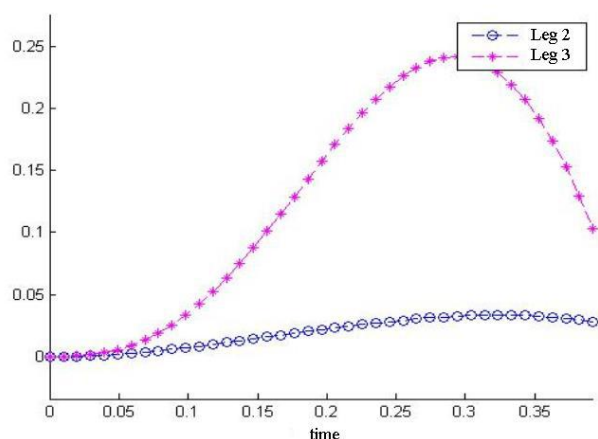


Fig. 8: Vertical position of the feet of the legs in the swing phase

It is important to emphasize that the initial estimate was constructed keeping in mind the condition of symmetry of the gait of the trot. Figures 9 and 10 show that the system cannot obtain periodicity over the initial estimated conditions, mainly because the legs 2 and 4 remain in the same state throughout the period.

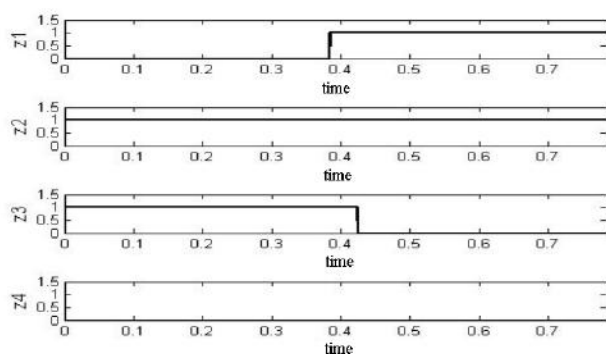


Fig. 9: Evolution of leg condition

For the proposed restriction conditions for the trotting gait in the quadruped system, it was not possible to obtain a positive result for the initial estimated conditions. The search process ends in the third iteration with residual of the violation of the restrictions of 2.862 (extremely superior value to the desired of  $10^{-8}$ ).

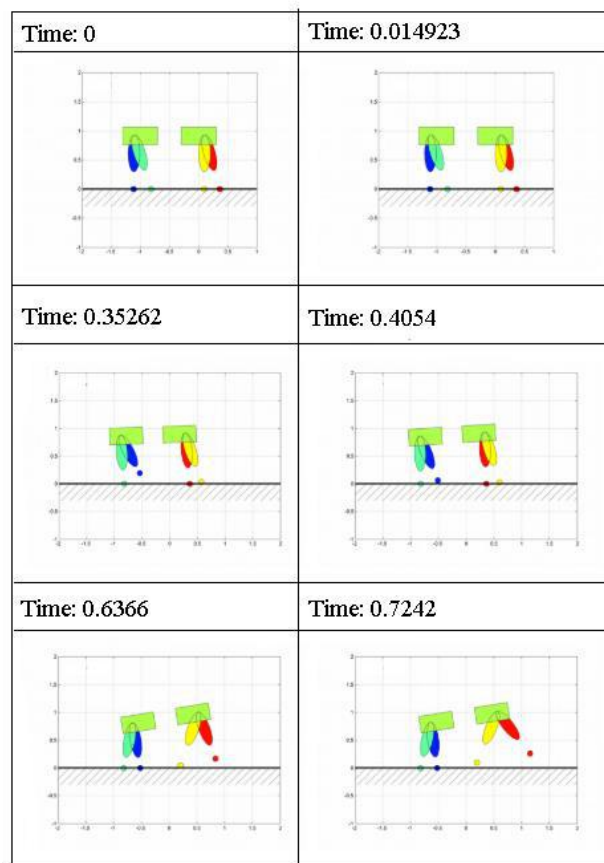


Fig. 10: Quadruple Animation

#### IV. CONCLUSION

This work presented a methodology for searching and optimizing the periodic gait of leg locomotion systems, more specifically a monopod and a quadruped, by the multiple shooting method.

The developed program was tested in the monopod for the search of the periodical gait of the jump in the same position and of jump with advance.

Although the leg restrictions for the quadruple system were in line with established conditions, it was not possible to find a periodic gait and perform an optimized gait search. Studies should be done to achieve a suitable initial condition for the search of periodic gait.

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